Prediction of parking space availability in real time

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Abstract

Intelligent parking reservation (IPR) systems allow customers to select a parking facility according to their preferences, rapidly park their vehicle without searching for a free stall, and pay their reservation in advance avoiding queues. Some IPR systems interact with in-vehicle navigation systems and provide users with information in real time such as capacity, parking fee, and current parking utilization. However, few of these systems provide information on the forecast utilization at specific hours – a process that requires the study of the competition between parking alternatives for the market share. This paper proposes a methodology for predicting real-time parking space availability in IPR architectures. This methodology consists of three subroutines to allocate simulated parking requests, estimate future departures, and forecast parking availability. Parking requests are allocated iteratively using an aggregated approach as a function of simulated drivers’ preferences, and parking availability. This approach is based on a calibrated discrete choice model for selecting parking alternatives. A numerical comparison between a one-by-one simulation-based forecast and the proposed aggregated approach indicates that no significant discrepancies exists, validating and suggesting the use of the less time consuming proposed aggregated methodology. Results obtained from contrasting predictions with real data yielded small average error availabilities. The forecast improves as the system registers arrivals and departures. Thus, the forecast is adequate for potential distribution in real-time using different media such as Internet, navigation systems, cell phones or GIS.

1. Introduction and literature review

Parking guidance information (PGI) and management systems based on Internet, cellular phones, PDA, and GIS technologies have been proposed by Oh, Lee, Kim, and Yang (2002) and Liu, Lu, Zou, and Li (2006) to provide effective parking information distribution and dissemination, integration with traffic information systems, advanced data processing technologies (e.g., data mining), and spatial data analysis capabilities. These systems assist drivers with their trip planning in searching and waiting for vacant parking stalls while mitigating driver frustration, and decreasing queues at parking entrances, the amount of miles traveled per vehicle, and average trip time, traffic congestion, energy consumption, and air pollution (Teodorovic & Lucic, 2006; Vianna, da Silva, & Ballassiano, 2004; Yang, Liu, & Wang, 2003).

1.1. Parking availability information

Some authors have studied real-time parking availability computation from different perspectives. For example, Martens and Benenson (2008) presented an agent-based model of driver parking behavior using a GIS simulation. In the model, drivers start searching for parking within 250 m from their destination considering real-time and expected parking availability, prices, and parking enforcement efforts. Further research is required to include sufficient driver surveys on parking behavior, traffic limitations, and spatial heterogeneity. Additionally, Teodorovic and Lucic (2006) presented an intelligent parking space inventory control system to decide whether to accept or reject new parking requests in real-time; the proposed system is based on a combination of simulation, optimization, and fuzzy logic. Decisions depended on the current state of the parking system such as parking space availability. The authors indicated that future applications of the proposed methodology to parking lots and garages in cities and airports are required including further evaluation of queues, distance traveled, trip time, etc.
1.2. Reservations

The term “availability” is closely linked to the term “reservations”. In parking topics reservations often refer to closing specific sections as needed in parking facilities, monthly payments or spaces required by regular clients. However, Wilbur-Smith Associates (2009) indicate that on-line reservations can improve customer experience and satisfaction; the authors report the city of Pittsburg as an example of best practice in advanced parking systems based on technology with a call-ahead reservation system. They also cite the case of Bay Area Rapid Transit (BART), in San Francisco/Oakland Metropolitan Area, California, with a centralized intelligent reservation and real time availability system which provided parking availability via telephone, Internet. This project offers a web interface to make single-days reservations and the capability to make en-route reservations and pre-trip reservations.

PGI and intelligent parking reservation (IPR) systems are part of intelligent transportation system (ITS) deployment in various cities of China, Japan, Europe, and United States (An, Han, & Wang, 2004; Teng, Qi, & Martinelli, 2008). These systems interact with in-vehicle navigation systems and Internet users to provide updated real-time parking information, such as location, capacity, parking fee, current utilization or availability, and performance of specific on-street parking or off-street parking facilities (Inaba, Naganawa, Ogiwara, & Yoshikai, 2001; Oh et al., 2002; Yang et al., 2003). Customers employ this information for improving the decision-making process by (i) selecting a parking facility according to their personal preferences, (ii) parking their vehicles in less time, and (iii) paying their parking reservations in advance avoiding queues. As a result, adverse impact on traffic congestion and pollution are diminished.

1.3. Potential benefits of forecasting parking availability

Multiple attributes influence the choice of a driver for a given parking alternative. Typical factors that affect the driver’s parking activity and decision-making are walking distance or distance to destination, driving and waiting time, parking fees, service level of parking lots, safety, etc. (An et al., 2004; Lam, Li, Huang, & Wong, 2006). Particularly, the number of available parking spaces is an important attribute in the driver’s parking decision-making process (Caicedo, 2009; Hendrickers and Outwater, 1998).

Drivers in private cars count on navigation systems or GPS applications mainly focused on finding the shortest path route toward the destination; unlike commuters that do count on transit trip information (Su & Chang, 2010) to improve the decision making process regarding mode, route or departure time. Once the drives arrives to the destination zone navigation systems are not of much use when it comes to parking, choosing among parking lot alternatives, or the process of searching for on-street free spaces.

Drivers make decisions based on the last experience in terms of availability of parking spaces in advance to the arrival at the destination zone; and even if they had information on current availability, it will not necessarily be the same at the time when they to the chosen parking facility. It is not had to imagine that having all drivers considering the same information on current availability will lead them towards the same most convenient alternative, which in turn will cause unaffordable wait times, queues and increase searching time.

However, if predicted parking availability information is correctly estimated and well managed, then enhanced informed decisions can be performed by selecting the most convenient parking facility and stall. As studied in Caicedo, Robuste, and Lopez-Pita (2006), drivers that possess information on parking availability are 45% more successful in their decisions than those without knowledge of this information when arriving at their parking facility.

Additionally, parking facility managers and operators may foresee the parking system performance and carry out short- and long-term preventive, strategic decisions to avoid system collapses. Reports from historical and forecasted events are useful in many processes of transportation management and planning. For example, public agencies such as city traffic and planning department employ prior and predicted parking availability information for managing transportation demand, and traffic congestion.

Though we were capable of identifying potential benefits of forecasting parking availability, we found few cases in which this topic is faced. For example, Burns and Faurot (1992) developed an econometric forecasting model to estimate the monthly revenues of two parking facilities in Kansas City, Missouri. This model considers changing economic conditions as an explanatory variable and also common available data (e.g., local economic activity, seasonal factors, and facility-specific events) in large cities. Arnott (2006) examined parking policy from the economic theory perspective, and analyzed the parking market equilibrium using spatial competition between parking facilities. A study developed by Erhardt, Kurth, Sabina1, and Myung (2005) described a market-based framework for modeling parking supply and cost. This model equilibrates parking demand and supply taking into account actual parking costs paid by drivers, and estimates changes in parking costs over time using longitudinal data. The two last references presented here predict parking demand with dynamic methodologies. Dunning (2006) presented a method for forecasting parking availability. Though the author uses a database that stores information to projecting space availability at each parking facility (e.g., parking lot entries and exits, historical information of vehicle traffic to the parking lot over time, and duration of stays at the parking lot), the system operates with data obtained from ongoing survey for remote lots with high demand and the modeling/solving background is poor since it does not consider characterization of driver’s behavior in choice making. Finally, Yang et al. (2003) predicted available parking spaces using neural network of time series, and recent and historical information. The network input employs multi-variable data depending on road traffic flow, weather, events, road conditions, etc. The authors stated that large sample sizes require large amount of time to execute affecting real-time applications. Additionally, they declared that determining the adequate number of neural units is a difficult task.

1.4. Objective and organization of the paper

In view of the references consulted and the potential benefits of forecasting parking availability, the objective of this paper is to develop a methodology aimed to provide complete and effective real-time availability forecasting for IPR architectures in parking facility information systems with reasonable computing time. Instead of employing a single parking request allocation, this methodology allocates several requests in a probabilistic form yielding a simpler and less time consuming algorithm.

The paper is organized as follows. The following section studies the characterization of competition process of parking facilities within an area of concern as a foundation of the proposed methodology. Thereafter, the real-time availability forecast (RAF) algorithm is presented to predict the number of expected market share in each parking facility at a certain moment in time. Subsequently, the three main subroutines of the RAF algorithm are described. These subroutines are comprised of parking request allocations, future departures estimations, and parking availability predictions. The RAF algorithm is validated using a simulation exercise and real case data. Then, an algorithmic performance analysis is described. Finally, conclusions and future research are discussed.
2. Characterization of driver parking requests

The parking access and revenue control system (PARC) displays parking availability information through variable message signs (VMS) (Caicedo et al., 2006; Chrest, Smith, Bhuian, Monahan, & Iqbal, 2001; Viana et al., 2004). This system counts the number of vehicles that enter and exit a parking facility determining both general and local availability. For auditing purposes, modern PARC systems record a chronological data set of clients per day. Normally, when one arrives at a parking facility (see Fig. 1). This equation utilizes a multi-nominal logit model, belonging to the well-known family of discrete choice models, in which alternatives are associated with perceived utility $V_{ij}$ (Caicedo, 2009; Lambe, 1996; Marianov, Rios, & Icaza, 2008; Thompson, Takada, & Kobayakawa, 2001). Notice that the utilization of other types of discrete choice models to forecast availability and their respective evaluation are beyond the scope of this paper:

$$P_{ij}^{fb} = \frac{\exp(V_{ij}^{fb})}{\sum_{j=1}^{J} \exp(V_{ij}^{fb})}, \quad \forall j \in J, \quad t = 1, \ldots, T, \quad \forall i \in I,$$

$$\forall f \in F, \quad \forall b \in B.$$  (1)

From the perspective of each driver’s decision, the utility function of parking alternatives $V_{ij}^{fb}$ is described according to a discrete choice theory as a function of the distance $d_{ij}$ between parking alternative $j$ and destination $b$, driving time $t_{ij}$ to reach parking alternative $j$, fare $t_{ij}$ for each duration of stay $\phi_i$, availability or dynamic capacity $z_{ij}$ in parking alternative $j$ at time $t$, waiting time $C_j$ caused by high levels of parking occupancy, and budget $Y_i$ as shown in Eq. (2):

$$V_{ij}^{fb} = \theta_0 + \theta_1 \cdot d_{ij} + \theta_2 \cdot t_{ij} + \theta_3 \cdot \phi_i + \theta_4 \cdot z_{ij} + \theta_5 \cdot C_j + \gamma_j \cdot \Theta_i \cdot Y_i,$$

$$\forall f \in F, \quad \forall b \in B,$$  (2)

where $\theta_0, \theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \gamma_j$; previously calibrated parameters:

$$\gamma_j = \begin{cases} 0 & : y \neq i, \\ 1 & : y = i. \end{cases}$$

The dynamic capacity $z_{ij}$ before assigning parking requests in period $t$ to parking facility $j$ is equal to the sum of the dynamic capacity $z_{ij-1}$ after parking requests in period $t - 1$ are assigned to parking facility $j$, and the departures from facility $j$ at time $t$ ($s_{jt}$) (See Eq. (4)). The latter represents the number of parking requests that have fulfilled their duration of stay within parking alternative $j$. Eq. (5) presents the capacity constraint, where $w_i$ is the static capacity in parking alternative $j$:

$$z_{jt} = z_{jt-1} + s_{jt}, \quad 0 \leq s_{jt} \leq w_i.$$  (4)

According to Caicedo (2005), parking facilities with high levels of demand patrons experience a waiting time, which do not depend on the number of servers located at the entrance (e.g., access lanes, servers or ticket booths). This additional waiting time is determined by the static capacity of the parking facility and departures behaving as one large server. A threshold was estimated to include the effect of waiting time in the decision-making process due to high levels of occupancy. When the facility reaches high levels of occupancy, a waiting time $C_j$ may easily be greater than 5 min. In practice, patrons of private parking facilities experience this additional waiting time once parking availability drops below 5%. The parameter $\gamma_j$, that multiplies the waiting time attribute $C_j$ in Eq. (2), is expressed by Eq. (6), where $a$ is the set of off-street parking facilities ($a \subset J$):

$$s_j = \begin{cases} 1 & : z_{jt} < 0.05w_i \quad \forall j \in a, \\ 0 & : z_{jt} \geq 0.05w_i \quad \forall j \notin a, \quad j \in J. \end{cases}$$  (6)

Summarizing, the total demand is distributed among feasible parking alternatives for each period and parking request type based on the probability computed with Eq. (1), which is determined by the utility function shown in Eq. (2). The remainder of the paper describes the proposed methodology to forecast the demand for future periods, based on the previously presented parking competition model employed to distribute requests.

3. Real-time availability forecast algorithm

The real-time availability forecast (RAF) algorithm predicts parking facility availability in real time using combined current (on-line) and historical information. This algorithm requires a discrete choice model (DCM) for incorporating the effect of receiving availability information to characterize and predict user selections, or allocate parking requests (Caicedo, 2009). This DCM must be previously calibrated with information from a study of parking preferences, which must include duration of stay, arrival and departure processes, and static capacity of each parking facility that operate in a specific zone of a city.

The expected outcome of the RAF algorithm is an availability forecast for all existing parking alternatives, which may be disseminated to users through the Internet, vehicle navigation systems, cell phones or VMS, as shown in Fig. 1. RAF algorithm operates with historical usage records of a typical operation day. This algorithm is fed over time with updated records of arrivals and departures at each parking alternative, in order to update parking availability forecast. This information is stored in a database and is obtained from a facility management system (FMS) at each parking alternative (see Fig. 1).

The RAF algorithm generates a random sample of $Q_i$ arrivals or parking requests for a time period $t$ (e.g., next 15 min) based on an aggregated historical average rate $\lambda_0$. These arrivals are randomly
disaggregated in terms of budget and destination according to historical aggregated proportions (i.e., expected number of users for each budget type \(i\) and destination \(b\)). Subsequently, the duration of stay for each request is simulated and discretized using a Gamma Distribution with historical pre-calibrated distribution parameters for time period \(t\).

After completing the request simulation for each time period \(t\), the algorithm executes the following three main subroutines, explained in the following section:

- **Parking request allocation (PRA):** This subroutine is aimed to allocate simulated requests \(Q_t\) in an aggregated manner among all existing parking alternatives.
- **Future departures estimation:** Departures in future periods, associated to recently allocated requests \(Q_t\), are estimated using simulated and discretized stays for all allocated requests on a one-by-one basis.
- **Availability forecast:** One-step availability prediction at each parking alternative is estimated using previously computed allocated requests for period \(t\), and expected departures during this period. These departures are comprised of previously computed future departures related to arrivals during periods prior to \(t\) \((Q_{t-1}, Q_{t-2}, \ldots)\).

According to the schema depicted in Fig. 2, the RAF algorithm operates with mixed information (real and simulated) until time period \(t = \pi\), and employs only simulated information such as request, user choices, and durations of stay after time \(\pi\). Existing parking space availability information is employed within a defined time interval \(\Psi\) (e.g., 15 min). At time \(t = \pi + \Psi\) (e.g., \(\pi + 15\)), a new record of entries and exits is required to update real and historical average parking arrivals, dynamic capacity, and so on.

### 4. Parking request allocation and availability forecast

This section explains the three main subroutines of the RAF algorithm: PRA algorithm, future departures estimations, and one-step parking availability forecasts.

#### 4.1. Parking request allocation (PRA) algorithm

The PRA algorithm is employed to determine the allocation of parking requests \(Q_t\) in time period \(t\) (previously simulated within RAF algorithm) amid all parking alternatives \((j \in J)\). This allocation algorithm relies on user preferences and parking alternative attributes for each request (e.g., fee and parking availability) to estimate aggregated probabilities \(P_{jt}\) for each alternative \(j\) and time period \(t\), in order to allocate all simulated requests \(Q_t\). These probabilities are modeled in aggregated terms, where demand probabilities for each alternative \(j\) and period \(t\) are calculated by adding individual probabilities defined for each request based on its simulated budget, destination, and duration stay type. Eq. (7) presents the aggregation approach, where \(P_{jt}(r)\) is the probability of a single simulated request \(r\) to be allocated to parking alternative \(j\) at time period \(t\). This probability is computed for each request/user \(r\) using the previously described DCM and is simulated by the RAF algorithm prior to the execution of the PRA algorithm, taking into account its attributes of budgets, destination, and stay duration:

\[
P_{jt} = \frac{\sum_{r=1}^{Q_t} P_{jt}(r)}{Q_t}.
\]  

The above procedure would be sufficient for allocating all parking requests in absence of capacity constraints. However, when capacity is considered, some requests may not be allocated to a specific parking alternative, and probabilities \(P_{jt}\) should be corrected to reflect this fact. Accordingly, the PRA algorithm executes \(X\) iterations necessary to allocate all parking requests among \(J\) parking alternatives, where \(x\) represents an intermediate iteration \((x \leq X)\). In other words, the algorithm will increase the required number of iterations until all parking requests are allocated, as shown in the Appendix and flowchart in Fig. 3. Note that the algorithm at most requires as many \(X\) iterations as \(J\) parking alternatives.

The PRA algorithm computes elements \(a_{xt}\) of the allocation matrix \(A_x\), shown in Eq. (8), that represent partial parking request allocation probabilities for each iteration \(x (x = 1, \ldots, X)\) and parking alternative \(j\) for each time period \(t\). Eqs. (9) and (10) indicate
At Eq. (9) indicates that each partial allocation probability must have values between 0 and 1, while constraints (10) assure that all parking requests must be allocated:

\[
\begin{align*}
0 & \leq a_{jt} \leq 1 \quad \forall j \in J; \quad x = 1, \ldots, X; \quad t = 1, \ldots, T, \\
\sum_{x=1}^{X} \sum_{j=1}^{J} a_{jt} & = 1 \quad \forall t = 1, \ldots, T.
\end{align*}
\] (8) (10)

After computing matrix \( A_t \), the total number of expected requests \( d_{jt} \) allocated to the parking alternative \( j \) at time period \( t \) is computed using Eq. (11). This expression depends on the expected parking request \( Q_t \) at time period \( t \), and the summation of partial parking request allocation probabilities for \( X \) iterations and parking alternative \( j \).

\[
d_{jt} = Q_t \sum_{x=1}^{X} a_{jt} \quad \forall j \in J; \quad t = 1, \ldots, T,
\] (11)

\[
d_{jt} \leq z_{jt} \quad \forall j \in J; \quad t = 1, \ldots, T.
\] (12)

If no availability exists to allocate all requests among the \( J \) parking alternatives, then an additional parking alternative is considered in the PRA algorithm. Therefore, a total of \( J + 1 \) parking alternatives are defined, in order to accommodate all parking requests.

### 4.2. Future departures estimation and parking availability forecast

Once the PRA algorithm is executed, the number of requests allocated to parking alternative \( j \) at time period \( t \) with a stay type \( f \) is computed, yielding future departures from alternative \( j \) associated to requests arriving in period \( t \). This estimation is arranged in a future departure matrix \( M_t \), as shown in Eq. (13), where \( m_{jt} \) is the number of requests with a stay type \( f \) allocated to alternative \( j \) at period \( t \):

\[
M_t = \begin{bmatrix}
1 & \cdots & j & \cdots & J \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
m_{11t} & \cdots & m_{1jt} & \cdots & m_{1jt} \\
\vdots & \cdots & \cdots & \cdots & \cdots \\
m_{X1t} & \cdots & m_{Xjt} & \cdots & m_{Xjt}
\end{bmatrix}
\] (13)
The computation of matrix $M_t$ relies on previously computed allocated parking requests $d_{jt}$ and discretized stay durations simulated by the RAF algorithm. Consequently, conditions (14) and (15) hold. Eq. (14) states that the summation of all computed future departures must be equal to requests $Q$, Eq. (15) indicates that requests arriving to parking alternative $j$ at a time $t(d_{jt})$ is the summation of all future departures depending on stay types $f$ for the same parking alternative $j$:

$$Q_t = \sum_{f=1}^{F} \sum_{j=t}^{T} m_{jf} \quad \forall \, t = 1, \ldots, T,$$

$$d_{jt} = \sum_{f=1}^{F} m_{jf} \quad \forall \, t = 1, \ldots, T, \quad \forall \, j = 1, \ldots, J.$$  

Matrix $M_t$ will be employed to compute departures on each period after time $t$, or future departures. In contrast, departures $s_{jt}$ at time period $t$ from parking facility $j$ depends on parking arrivals to alternative $j$ for periods prior to time $t$, the duration of their stays, and the computation of future departures for all previous periods $u < t$. Let $\phi(m_{ju})$ be the stay duration of parking requests with a stay type $f$ assigned to alternative $j$ at period $t$, and let $c_{jf}(t)$ be the number of requests allocated to alternative $j$ with a stay type $f$ arriving at time $t$ and exiting at period $t$. Therefore, departures $s_{jt}$ are calculated using Eqs. (16) and (17):

$$c_{jf}(t) = \begin{cases} m_{ju} : u + \phi(m_{ju}) = t & \forall \, t = 1, \ldots, T, \quad \forall \, u < t, \\ 0 & \text{Otherwise} \end{cases} \quad f = 1, \ldots, F, \quad j = 1, \ldots, J.$$  

$$s_{jt} = \sum_{u=1}^{t-1} \sum_{j=1}^{J} c_{ju}(t) \quad \forall \, t = 1, \ldots, T, \quad \forall \, j = 1, \ldots, J.\)  

On-line parking space availability information can be employed to prepare periodical updated parking utilization predictions. The PARC system of each parking alternative (connected to an IPR system) prepares information regarding the availability at time $t-1$ (i.e., $z_{jt-1}$). Finally, knowing this availability, the total number of expected requests $d_{jt}$ allocated to the parking alternative $j$, and the number of departures $s_{jt}$ at time $t$, the RAF algorithm computes the prediction of parking utilization $z_{jt}$ for period $t$ using Eq. (18):

$$z_{jt} = z_{jt-1} + s_{jt} - d_{jt}.$$  

5. Analysis and validation of RAF algorithm

Two numerical validations of the proposed methodology are provided in this section. Firstly, a comparison of the methodology with a one-by-one simulation approach is presented to justify the utilization of the aggregated allocation approach, yielding a less time-consuming methodology. Secondly, the performance of the RAF algorithm is evaluated for a real case scenario. Additionally, an algorithmic time execution analysis is presented.

5.1. Validation of RAF using simulation exercise

This section demonstrates that results from a complex one-by-one simulation approach are not significantly different from the results obtained from the proposed aggregated PRA algorithm, with strongly less time computing.

Notice that the proposed aggregated methodology neglects any interaction between waiting and searching time parking performance and user choice process. In contrast, in a more complex one-by-one simulation approach, the availability, and waiting and searching times are updated after each arrival or/and departure, and the subsequent arrival will face a different choice scenario than previous arrivals. The one-by-one simulation approach models the interaction commented above. However, this study shows that the effect of not considering interaction between parking performance and choice process is not quite relevant, justifying the use of the aggregated approach instead of a one-by-one simulation.

A simulation environment with two parking alternatives was constructed to simulate choice behavior in a spreadsheet, and evaluate the effectiveness of the PRA algorithm. Simulations are based on the studies of Caicedo (2005), Caicedo et al. (2006), and Caicedo (2009) using the concept of time-steps. One unit of time-step is equivalent to a vehicle traveling 2.4 m (a typical width of a parking space) at an average vehicle speed of 15 km/h, i.e., there are 6250 time-steps in one hour. The simulation iterates from 1 to 6250 representing the passage of time. Vehicles in the parking system may change their status for each time-step.

The simulation environment considers that users arrive, choose, remain parked or depart within a period of one hour. These choices condition the future parking availability. Each simulation scenario is defined with a combination of attribute levels presented in Table 1. For example, in a single simulated scenario, the arrival time of each request is generated using an arrival rate value of $\lambda$ (e.g., 72 veh/h), departure times are simulated using departure rates $\lambda_j$ and $\lambda_2$ (e.g., 36 and 84 veh/h), and so on. Note that in these particular simulations the duration of stay for each parking request is greater than one hour.

This exercise employs choice probabilities of Eq. (1), and the utility function expressed in Eq. (19). Expression (19) is a particular case of the utility function stated in Eq. (2):

$$V_j = \theta_0 J + \theta_1 z_j + v C,$$

where $\theta_j$, $\theta_j$, $\theta_j$: previously calibrated parameters. $C$ waiting time caused by high levels of occupancy within the parking facility, $z_j$ historical or perceived value of the availability in alternative $j$.

Upon evaluating each request, parking space availability at each alternative is updated as a function of prior arrivals and departures. When an alternative is chosen using choice probabilities of Eq. (1), a unit is subtracted from the availability. If during the execution of a simulation a driver chooses a full parking facility, then he/she will be forced to decide again in a later period of time. In the simulations, drivers eventually park in a free parking space without aborting the searching process or exiting the facility. Finally, the market share or equivalent availability in the simulations is computed by summing the number of vehicles that parked in each alternative.

Fig. 4 represents one hour of operation at a parking facility. The average arrival rate $\lambda$ is 120 user/h, the average departure rate $\lambda_1$ at alternative 1 is equal to 36 user/h, and the average departure rate $\lambda_2$ at alternative 2 is 108 user/h. As a result of

<table>
<thead>
<tr>
<th>Table 1 Parameters considered in the simulations.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arrival rate $\lambda$</td>
</tr>
<tr>
<td>(veh/h)</td>
</tr>
<tr>
<td>24</td>
</tr>
<tr>
<td>72</td>
</tr>
<tr>
<td>120</td>
</tr>
<tr>
<td>168</td>
</tr>
<tr>
<td>216</td>
</tr>
</tbody>
</table>

Note: parameters and experiments defined in Caicedo (2009), $w_1 = 100; \theta_02 = 0; r_1 = -0.2167; C_j = 5; \theta_{01} = +0.0425.
The performance of the PRA algorithm is evaluated with different scenarios that consist of attribute level combinations. A sample of 1000 random parameter combinations/scenarios was generated from a total of 15,000 combinations. One hundred simulations of arrival, departure, and choice processes were executed for each scenario. The PRA algorithm was carried out in an independent process for the same scenarios, and thus, guaranteeing contrast consistency. The market share of each alternative calculated by the PRA algorithm was recorded for all 100 exercises and then compared with the simulation results. Thus, the following null hypothesis is formulated:

\[ H_0 \]

The allocated requests resulting from the simulation exercises are not significantly different from the allocated requests calculated using the PRA algorithm.

Statistical values such as mean and the standard deviation are calculated from market share samples. The appropriate technique to ratify this hypothesis is the application of the T-test (Canavos, 1995) with the degrees of freedom associated to the size of each sample \( (DF = n_1 + n_2 - 2) \), as shown in Eq. (20):

\[
\hat{t} = \frac{X_1 - X_2}{\sqrt{\frac{SD_1^2}{n_1} + \frac{SD_2^2}{n_2}}},
\]

where:

- \( X_1 \): Accept
- \( X_2 \): Reject
- \( \lambda_a \): arrival rate
- \( \lambda_{s1} \): departure rate at alternative 1
- \( \lambda_{s2} \): departure rate at alternative 2
- \( P1(\%) \): Preference for parking facility 1
- \( z_{01} \): initial availability in parking facility 1
- \( z_{02} \): initial availability in parking facility 2

**Table 2**

<table>
<thead>
<tr>
<th>Time</th>
<th>Hour 10</th>
<th>Hour 11</th>
<th>Hour 12</th>
<th>Hour 13</th>
<th>Hour 14</th>
<th>Hour 15</th>
<th>Hour 16</th>
<th>Hour 17</th>
<th>Hour 18</th>
<th>Hour 19</th>
<th>Hour 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shape parameter of gamma distribution ( (\alpha) )</td>
<td>0.976</td>
<td>1.383</td>
<td>1.321</td>
<td>1.424</td>
<td>2.369</td>
<td>1.581</td>
<td>1.953</td>
<td>1.912</td>
<td>1.317</td>
<td>1.069</td>
<td>0.912</td>
</tr>
<tr>
<td>Scale parameter of gamma distribution ( (\beta) )</td>
<td>3.001</td>
<td>1.095</td>
<td>2.266</td>
<td>1.608</td>
<td>1.514</td>
<td>2.390</td>
<td>1.336</td>
<td>1.102</td>
<td>2.093</td>
<td>3.673</td>
<td>3.625</td>
</tr>
<tr>
<td>Expected arrivals ( (\lambda_a) )</td>
<td>98</td>
<td>87</td>
<td>53</td>
<td>67</td>
<td>57</td>
<td>38</td>
<td>75</td>
<td>76</td>
<td>71</td>
<td>87</td>
<td>50</td>
</tr>
<tr>
<td>Expected departures ( \lambda_{s1} )</td>
<td>1</td>
<td>12</td>
<td>7</td>
<td>20</td>
<td>71</td>
<td>1</td>
<td>0.5</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Note: Assume initial availability in each alternative: \( z_{01} = 2; z_{02} = 73; z_{03} = 132; z_{04} = 146 \).
where \( n_1, n_2 \): sample sizes (100); \( \bar{X}_1, \bar{X}_2 \): means of final utilizations; \( SD_1, SD_2 \): standard deviations of final utilizations.

Thirteen percent of the evaluated scenarios were non-successful, requiring a third alternative since the arrival rate was too high relative to the departure rate in each parking facility. Only successful exercises were employed in the comparison analysis (i.e., 87% of the evaluated scenarios). Fig. 5 presents a summary of the results obtained in the simulations. The dots (•) represent the scenarios where the hypothesis \( H_0 \) was rejected, and the rectangles indicate that the hypothesis \( H_0 \) was accepted, validating the utilization of the PRA algorithm.

Assuming that parking requests allocated to each alternative describe a Poisson distribution, the hypothesis \( H_0 \) is accepted in 97.36% of the successful cases with a 95% confidence. Whereas, the dots refer to 2.64% of the successful scenarios in which hypothesis \( H_0 \) is rejected, particularly when departure rate ratios, \( \lambda(\bar{s}_1)/\lambda(\bar{s}_2) \), have values between 0.11 and 3. Approximately 885 out of 1000 scenarios were generated within this departure ratio range, concluding that rejected cases do not show significant correlation with the attributes of Table 1 (i.e., approximately 3% of error within the aforementioned range).

5.2. Validation of RAF in a real case scenario

The performance of the proposed RAF algorithm was evaluated employing the database containing records of user behaviors (i.e., decisions and utilization) in the SABA underground parking facility at the Cathedral Plaza, Barcelona, on Saturday 9th of November, 2002 (see Caicedo et al., 2006). This facility has a capacity of 665 vehicles among four levels (97, 152, 152, and 164 parking spaces, respectively), and it is provided by a level-4 PARC system, which continuously supplies real-time information about detailed

<table>
<thead>
<tr>
<th>Table 3</th>
</tr>
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<tbody>
<tr>
<td>Difference between the RAF forecast and actual availability for a ten-hour period.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Anticipated number of hours (h)</th>
<th>Alternative 1 (level 1)</th>
<th>Alternative 2 (level 2)</th>
<th>Alternative 3 (level 3)</th>
<th>Alternative 4 (level 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
<td>Sx</td>
<td>k</td>
<td>Sx</td>
<td>k</td>
</tr>
<tr>
<td>10</td>
<td>−0.31 0.83</td>
<td>−2.62 2.52</td>
<td>−4.86 3.78</td>
<td>−8.26 0.24</td>
</tr>
<tr>
<td>9</td>
<td>−0.63 0.76</td>
<td>−0.52 4.95</td>
<td>−5.80 5.56</td>
<td>−4.82 4.13</td>
</tr>
<tr>
<td>8</td>
<td>−0.12 1.47</td>
<td>0.50 3.22</td>
<td>−2.35 3.80</td>
<td>−4.74 4.53</td>
</tr>
<tr>
<td>7</td>
<td>0.22 0.98</td>
<td>5.77 9.95</td>
<td>−3.94 5.12</td>
<td>−3.73 3.89</td>
</tr>
<tr>
<td>6</td>
<td>0.12 1.69</td>
<td>5.53 7.35</td>
<td>−5.25 7.53</td>
<td>−3.18 3.75</td>
</tr>
<tr>
<td>5</td>
<td>0.56 1.54</td>
<td>4.96 8.95</td>
<td>−7.64 10.44</td>
<td>−4.30 5.27</td>
</tr>
<tr>
<td>4</td>
<td>0.18 1.48</td>
<td>3.80 9.92</td>
<td>−8.46 9.35</td>
<td>−4.13 5.01</td>
</tr>
<tr>
<td>3</td>
<td>0.27 1.59</td>
<td>3.92 9.52</td>
<td>−9.11 8.98</td>
<td>−4.23 3.50</td>
</tr>
<tr>
<td>2</td>
<td>0.34 1.60</td>
<td>3.04 9.00</td>
<td>−7.78 7.70</td>
<td>−3.25 2.74</td>
</tr>
<tr>
<td>1</td>
<td>0.12 1.13</td>
<td>1.51 6.40</td>
<td>−5.06 6.58</td>
<td>−2.08 1.41</td>
</tr>
<tr>
<td>Capacity</td>
<td>97 veh</td>
<td>152 veh</td>
<td>152 veh</td>
<td>164 veh</td>
</tr>
</tbody>
</table>
parking space availability. As previously mentioned, the RAF algorithm requires a calibrated DCM that is specified from a global study of parking preferences, described in Caicedo (2009). The author redefined the DCM presented by Caicedo et al. (2006), in order to model the SABA Cathedral application. In this DCM, users choose one of the four levels of the parking facility.

The RAF algorithm was implemented to forecast parking availability starting at 10:30 am of a typical day using historical decision and utilization records of the SABA Cathedral parking facility. These records were employed to estimate the parameters presented in Table 2. The RAF algorithm utilizes these parameters to generate a random sample of $Q_t$ arrivals based on an average rate $\lambda_t$ and duration of stay generated using Gamma distribution parameters $\delta_t$ and $\beta_t$ for each forecast period $t$, with $t = 1, \ldots, T$.

For example, Table 2 indicates that the expected number of vehicles arriving at the 15th hour is 38. The RAF algorithm will predict 27 arrivals or parking requests for this hour, and other arrival values for the 16th hour, the 17th hour, and so on using a Poisson distribution. At time 16:00:01, the IPR system determined, based on parking reservations, that the number of arrivals at the 15th hour was not 27, but rather 40 vehicles had arrived requiring the system to compute the forecast once again. At time 16:00:02, a user solicits an availability forecast for the 17th hour, and the system will indicate that there will be 14 parking stalls available.

The RAF algorithm employs real stored records of arrivals and departures to calculate the number and time of arrivals and departures that have occurred prior to the current time. This data is used to update predictions with recent and updated information. Fig. 6(a) and (b) show the comparison between real availability for two garage levels of the Cathedral parking facility and the predicted information employing 3, 2, 1 h, and 15 min of anticipation. As expected, the example in these figures illustrates that the availability forecast improves when the historical information employed in the forecast process is increased and updated.

Table 3 summaries the result analysis of differences between forecast and actual availability for each of the four parking levels during a ten-hour period. This table presents average $\bar{x}$ and standard deviation $(Sx)$ forecast errors for each level and anticipated number of hours ranging from 10 to 1 h. Notice that the errors in the forecast possess very little variability, tend to stabilize at small values, particularly between 1 and 4 h of anticipation, and present different bounds for each level. These results may be conditioned to the capacity for each garage level, as observed at the bottom of the table. However, in relative terms, discrepancies remain very small (less than 3% in the case of 1 h of anticipation).

The results of the validation presented in this section are encouraging, considering that the RAF algorithm employs the same type of information that any FMS may request, and bearing in mind that vehicles duration of stay are unknown. Once an extensive period of time has elapsed such as one week, the RAF algorithm obtains durations of stay at each parking level from the user records provided by the FMS.

5.3. Algorithm time execution

Both RAF and PRA algorithms were programmed in Visual Basic (VB) under MS-Excel to access, read, and store data. Fig. 7 shows a chart with the performance of the algorithms in the prediction process without taking into account the time needed to update initial parking availability. This chart illustrates that the performance worsens as forecasts are increased due to the number of stored entrance or departure events. The thick line represents the time in seconds required to process an event. The thin line with rhombuses indicates the forecast rate (i.e., events/time). For example, 171 events may be forecasted in 1 min, and 686 events may be forecasted in 12 min, which is equivalent to anticipating 1.5 and 2 h of operation. The RAF algorithm forecasts 10.5 h of operations (i.e., more than 1100 events of entrance-choice or departure) in approximately 40 min using a laptop computer with a 1.86 GHz Pentium T2130 dual core processor and 1 MB RAM. If these algorithms are programmed in more powerful languages without interfaces and are executed on supercomputers, then the time required to forecast parking availability may be reduced significantly.

6. Conclusions

IPR systems lead to a more effective management of parking availability, while improving customer satisfaction and parking service productivity. These systems provide drivers with real-time
information on parking location and current space availability, in order to reserve the most convenient parking facility, and thus, to avoid traffic circulating in search of empty stalls. The information presented to the user is valuable for decision-making independent of whether the user decides or not to reserve a parking space. However, user and operator decisions are enhanced if forecasted parking availability is known in advance.

This paper proposes a methodology for predicting space availability in an IPR architecture for parking facility information systems. This methodology consists of a real-time availability forecast (RAF) algorithm, which evaluates each parking request and uses an aggregated approach to allocate iteratively parking requests as a function of simulated drivers’ preferences, and parking availability. The aggregated allocation approach is based on a previously calibrated DCM for selecting alternatives, which is defined as a function of duration of stays, user budgets and destinations, parking availability, waiting times, and fees. The RAF algorithm employs historical information of arrivals and departures to update and predict the dynamic capacity or availability for each parking alternative.

A one-by-one simulation-based forecasting algorithm was developed, in order to be compared with the proposed aggregated approach. An exhaustive numerical comparison between these two approaches shows that there are no significant differences, validating and suggesting the use of the less time consuming proposed aggregated methodology.

The RAF algorithm was applied to a real case instance at the SABA Cathedral parking facility in Barcelona, where real availabilities were compared to predicted results of the RAF algorithm. Due to the probabilistic nature of this algorithm, it was executed several times. The comparison yielded very satisfactory results with small average error availabilities (less than 3% in the case of 1 h of anticipation) for a four level parking facility with parking capacities of 97, 152, 152, and 164 parking stalls.

Acknowledgements

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Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at doi:10.1016/j.eswa.2012.01.091.

References


