An upwind numerical approach for an American and European option pricing model

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Abstract

The numerical solution of several mathematical models arising in financial economics for the valuation of both European and American call options on different types of assets is considered. All the models are based on the Black-Scholes partial differential equation. In the case of European options a numerical upwind scheme for solving the boundary backward parabolic partial differential equation problem is presented. When treating with American options an additional inequality constraint leads to a discretized linear complementarity problem. In each case, the numerical approximation of option values is computed by means of optimization algorithms. In particular, Uzawa's method allows to compute the optimal exercise boundary which corresponds to the classical concept of moving boundary in continuum mechanics. © 1998 Elsevier Science Inc. All rights reserved.

Keywords: Option pricing; Black-Scholes equation; Characteristics method; Linear complementarity problems; Uzawa's algorithm

1. Introduction

A traded option is a contract which gives to its owner the right to buy (call option) or to sell (put option) a fixed quantity of assets of a specified common stock at a fixed price (striking price) on or before a given date (expiry date). The market price of the rights to buy or sell are termed call price and put price, respectively. When the transaction involved in the option takes place the option has been exercised. In this paper we deal with European and American call

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PII: S0096-3003(97)10122-9
options on shares which may pay continuous dividends. In the first case the option may be exercised only on the expiry date while the American options may be exercised on any moment before the expiry date. In any case, when the option is exercised the owner pays the striking price and receives the underlying stock (share, for example). The decision of the owner depends on the current price of the stock. Thus, if we note by $C^*, S^*$ and $K$ the values on the expiry date of the call, the share and the striking price respectively, then clearly $C^* = \max(0, S^* - K)$.

In this work we shall present a mathematical model which gives the value $C$ of the call option in any moment prior to the expiry date. The main interest of options on assets, as well as other derivative securities, comes from limiting the risk due to unexpected fallings of the asset price. Moreover, different strategies (hedge, spread, combination, ...) including the properties of options and the corresponding stock lead to more conservative or risky positions (see [1], for more details).

Nevertheless, the increasing success of option markets is not only due to the previously described advantages they represent for the individual investors but also to the continuous improvements in the option markets organization in the 1970s and 1980s. In any case, this success in worldwide stock exchange markets motivates the scientific interest to establish well suited mathematical models to obtain theoretical numerical values of option prices in terms of the different involved financial factors.

In this last sense, the most widely extended numerical methods for valuing derivative securities can be classified in lattice and finite differences approaches. The first ones were introduced by Cox et al. [2] and extended by Hull and White [3]. The finite difference approach is applied to the backward parabolic differential equation introduced by Black and Scholes [4] to model the evolution of call option prices. The existence of a formal expression for the solution of this partial differential equation allows to evaluate the different numerical methods in European call options. Finite differences were suggested by Schwartz [5] for the valuation of warrants and extended by Courtadon [6] to approximate European option values. This last work is based on explicit second-order schemes in time and space.

In this work we consider different implicit finite difference schemes in space and a particular upwind scheme for the joint discretization of the first-order time and space derivatives.

In Section 2, the mathematical models for the European and American call option pricing problems are posed. In the first case, the closed-form of the solution for the Black–Scholes equation including continuous dividend distribution was proposed by Dewyne et al. [7] in terms of the kernel of the heat equation. In the American call option the additional constraint gives place to a free boundary problem whose analytic solution cannot be obtained.

In Section 3, the numerical method here proposed for the discretization of Black–Scholes equation is described. This method is extended in Section 4 to
discretize the parabolic free boundary problem associated to the Black–Scholes equation in American options. The solution of the resulting linear complementarily discretized problems is performed by different quadratic programming algorithms. In Section 5, several test examples are presented and discussed to illustrate the good performance of the numerical methods. Finally, in Section 6, some conclusions and further future applications of the method are suggested.

2. The mathematical models

In this work we are concerned with the numerical valuation of call options. The departure point is the mathematical model proposed in Black and Scholes [4] which is actually used in real markets to obtain the current theoretical option value. The unknown of the model is the value of the call option as function of time and the market price of the underlying security. In a wider sense, the model assumes that the present value $C$ of the call depends on the current value $s$ of the underlying asset, the striking price $K$, the time to expiration $T - t$ ($T$ is the expiry date and $t$ is the present moment), the stock volatility $\sigma$, the risk free interest rate $r$ and the continuous dividend rate $d$. If we consider a ceteris paribus analysis, the value $C$ increases with $s$, $T - t$, $\sigma$ and $r$ and decreases with $K$. The dependence on dividends is more complex. Here we shall consider a continuous dividend, so it has the same effect as a negative interest rate (the higher the continuous dividend rate the lower the call current price).

Next step in modelling option pricing phenomena is based on what is known as general arbitrage relationships which avoid the presence of riskless arbitrage opportunities, that is, situations where without initial investment we can obtain immediately a positive profit and a nonnegative one in every future date. From a mathematical point of view these assumptions lead to the desired qualitative properties of the solution and provide several conditions for the model.

In order to deduce the following Black and Scholes equations it is assumed that the underlying security prices follow a particular random walk process which consists of a Brownian motion [8]. Moreover, the random variation of stock price at each moment is supposed to be independent of its variation in the immediately previous or future moment. Although, in practice, the properties of stock prices movements result from the interaction of different economic causes, the probabilistic description by using the Brownian motion is well suited for short term situations and for shares whose value changes are small and smooth (i.e. we assume that sudden jumps are not allowed). A simple way to obtain the Black–Scholes partial differential equation from probabilistic arguments based on binomial model is presented by Cox and Rubinstein for
European call options [1]. An alternative approach issued from stochastic differential calculus can be found in [7].

In order to state the models treated here, let \( D = [0, T] \times [0, S] \) be the time-share value domain. Then the whole corresponding set of equations is:

\[
\frac{\partial C}{\partial t}(t,s) + rs \frac{\partial C}{\partial s}(t,s) + \frac{1}{2} \sigma^2 s^2 \frac{\partial^2 C}{\partial t^2}(t,s) - rC(t,s) = 0 \quad \text{in } D, \tag{1}
\]

\[
C(t,0) = 0, \quad t \in [0, T], \tag{2}
\]

\[
C(t,S) = S \exp(d(t - T)) - K \exp(r(t - T)), \quad t \in [0, T], \tag{3}
\]

\[
C(T,s) = \max(0, s - K), \quad s \in [0, S], \tag{4}
\]

where \( t \) is the time, \( s \) is the share value, \( C \) is the unknown call value, \( r \) is the interest rate, \( \sigma \) denotes the volatility of the share and \( K \) is the striking price. The boundary condition for \( s = 0 \) means that when the asset value is zero the call value is zero too. Even, from the mathematical point of view, the existence of solution is more easily achieved if the same homogeneous boundary condition is posed for a small positive share value which seems financially reasonable. On the other hand, from reasonable financial arguments, it is assumed that the call price tends to the asset value when this one tends to infinity. In accordance to this fact, and in order to work with finite domains, the boundary condition (3) is considered. The final condition (4) to this backward parabolic problem translates the call value on expiry date. In [7] a simple extension of the previous model is proposed to include the possibility of a continuous dividend distribution by the share:

\[
\frac{\partial C}{\partial t}(t,s) + (r - d)s \frac{\partial C}{\partial s}(t,s) + \frac{1}{2} \sigma^2 s^2 \frac{\partial^2 C}{\partial t^2}(t,s) - rC(t,s) = 0 \quad \text{in } D, \tag{5}
\]

\[
C(t,0) = 0, \quad t \in [0, T], \tag{6}
\]

\[
C(t,S) = S \exp(d(t - T)) - K \exp(r(t - T)), \quad t \in [0, T], \tag{7}
\]

\[
C(T,s) = \max(0, X - K), \quad s \in [0, S], \tag{8}
\]

where the new constant parameter \( d \) denotes the dividend rate. From the financial point of view, a realistic hypotheses on this parameter is that \( r > d > 0 \).

Both previous models correspond to European options which cannot be exercised before the expiry date. They are analogous to those arising in termic problems in fluid mechanics [9]. In this sense, the value of the option can be associated to a fluid temperature governed by a classical backward heat equation. The boundary backward parabolic partial differential equation problem (5)-(8) is well posed and the formal expression of the exact solution is given by

\[
C(s,t) = sf(d_1) \exp(-d(T - t)) - Kf(d_2) \exp(-d(T - t)) \tag{9}
\]
with

\[ f(y) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{y} \exp(-x^2/2) \, dx, \]

\[ d_1 = \ln(s/K) + \frac{(r - d + \sigma^2/2)(T - t)}{\sigma\sqrt{T - t}}, \]

\[ d_2 = d_1 - \sigma\sqrt{T - t}, \]

where the function \( f \) has a classical interpretation in probabilistic and classical heat equation contexts.

In American call options, the owner can exercise them and obtain the stock in any time before expiry date. So, the mathematical model takes this additional fact into account by means of the inequality constraint:

\[ C(t, s) > \max(0, s \exp(d(t - T)) - K \exp(r(t - T)), (t, s) \in D. \quad (10) \]

The above condition means that if at any moment the owner finds it better to exercise the option, buy the stock at the striking price and sell it immediately, he will proceed to do so. Mathematically, the condition (10) leads to a moving boundary problem whose time dependent coincidence set is:

\[ D^0(t) = \{s \in [0, S]/C(t, s) = \max(0, s \exp(d(t - T)) - K \exp(r(t - T))), t \in [0, T]. \]

At each time \( t \), there is a point \( s^*(t) \) such that for \( s < s^*(t) \) the Black–Scholes equation is valid and for \( s > s^*(t) \) the point \( (s, t) \) belongs to the coincidence set (so the Black–Scholes equation is no longer valid). Therefore, the curve \( s = s^*(t) \) on the plane \( t - s \) describes a part of the moving boundary of the problem which is termed in the financial economic context as optimal exercise boundary.

This paper is mainly devoted to the numerical solution of the previous models by using their analogy with the well-known convection–diffusion-reaction equations in termic problems. In this case, the reaction terms are linear but the presence of a moving boundary involves a certain nonlinearity in American options. A very interesting presentation of this last problem as a one phase Stefan problem can be found in [7] but with a slightly different condition instead of (10). In next sections we propose different numerical finite-difference discretization schemes to approximate the solution of Eqs. (5)–(8) and Eqs. (5)–(10).

3. European options: a discretized Black–Scholes equation

We shall now introduce several finite-difference discretization of Eqs. (5)–(8). To this end let \( N \) be the number of time intervals, let \( k = T/N \) be the constant time step and let \( t_n = kn, n = 0, \ldots, N, \) be the finite-difference time mesh points. Moreover, let \( M \) be the number of share value intervals, let
\[ h = \frac{S}{M+1} \]
be the constant share value step and let \( s_i = h_i, \ i = 0, \ldots , \]
\( M + 1, \) be the discrete set of share prices. Then, several possible well-known
finite-difference schemes for Eqs. (5)-(8) are based on the derivative approximations
\[
\frac{\partial C}{\partial t} (t_n, s_i) \approx \frac{C(t_n, s_i) - C(t_{n+1}, s_i)}{k},
\]
\[ (r - d)s_i \frac{\partial C}{\partial s} (t_n, s_i) \approx \frac{(r - d)s_i C(t_n, s_i) - (r - d)s_i C(t_n, s_{i-1})}{h}, \]
\[
\frac{\partial^2 C}{\partial s^2} (t_n, s_i) \approx \omega \left( \frac{C(t_n, s_{i+1}) - 2C(t_n, s_i) + C(t_n, s_{i-1})}{h^2} \right)
+ (1 - \omega) \left( \frac{C(t_{n+1}, s_{i+1}) - 2C(t_{n+1}, s_i) + C(t_{n+1}, s_{i-1})}{h^2} \right),
\]
where \( \omega \in [0, 1] \) is a constant parameter. The resulting methods are classical in
numerical solution of parabolic partial differential equations (see [10], for example). For \( \omega = 0 \) the resulting method is explicit and for \( \omega > 1 \) is implicit. It
is well-known that the convergence of the method for \( \omega = 0 \) is limited by
stability conditions which restrict the choice of the time and share price step
parameters. Although the implicit methods are unconditionally stable they
involve the solution of a tridiagonal system at each time step. The choice
\( \omega = 0.5 \) corresponds to Crank–Nicholson method which is order two in \( k \) and \( h. \)

Another upwind method which has been used in fluid mechanics for convection–diffusion equations in time dependent problems is the characteristics
method (see [11], for example). The departure point is the total derivative
concept defined in our financial model by
\[
\frac{DC}{Dt} (t, s) = \frac{\partial C}{\partial t} (t, s) + (r - d)s \frac{\partial C}{\partial s} (t, s).
\]
The total derivative can be interpreted in the frame of our option pricing
problem as it has been previously interpreted as a material derivative in continuous
mechanics. At this point we must distinguish between two different
phenomena which motivate call price variation in Black–Scholes model. A first
fact is the Brownian motion of the stock price (with a volatility \( \sigma \)) that is mainly
considered by the second-order diffusion term. If the riskless interest rate and
the deterministic continuous dividend rate were both zero, the previous fact
would lead to a pure diffusion equation (see [8] for a simple description of the
relationship between diffusion equation and Brownian motion). Nevertheless,
the presence of interest and dividends, that is the presence of riskless alternative
deterministic sources of profit, must be taken into account in order to achieve a
more realistic model. This motivates the appearance of convection and linear
reaction terms. In this sense, the total derivative measures the call price variation
with time and the variation due to the profit flux associated to the existence
of dividends and riskless interest rates. Moreover, several important financial concepts involved in option values are present in the different terms of Black–Scholes model. Thus, the neutral hedge ratio which measures changes in the value of the call relative to the change in the stock value is given by

\[ \Delta(t,s) = \frac{\partial C}{\partial s}(t,s), \]

(16)

the option's elasticity is

\[ \Omega(t,s) = \frac{s}{C} \frac{\partial C}{\partial s}(t,s), \]

(17)

and the option's theta which measures the sensitivity of the call price with time is

\[ \Theta(t,s) = \frac{\partial C}{\partial t}(t,s). \]

(18)

So, the total derivative can be written as

\[ \frac{DC}{Dt}(t,s) = \Theta(t,s) + (r - d)s\Delta(t,s), \]

(19)

in terms of the previous financial concepts.

Next step in the numerical characteristics method is the approximation of the total derivative (15) at each point of the finite-difference mesh by the quotient

\[ \frac{DC}{Dt}(t_{n},s_{i}) \approx \frac{C(t_{n+1},\chi^{k}(s_{i})) - C(t_{n},s_{i})}{h}, \]

(20)

where

\[ \chi^{k}(s_{i}) = s_{i} \exp (k(r - d)). \]

(21)

The previous expression for \( \chi^{k}(s_{i}) \) comes from the exact solution of the final value problem

\[ y'(\tau) = (r - d)y(\tau), \quad y(t_{n}) = s_{i}, \]

(22)

so that \( \chi^{k}(s_{i}) = y(t_{n+1}) = s_{i} \exp (k(r - d)). \) Therefore \( \chi^{k}(s_{i}) \) can be viewed as the value at time \( t_{n+1} \) of a share whose market price at time \( t_{n} \) is \( s_{i} \) when the interest and dividend rates are \( r \) and \( d \), respectively. Moreover, the solution of the final value problem (22) can be interpreted as the path described by the share value from time \( t_{n} \) to time \( t_{n+1} \) due to the balance between the continuous interest and dividend rates.

Next step to obtain the discretized problem is the approximation of the value \( C(t_{n+1},\chi^{k}(s_{i})) \) by using the linear interpolation from call values at finite-difference mesh points. For this we select the time step \( k \) verifying the restriction:

\[ k < \frac{\log(S/(S-h))}{r - d}, \]

(23)
which implies that \( s_i < \chi^k(s_i) < s_{i+1} \) for all values of \( i \). Then we consider the approximation

\[
C(t_{n+1}, \chi^k(s_i)) \approx \alpha_i C(t_{n+1}, s_i) + (1 - \alpha_i) C(t_{n+1}, s_{i+1})
\]

with

\[
\alpha_i = \frac{(s_{i+1} - \chi^k(s_i))}{h}, \quad i = 0, \ldots, M.
\]

The time step constraint (23) can be relaxed if we search the finite-difference interval of share values where \( \chi^k(s_i) \) belongs to.

The substitution of either (11)–(14) or (14)–(20) approximations in Eqs. (5)–(8) leads to a tridiagonal linear system at each time step which can be solved by the well-known Thomas algorithm (see [12], for example). The solution of this linear system is the corresponding finite-difference option value approximation at times \( t_n \) for \( n = 1, \ldots, N \). In the characteristics method, the linear system to solve is:

\[
AC^n = b^{n+1}
\]

with the nonzero coefficients of the tridiagonal matrix \( A = (a_{ij}) \) given by

\[
a_{ii} = -\frac{(\omega k \sigma^2 S_i^2)}{h^2}, \quad i = 1, \ldots, M,
\]

\[
a_{i,i+1} = \frac{(\omega k \sigma^2 S_i^2)}{(2h^2)}, \quad i = 1, \ldots, M - 1,
\]

\[
a_{i-1,i} = \frac{(\omega k \sigma^2 S_i^2)}{(2h^2)}, \quad i = 2, \ldots, M,
\]

and the time dependent vector \( b^{n+1} \) is:

\[
b_1^{n+1} = -\frac{((1 - \omega)k \sigma^2 S_1(-2C_1^{n+1} + C_2^{n+1}))}{h^2} - \alpha_1 C_1^{n+1} - (1 - \alpha_2) C_2^{n+1},
\]

\[
b_i^{n+1} = -\frac{((1 - \omega)k \sigma^2 S_i(-2C_i^{n+1} + C_{i+1}^{n+1}))}{h^2} - \alpha_i C_i^{n+1} - (1 - \alpha_{i+1}) C_{i+1}^{n+1}, \quad i = 2, \ldots, M - 1,
\]

\[
b_M^{n+1} = -\frac{((1 - \omega)k \sigma^2 S_M(C_{M-1}^{n+1} - 2C_M^{n+1} + C_{M+1}^{n+1}))}{h^2} - \alpha_M C_M^{n+1} - (1 - \alpha_M) C_{M+1}^{n+1} - \omega k \sigma^2 S_M C_{M+1}^{n+1}/(2h^2),
\]

where \( C_{M+1}^{n+1} \) and \( C_M^{n+1} \) are given by the boundary condition (7). The \( i \)th coordinate of the vector \( C^n \) is the approximation of the value \( C(t_n, s_i) \).

4. American options: a discretized Black–Scholes equation with obstacle

As it has been pointed out in Section 2, the possibility of exercising the American call options before expiry date leads to a free boundary problem related to the inequality constraint (10). Moreover, the free boundary only exists if there is a strictly positive continuous dividend rate. That is, if \( d = 0 \) the optimal exercise boundary for both option problems would be the expiry date for every value of the stock price as it was also found in the numerical tests.
Mathematical models like (5)–(8) and (10) are termed as parabolic obstacle problems in free-boundary literature (see [13], for example). In a first step, its numerical solution is based on finite-difference and finite-elements discretizations (see Elliot and Ockendon [14]). This results in an algebraic constrained minimization problem for each time step. In our case the characteristic discretization leads to a finite set of discrete quadratic programming problems with linear constraints which can be written as linear complementarity problems. Several algorithms issued from constrained optimization theory have been adapted to this particular option pricing problem and their performances have been compared.

Thus, once the combination of characteristics and finite-difference discretization explained in the previous section has been applied to the American option problem (Eqs. (5)–(8) and (10)), the following linear complementarity problems are posed:

For \( n = N - 1, N - 2, \ldots, 0 \), find \( C^n \in \mathbb{R}^M \) such that

\[
C^n \geq F^n, \quad AC^n - b^{n+1} \geq 0, \quad (C^n - F^n)^T(AC^n - b^{n+1}) = 0, \quad (27)
\]

where

\[
F^n_i = \max(0, s_i \exp(-dk(N - n) - K \exp(-rk(N - n))).
\]

In order to solve the quadratic programming problems (27) several projection methods have been used. For example, overrelaxation and projected steepest descent are simple and easy to code. Both consist of iteration processes with two parts: the minimization and the projection onto the coincidence set. Nevertheless, in order to obtain a better approach of the optimal exercise boundary, the Kuhn–Tucker multipliers computed in Uzawa’s duality method has been performed. We address the reader to [15,16] for a detailed description of the quoted methods and we shall recall here a brief presentation of Uzawa’s algorithm applied to Eq. (27). For this method, the departure point is the introduction of the Lagrangian function:

\[
L^n(X, \lambda) = \frac{1}{2}X^TAX - X^Tb^{n+1} + (F^n - X)^T\lambda, \quad (X, \lambda) \in \mathbb{R}^M \times \mathbb{R}^M. \quad (28)
\]

Then, at each time the algorithm consists of:

1. Initialize \( \lambda^0 > 0 \).
2. At step \( k \), solve the linear system:

\[
AX^k = b^{n+1} + \lambda^k.
\]
3. Update of the Kuhn–Tucker multiplier

\[
\lambda_i^{k+1} = \max(0, \lambda_i^k + \rho(F^n_i - X_i^k)).
\]

Step 2 searches for the primal solution while Step 3 is related to the dual problem associated to the Lagrangian formulation. The limit values of the
sequences \( \{X^n_k\} \) and \( \{\lambda^n_k\} \) are the approximation vector of call values \( C^n \) and its Kuhn–Tucker multiplier \( \theta^n \), respectively. The convergence results of the previous algorithm under certain conditions on the parameter \( \rho \) are detailed in [15], for example. The optimal exercise boundary can be obtained from Kuhn–Tucker multipliers taking into account that they only vanish outside the co-incidence set.

5. Numerical results

As an illustration of the good performance of the algorithms, in this section we shall present the numerical results obtained for different types of options with different data sets. First test corresponds to the valuation of a call option on a stock whose volatility is \( \sigma = 30\% \) and which pays a continuous dividend \( d = 7\% \) p.a. The interest rate is supposed to be \( r = 10\% \) p.a., the striking price is \( K = 500 \), the time to expiry is \( T = 0.5 \) and the security maximum price \( S = 800 \) has been chosen. The time and share value steps have been taken to \( k = 1/360 \) and \( h = 4 \), respectively. Figs. 1 and 2 show the optimal exercise boundary for the data set: \( \sigma = 30\% \), \( d = 7\% \) p.a., \( r = 10\% \) p.a., \( K = 500 \), \( S = 800 \) and \( T = 0.5 \). The computation has been obtained with the numerical approximation scheme (1)–(4) for \( \omega = 0.5 \) with \( k = 1/360 \) and \( h = 4 \).
Fig. 2. Optimal exercise boundary for the data set: $\sigma = 30\%$, $d = 7\%$ p.a., $r = 10\%$ p.a., $K = 500$, $S = 800$ and $T = 0.5$. The computation has been obtained with the characteristics scheme and $\omega = 1$, with $k = 1/360$ and $h = 4$.

boundary for the last six months before expiry date computed with a classical forward scheme in time and characteristics, respectively. In both cases, the free boundaries are obtained from the Kuhn–Tucker multipliers issued from Uzawa’s algorithm.

In Fig. 3 the computed solutions with the different discretization methods (Crank–Nicholson or characteristics) and several quadratic programming algorithms for the linear complementarity problem are shown.

Besides the first test, an option whose underlying asset pays no dividends has been considered by taking $d = 0$. In this second case, there is no free boundary and the optimal exercise would correspond to the expiry date. From this point of view the evolution of the resulting prices for European and American call options is the same. In Fig. 4, shows the comparison between the numerical solution for the first test and the one corresponding to an option with the same financial data set but paying no dividends. As it is pointed out in the financial literature, the presence of dividends is crucial for the existence of an optimal exercise time prior to expiry date [1]. Mathematically, the nonzero continuous dividend rate leads to the appearance of the free boundary.
Fig. 3. Computed call values with different numerical methods for the data set: $\sigma = 3\%$, $d = 7\%$ p.a., $r = 10\%$ p.a., $K = 500$, $S = 800$ and $T = 0.5$. The discretization steps have been taken to be $k = 1/360$ and $h = 4$.

Fig. 4. Computed call values with characteristics plus Uzawa's methods for the data set: $\sigma = 30\%$, $r = 10\%$ p.a., $K = 500$, $S = 800$ and $T = 0.5$. The upper and lower curves correspond to $d = 0\%$ and $d = 7\%$, respectively. The discretization steps have been taken to be $k = 1/360$ and $h = 4$. 
6. Conclusions

The presentation of an alternative discretization method for the valuation of American call options with the Black and Scholes model with obstacle is the main task of this work. The proposed method has already been used in several problems of heat transfer and continuum mechanics. After the description of its adaptation to financial economic models, the resulting discretized complementarity problems are solved by different algorithms. Several numerical tests show the good performance of characteristics technique when compared with the computed solution by other discretization and projection methods.

Although this characteristic numerical scheme has been used in this work to treat call options on shares distributing a constant continuous dividend in a financial market whose risk free interest is also constant, the extension of its usage to time dependent dividends, rates of interests and share volatility requires some minor modifications such as the numerical integration of the final value problem instead of its exact solution. On the other hand, it seems an interesting future scope, the application of the scheme to more complex financial options such as exotic or path dependent options described in [17], for example.

References