Abstract. We have described previously a method of automatically constructing statistical models of shape. The method treats model-building as an optimisation problem by re-parameterising each shape so as to minimise the description length of the training set. The approach requires an explicit parameterisation of each shape, which is straightforward in 2D, but non-trivial in 3D. It is necessary to provide some parameterisation of the training set to initialise the optimisation. An inappropriate initial parameterisation can cause the optimisation to converge at a slower rate or stop it from converging to a satisfactory solution. In this paper we describe a method of producing a consistent parameterisation for a given set of surfaces. The consistent parameterisations were used to initialise the model-building algorithm and produced results that were significantly better than alternative approaches.

1 Introduction

Statistical shape models (SSMs) have proven to be a powerful basis for image segmentation and shape analysis. Model-building involves establishing, from a training set, the pattern of ‘legal’ shape variation for a given class of object. Statistical analysis is used to provide an efficient parameterisation of this variability. The resulting models have good specificity (they can only represent valid instances of the class of object) and generalisability (they can represent any instance of the class of modelled object).

A key issue in constructing a shape model is establishing a dense correspondence between shape boundaries/surfaces over a reasonably large set of training images. It is important to establish the ‘correct’ correspondences, otherwise an inefficient parameterisation of shape can result, leading to non-specific models and non-general models. We have shown previously [1–3] how the correspondence problem can be solved by treating it as an integral part of the shape modelling process. The approach involves optimising the description length of the training set by explicitly manipulating the parameterisation of each training shape, leading to a training set with minimum description length (MDL).

The MDL method works robustly in 2D [1], but we have found that it can be sensitive to initialisation in 3D. The problem arises since the model-building algorithm requires an explicit parameterisation of each shape, which is straightforward in 2D, but non-trivial in 3D. If the initial parameterisations are inconsistent across the training set, then the initial correspondence is poor, which can slow down convergence of the MDL method or stop it from converging at all.

There are many methods of surface parameterisation; a review can be found in [4]. Each method produces a parameterisation with different properties such as a harmonic mapping or a conformal mapping, but none of the methods produce a parameterisation with the properties that we desire, namely one that is consistent between different instances of the same class of object.

In this paper, we treat surface parameterisation as an explicit optimisation problem. This approach to parameterisation allows us to construct consistent parameterisations that are guaranteed to have pre-determined desirable properties. The approach is simple yet effective and, as reported in section 5, leads to improved shape models.

2 Statistical Shape Models

A statistical shape model is built from a training set of example shapes \( \{S_i : i = 1, \ldots, n_s\} \), aligned to a common frame of reference. Each shape, \( S_i \), can – without loss of generality – be represented by a set of \( n \) corresponding points regularly sampled on the shape. The concatenated coordinates of these points form the representation of the shape, an \( n_p \) dimensional shape vector \( x_i \). Using Principal Component analysis, each shape vector can then be expressed using a linear model of the form:

\[
x_i = \bar{x} + Pb_i = \bar{x} + \sum_{m} p^m b_i^m,
\]  

\( x_i \) is the shape vector of the \( i \)-th shape, \( \bar{x} \) is the mean shape, \( P \) is the PCA matrix, \( b_i \) are the shape coefficients for the \( i \)-th shape.

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where $\bar{x}$ is the mean shape vector, $P = \{p^m\}$ are the eigenvectors of the covariance matrix that describe a set of orthogonal modes of shape variation and $b = \{b^m\}$ are shape parameters.

The ability of the model to represent the shape variation depends crucially on the set of corresponding points chosen. The approach we will follow is to parameterise each shape. Corresponding points are then points on different shapes with the same parameter values.

3 Automatic Model-building by Direct Optimisation of Description Length

We have shown previously that the correspondence problem can be solved by treating it as an integral part of the shape learning process both in 2D [1, 2] and in 3D [3]. The basic idea is to choose the correspondences that build the ‘best’ model, treating model building as an optimisation task. This requires a framework involving:

- a method of manipulating correspondences,
- an objective function to assess the ‘quality’ of the model built from a given set of correspondences,
- a method of optimising the objective function with respect to the set of correspondences.

As explained in the previous section, we parameterise each shape. For a 3D surface with spherical topology, this involves mapping the surface of the shape to the surface of a sphere, with no folds or tears. The parameters are then just the usual spherical polar coordinates, and corresponding points across the set of shapes are points with the same values of the parameters. So, we now see how we can change the correspondence of a single shape $S_i$; we just need to re-parameterise that shape. That is, we need to apply a smooth, continuous transformation to the entire surface of the parameter sphere for just that shape. In what follows, we generate such transformations using a composition of clamped plate spline (CPS) transformations [5]. These are transformations which apply a homeomorphic mapping (no folds or tears) which deforms a cap-shaped region of the sphere. The parameters of a single transformation are the position of this region ($P$), the size of this region ($\omega$), and the amount of deformation ($a$).

The parameterisation, and hence the correspondence, of each shape can then be individually adjusted. The objective function for this re-parameterisation is one based on the information-theoretic concept of Minimum Description Length (MDL). It has been shown previously that this produces models with good generalisation ability, specificity and compactness. Full details can be found in [1].

Optimisation proceeds in an iterative manner. At each iteration, we add $N_K$ CPS warps to the parameterisation of each shape. The widths $\{\omega\}$ and positions $\{P\}$ are chosen stochastically and fixed during optimisation; the amounts of deformation $\{a\}$ are used as the parameters of the optimisation. The position of the clamped plate spline is chosen from a uniform distribution over the sphere and the width is chosen from the positive half of a Gaussian distribution with zero mean and standard deviation $\sigma_G$. The convergence of the algorithm is relatively insensitive to the value of $\sigma_G$. A value of $\sigma_G = \frac{1}{2}$ was used in the experiments reported below. Optimisation is achieved by estimating the gradient of the objective function with respect to the set $\{a\}$ [6] and performing a line search along this gradient to find the optimal objective function value.

As in many optimisation tasks, the initialisation is important; in our case, this is the initial parameterisation of the set of shapes. It is important to choose a consistent set of initial parameterisations, otherwise convergence of the optimisation is slowed down. The method of selecting a suitable initial parameterisation is the main contribution of this paper.

4 Consistent Surface Parameterisation

We start from a set of training shapes $\{S_i\}$ with spherical topology. Each shape is a triangulated surface in 3D, with vertices $\{v_j\}$ and triangles $\{t_k\}$, where $v(t_k)$ are the vertices of triangle $k$. A parameterisation of a surface then consists of assigning a parameter value $u_j \in S^2$ to each vertex $v_j$. Barycentric coordinates are used to interpolate this parameterisation in between vertices.

An initial discrete parameterisation of the training surfaces can be achieved using many methods [4]. These parameterisation methods do not, however, consider the training set as a whole, hence the parameterisations are likely to be inconsistent across the group. Figure 1 shows a comparison between a SPHARM parameterisation, produced by the
method of Brechbühler et al. [7], and a consistent parameterisation, produced by the method of Section 4.1.

Consistency of the parameterisation across the group is not the only consideration – parameterisation can also lead to (possibly severe) area distortion. This becomes a problem when model-building, since area distortion in the parameterisation leads to under- or over-sampling of the surface, giving a poor representation of the shape. This is because sampling is performed evenly on the parameter space (the sphere), rather than directly on the shapes. The benefit of correcting area distortion is shown in figure 2, which compares a surface sampled using two different parameterisations: a parameterisation with area distortion and a parameterisation that has been corrected using the method described below (Section 4.2).

4.1 Groupwise-consistent Parameterisation

We wish to manipulate the set of parameterisations so as to produce a good correspondence across the training set. A quick solution is to take each training surface in turn, identify a correspondence between it and the surface of a ‘reference shape’, and then manipulate the parameterisation to match this correspondence.

For the results reported below, the first training example was chosen as the reference shape, and its triangulated surface was decimated to give a new triangulation with 20% of the original vertices remaining.

Let \( \{ v_i^A \} \) be the vertices of the decimated reference shapes and let \( \{ v_j^B \} \) be the vertices of another of the training shapes. We assign a correspondence between each point, \( v_j^B \), and the closest point in \( \{ v_j^B \} \). The result is an index, \( I \), that maps the two sets such that \( v_j^B_{I(i)} \) is the closest point to \( v_i^A \).

We now wish to transform the parameterisation of shape \( B \) so that the parameter values \( \{ u^B_{I(i)} \} \) best match the values of \( \{ u_i^A \} \). This is achieved by manipulating \( \{ u^B_{I(i)} \} \) so as to minimise a sum of squares objective function:

\[
F = \sum_i |u_i^A - u^B_{I(i)}|^2. \tag{2}
\]

We manipulate \( \{ u^B_{I(i)} \} \) using a clamped plate spline-based representation of transformation, using the same algorithm as described in Section 2 above.
4.2 Minimising Distortion

Since the parameterisations are in correspondence, area distortion can be corrected by transforming the parameterisation on a single shape and then applying the same transformation to all members of the training set.

Area distortion is corrected by manipulating the parameterisation so as to minimise the following objective function:

\[
F = \sum_{k} \left( \frac{\text{area}(v(t_j))}{\sum_{j} \text{area}(v(t_j))} - \frac{\text{area}(u(t_j))}{\sum_{j} \text{area}(u(t_j))} \right)^2,
\]

where \( \text{area}(v(t_j)) \) is the area of the \( j^{th} \) triangle in \( \mathbb{R}^3 \) and \( \text{area}(u(t_j)) \) is the area of the same triangle in \( S^2 \); \( N_T \) is the number of triangles. Manipulation of parameterisation and optimisation are as described in Section 2.

Note that the SPHARM method also produces parameterisations with no area distortion. In the results reported below, SPHARM was used as an initial estimate of parameterisation, hence there was very little distortion to remove. The distortion-minimising step is, however, likely to be critical if other methods are used as an initial estimate of parameterisation.

5 Results

MDL shape models of brain ventricles were constructed from 15 examples, using two different parameterisations: one obtained using the SPHARM method [7] and one using the Groupwise-consistent method of parameterisation proposed in this paper. To distinguish between the two, we will refer to them as ‘SPHARM’ and ‘Groupwise-consistent’ models respectively.

Qualitative results were produced by varying the first mode of variation by \( \pm 2 \) [standard deviations found over the training shapes]. A quantitative evaluation of the models was performed using two objective measures of model quality: generalisation ability and specificity. The derivation of each measure is given in [8]. Generalisation ability was measured by leave-one-out reconstruction, and specificity was assessed by generating a population of instances using the model and comparing them to the members of the training set. In both cases, lower values are preferable. The standard error of each measure can also be calculated, giving the significance of differences between models produced by different approaches.

Qualitative results in figure 3 show that the Groupwise-consistent model displays modes of shape variation that would be expected from the brain ventricle. The value of the MDL objective function is 80.3 for the SPHARM model and 68.4 for the Consistent Parameterisation model, suggesting that the Groupwise-consistent model performed better. The quantitative results in figure 4 confirm this by showing that the Groupwise-consistent model has significantly better specificity and generalisation ability than the SPHARM model.

6 Conclusions

We have described a simple yet effective method of achieving a consistent parameterisation of a set closed surfaces. The new method of parameterisation produces significantly better MDL shape models than alternative approaches to parameterisation.

We have also run the method described here on a set of proximal femurs and the resulting model was a vast improvement on the initial parameterisation. Future work will see us test the robustness of the method on a wider range of objects.

The MDL model-building algorithm would ideally be able to converge quickly and reliably from any given parameterisation of the training surfaces. The algorithm can, however, get trapped in local minima, which makes a consistent initial parameterisation essential. The focus of our current work is on investigating ways of making the model-building algorithm more robust to local minima.
Figure 3. The first mode of variation of models built from the groupwise consistent parameterisation. Variation is plus and minus two standard deviations about the mean. The main variation is a change in curvature of the body and ‘rotation’ of the anterior horn.

Figure 4. A quantitative comparison of the SPHARM and Groupwise-consistent models.

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