Topography Independent InSAR Coherence Estimation in a Multiresolution Scheme

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Abstract— The main advantage of multidimensional SAR data is the possibility to perform quantitative remote sensing in order to characterize the Earth surface. In most of the cases, the quantitative retrieval of physical parameters is performed on the basis of the correlation properties of the set of SAR images, measured by the correlation coefficients and coherence parameters. As demonstrated, coherence estimation is affected by several problems as the presence of biases or the loss of spatial resolution with respect to the original SAR images. The scope of this paper is to characterize accurately the coherence parameter, especially in the case of interferometric SAR data, by considering a new model for speckle noise in multidimensional SAR data proposed by the authors. Results with experimental data are presented.


I. INTRODUCTION

In dual- or multidimensional complex SAR imagery, the complex correlation coefficient between two SAR images has been demonstrated as an important source of physical information about the scene being imaged by the SAR sensor. For dual or multiple baseline SAR interferometry (InSAR), the phase component of the correlation coefficient is proportional to the vertical structure of scene under observation. In particular, the relief topography can be obtained by means of two SAR images. On the other hand, the amplitude, known as coherence, contains information concerning the similarity between the pair of images. Hence, it presents information about the phase quality, i.e., about the noise content [1]. For multidimensional SAR imagery, e.g., SAR polarimetry (PolSAR), the coherence parameters obtained from the different SAR images have been shown to contain information concerning the scattering process occurring at the surface. Indeed, physical parameters associated with surface or volume scattering processes can be retrieved from these coherence parameters.

The use of coherence to perform quantitative studies of physical parameters, especially in InSAR applications, has been reported in the literature by different authors. All these methods rely, consequently, on a correct estimation of the coherence parameter. Nevertheless, this estimation process presents some drawbacks precluding an accurate quantitative estimation of the coherence. The fist of these problems is the presence of a bias in the estimation of the coherence value. Indeed, there are two different biases when coherence is estimated. On the one hand, the bias induced by the topographic component, which is noticeable in the complete range of coherence values. On the other hand, for low coherence values, the coherence parameter is overestimated with respect to the true value. The second problem of coherence estimation is the loss of spatial resolution due to the fact that coherence needs to be estimated from the available SAR data.

Recently, the authors have proposed a new model of speckle noise for multidimensional SAR imagery [2]. The scope of this paper is to use this noise model in order to derive a more precise characterization of coherence with especial attention to InSAR data. Hence, the paper is divided as follows. The model is presented in Section II and employed in Section III to study of the coherence parameter. Section IV considers multiresolution techniques to estimate the coherence parameter. Results are presented in Section V and the conclusions are presented in Section VI.

II. HERMITIAN PRODUCT SPECKLE NOISE MODEL

The coherence parameter refers to the amplitude of the complex correlation coefficient between two SAR images. Given the SAR images $S_1$ and $S_2$, coherence is defined as [3]

$$\rho = \frac{E\{S_1 S_2^*\}}{\sqrt{E\{|S_1|^2\} E\{|S_2|^2\}}}$$

(1)

where $^*$ refers to the complex conjugation and $E\{\cdot\}$ is the expectation operator. Under the assumption that $S_1$ and $S_2$ are ergodic in mean and in correlation, the ensemble average in (1) can be substituted by a spatial averaging

$$\hat{\rho}_{MLT} = \frac{\sum_{m=1}^{M} \sum_{n=1}^{N} S_1(m,n) S_2^*(m,n)}{\sqrt{\sum_{m=1}^{M} \sum_{n=1}^{N} |S_1(m,n)|^2 \sum_{m=1}^{M} \sum_{n=1}^{N} |S_2(m,n)|^2}}$$

(2)

where $MN$ are the number of averaged samples. Additionally, this spatial averaging makes only sense if the processes $S_1$, $S_2$ and $S_1 S_2^*$ are wide-sense stationary in mean. The calculation of the coherence $|\rho|$ is possible for any type of multidimensional SAR data, as for instance: InSAR or PolSAR.
As observed in (2), the calculation of coherence $|\rho|$ depends on the complex Hermitian product of a pair of SAR images $S_1 S_2^*$. Thus, the characterization of the coherence parameter follows from a characterization of this complex Hermitian product.

### A. Complex Hermitian Product Speckle Noise Model

Speckle noise is one of the most important problems of SAR data. This noise component is originated by the inherent complex nature of the SAR image formation process. For a particular SAR image, under the hypothesis that it can be described by the complex, circular, Gaussian probability density function (pdf), speckle noise can be modeled as a multiplicative noise component. Nevertheless, this noise model can not be considered for multidimensional SAR data since it is not able to completely characterize the data. In [2], the authors have proposed a speckle noise model for the complex Hermitian product of two SAR images, which has been extended to model speckle noise in multidimensional SAR data.

The complex Hermitian product of two SAR images can be written in the complex plane as

$$S_1 S_2^* = |S_1 S_2^*| e^{i(\theta_1 - \theta_2)} = z e^{i\phi}$$

where $z$ represents the amplitude of the product and $\phi$ corresponds to the Hermitian product phase. For instance, in the case of InSAR data, the phase component $\phi$ contains information concerning the topography of the terrain, whereas for PolSAR data it contains information concerning the scattering process occurring at the resolution cell. In [2] the authors have demonstrated that the complex Hermitian product can be described by the following speckle noise model

$$S_1 S_2^* = \Psi \bar{z}_n n_m N_c e^{i\phi} + \Psi (|\rho| - N_c \bar{z}_n) e^{i\phi} + \Psi (n_a + jn_m).$$

The following list details the parameters of the noise model:

- $\Psi$ is the average power in the two SAR images defined as

$$\Psi = \sqrt{E\{|S_1|^2\} E\{|S_2|^2\}}.$$  

- $\bar{z}_n$ corresponds to the normalized average intensity

$$\bar{z}_n = \frac{\pi}{4} F_1\left(-\frac{1}{2},-\frac{1}{2};1;|\rho|^2\right).$$

- $N_c$ is a parameter described by the expression

$$N_c = \frac{\pi}{4} |\rho|^2 F_1\left(1,\frac{1}{2},\frac{1}{2};2;|\rho|^2\right).$$

This parameter is of special importance since it is directly related with the coherence value, as it can be observed in Fig. 1.

- The coherence $|\rho|$ corresponds to the amplitude of the normalized correlation coefficient at (1), whereas $\phi$ is its phase component. The phase $\phi$ is also known as the average phase. In the particular case of InSAR data, the topography information is contained in $\phi$.

As it can be observed at (4), it is possible to identify two speckle noise sources in the case of the complex, Hermitian product of two SAR images:

- The term $n_m$ represents the real multiplicative speckle noise term. This noise component is characterized by

$$E\{n_m\} = 1 \quad \text{var}\{n_m\} = 1. \quad (8)$$

- The term $n_a + jn_m$ corresponds to the complex additive speckle noise term, characterized by the following statistical models

$$E\{n_a\} = 0 \quad \text{var}\{n_a\} = \frac{1}{2} (1 - |\rho|^2)^{1.32}$$

$$E\{n_m\} = 0 \quad \text{var}\{n_m\} = \frac{1}{2} (1 - |\rho|^2)^{1.32}.$$  

For the complex Hermitian product of SAR images, the speckle noise is due to the combination of the multiplicative and the additive noise sources. In order to determine the behavior of the final speckle noise, it must be considered that the multiplicative component $n_m$ is multiplied by the parameter $N_c$. Hence, for low coherence values, the dominant component is the additive term $n_a + jn_m$, since these terms present maximum variance values, see (9), and the term $N_c$ is close to zero. On the contrary, for high coherence values, the behavior of speckle noise is mainly determined by the multiplicative component of speckle $n_m$. Additionally, the multiplicative speckle noise component $n_m$ depends also on the phase term $\phi$. This dependence provokes that the real and imaginary parts of the complex Hermitian product can present different speckle behaviors depending on the particular value of the phase $\phi$.

### III. COHERENCE BIAS MODEL

As stated at Section I, one of the main problems in estimating coherence is the fact that its estimation by means of multilook techniques (2) is biased. Thus, in order to make a quantitative use of $|\rho|$, it is important to establish a signal model for these bias components.
A. Topographic Bias
The first source of bias is induced by the topography of the terrain [4]. A first alternative to eliminate this coherence bias is to compensate (2) for the topographic phase $\phi_t$

$$\hat{\rho}_{PHC} = \frac{\sum_{m=1}^{M} \sum_{n=1}^{N} S_1(m,n) S_2^*(m,n) e^{-j\phi_t}}{\sqrt{\sum_{m=1}^{M} \sum_{n=1}^{N} |S_1(m,n)|^2 \sum_{m=1}^{M} \sum_{n=1}^{N} |S_2^*(m,n)|^2}}. \quad (10)$$

As it can be deduced, the drawback of this approach is that the topographic phase $\phi_t$ needs to be estimated from the data or by means of external digital elevation models of the terrain under consideration.

B. Speckle Induced Bias
It is well known that apart from the bias induced by the topographic component of the data, the coherence estimated by means of multilook techniques, with (2) or without (10) topography compensation is overestimated for low coherences. This bias decreases with respect to the number of averaged samples. Nevertheless, the origin of this bias is still not clear. In what it follows, the speckle noise model presented in (4) will be considered in order to study this bias term. As it shall be demonstrated, the bias for low coherences is induced by the additive speckle noise component $n_{aw} + jn_{ai}$.

The speckle noise model for the complex Hermitian product of SAR images presented in (4) corresponds to a model for single-look data. It is worth to mention that despite multidimensional SAR data are described by the so-called Wishart pdf, the expression presented in (4) can not be derived from it due to the inability of this pdf to consider single-look data [5].

In order to evaluate (2), the noise model for the denominator is first considered. In this case, the intensity of the SAR images can be modeled by the multiplicative speckle noise model

$$\sqrt{\{S_1^2\} \{S_2^2\}} = \psi \sqrt{|n_{al}| m_{a2}} \quad (11)$$

where $n_{al}$ and $n_{a2}$ are the multiplicative noise sources. Thus, introducing (4) and (11) into (2), it is possible to derive a signal model for the complex correlation coefficient

$$\hat{\rho} = \frac{|\rho| e^{j\phi} + \overline{\chi_n N_c(n_{aw} - 1)} e^{j\phi} + (n_{aw} + jn_{ai})}{\sqrt{n_{al} m_{al}}} \quad (12)$$

The model in (12) represents the noise model for single-look data. A signal model for multilook data can be considered as follows

$$\hat{\rho}_{MLT} = \frac{|\rho| e^{j\phi} + \overline{\chi_{n_{aw}} N_c(n_{aw} - 1)} e^{j\phi} + (n_{aw} + jn_{ai})}{\sqrt{n_{al} m_{al} m_{a2}}} \quad (13)$$

where the new noise sources can be characterized by

$$E\{n_{aw}^2\} = E\{n_{aw}'\} = 0$$
$$E\{n_{aw}'\} = E\{n_{aw}'\} = E\{n_{a2}'\} = 1. \quad (14)$$

The analysis of (13) results difficult due to the complexity to jointly consider the numerator and denominator. In order to facilitate this study, the denominator is not considered in the following as it can be taken as a normalizing factor. Additionally from (14), one can notice that the expectation of its two components is equal to one. The coherence, which represents the amplitude of (13), can be then analyzed as

$$|\hat{\rho}_{MLT}| = |\rho| e^{j\phi} + \overline{\chi_n N_c(n_{aw} - 1)} e^{j\phi} + (n_{aw} + jn_{ai}). \quad (15)$$

The previous expression can be simplified by taking into consideration the fact that the expectation of the multiplicative component of speckle $n_{aw}'$ equals one independently of $MN$. Thus, (15) can be approximated by the following expression

$$|\hat{\rho}_{MLT}| = |\rho| e^{j\phi} + (n_{aw} + jn_{ai}). \quad (16)$$

A direct analysis of the coherence model presented in (16) is not feasible due to the absolute value operation. However, the analysis turns easier if the intensity is considered. In this case, it is possible to obtain the expectation of the coherence value

$$E\{|\hat{\rho}_{MLT}^2\} = E\{|\rho| e^{j\phi} + (n_{aw} + jn_{ai})^2\}$$
$$= |\rho|^2 + E\{n_{aw}^2\} + 2|\rho| \cos(\phi) E\{n_{aw}\} + 2|\rho| \sin(\phi) E\{n_{ai}\}. \quad (17)$$

Introducing the moments indicated in (14) into (17)

$$E\{|\hat{\rho}_{MLT}^2\} = |\rho|^2 + (1 - |\rho|^2)^{3.2} \quad (18)$$

As it can be deduced from (18), the coherence parameter is overestimated due to the presence of the complex additive speckle noise term $n_{aw} + jn_{ai}$. The behavior of this bias corresponds with the observations of other authors, in such a way that the bias increases with decreasing coherences and it decreases when the number of averaged samples $N$ is increased [3].

As it has been mentioned, (18) has been derived for the square value of the coherence. This expression could be also considered to correct the value of the coherence itself. Nevertheless, the expression (18) can not be employed for this purpose, especially for low coherence values, since the square root operation has to be applied to obtain the coherence value. The reason of this drawback has to be found in the fact that due to the variance present in the data, a correction based in (18) would lead to negative values, which the square root operation can not handle.

As a result, the sole way to reduce this coherence bias would be to increase the number of averaged samples $MN$. The first drawback of this approach is the clear loss of spatial resolution with respect to the original SAR images. The second problem is that for large areas, one can not longer assume the homogeneity condition to estimate coherence.

IV. MULTiresolution COherence ESTIMATION
A demonstrated, the model presented in (13) has allowed determining that the low coherence bias of the coherence parameter is basically induced by the zero-mean, complex additive speckle noise term. Nevertheless, a preliminary study of this model has not offered a feasible algorithm to correct this bias term. Hence, the bias must be reduced by increasing the
number of averaged samples $MN$, at the expense of spatial resolution and the risk to average non homogeneous samples. Multiresolution signal processing techniques offer the possibility to face this trade-off. These techniques permit to estimate the useful signal component by means of large windows while reducing to the minimum the spatial resolution losses. There exist different techniques to perform a multiresolution analysis of a given signal. In what it follows, the wavelet transform is considered. We direct the reader to [6] for a detailed presentation of this tool.

A. Coherence Estimation Based on the Wavelet Transform

In this section a novel approach to estimate the coherence parameter, based on the wavelet transform is proposed.

As observed in (7) and Fig. 1, the parameter $N_c$ contains the same information as the coherence parameter. Then, $N_c$ will be estimated instead of $|\rho|$ since a simpler signal noise model can be considered. Based on the same arguments employed to derived the coherence model (13), one can demonstrate that if only the phase component of the complex Hermitian product $\phi$ is considered, it is possible to write [7]

$$e^{i\phi} = N_c e^{i\phi} + (v + jv)$$

(19)

where

$$E\{v\} = E\{v\} = 0$$

$$\text{var}\{v\} = \text{var}\{v\} = \frac{1}{2} \left(1 - |\rho|^2\right)^{0.685}.$$ (20)

As it can be seen, the speckle noise model of the complex phase component (20) is almost equivalent to the speckle noise model of the complex Hermitian product $S_jS^*_j$.

The coherence estimation algorithm is based on the 2D wavelet transform of the real and imaginary parts of (19). In the wavelet domain, it is possible to demonstrate that the amplitude of a wavelet coefficient corresponds to

$$E\left[|\text{DWT}_{2D} \exp (j\phi)|^2\right] = 2^i N_c^2 + \text{var}\{v\} + \text{var}\{v\}$$

(21)

where $i$ denotes the wavelet scales. Consequently, for $i$ high enough, the amplitude of the wavelet coefficients contains directly the correlation information. Therefore, the proposed algorithm detects those coefficients containing useful signal and multiplies the real and imaginary parts by 2, for every wavelet scale being inversely transformed. The final effect of this process is that the weight of those wavelet coefficients containing useful signal is increased with respect to noise. We direct the authors to [7] for a more detailed description of this approach.

V. RESULTS

The previous algorithm has been tested over an ESAR-DLR X-band interferogram of Mt. Etna (Italy). In this case, the coherence is calculated by means of (2), (10) and the previous algorithm based on the 2D wavelet transform. Results are shown in Fig. 2. In Fig. 2b, it can be observed that the proposed algorithm is able to estimate very low coherences (lower part of the interferogram) thanks to the selection of those coefficients which do not contain signal. Additionally, from Figs. 2c and 2d, it can be deduced that the wavelet transform allows mitigating the effect of the bias due to topography without the necessity to compensate for it.

VI. CONCLUSIONS

A novel speckle noise model for the coherence parameter has been derived. It is demonstrated that the coherence bias for low coherences is induced by an additive component of speckle noise. A novel approach to estimate coherence, based on the wavelet transform, is proposed. This algorithm allows mitigating the effects of the bias for low coherences, as well as the bias due to topography.

REFERENCES