INTERACTIVE SIMULATION OF SHIP MOTIONS IN RANDOM SEAS BASED ON REAL WAVE SPECTRA

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Abstract: The current paper presents a methodology to compute and represent the ship motions at interactive frame rates, when navigating on a virtual sea defined by a real wave spectrum. The Inverse Discrete Fast Fourier Transform algorithm is used to compute the sum of all ship motions components induced by the waves and get the final irregular ship motion. Currently, this is the most efficient method to simulate the ship motions in interactive ocean simulations using wave spectra to describe the sea states. The visual realism and physical accuracy of the simulations turns this method into a powerful tool for developing interactive ship simulators.

1 INTRODUCTION

Physically based ship motions simulation is a fundamental task for ship simulators used as training or as studying tools. In order to immerse the user in the Virtual Environment as much as possible, these tools require high levels of visual realism and physical accuracy at interactive frame rates. In fact, discrepancies between the sea state and the induced ship motions, even if small, are immediately detected by the user and the sense of immersion decreases substantially. Moreover, if manoeuvrability operations are to be studied with the simulator, it is fundamental to preserve the physical accuracy to obtain valid conclusions.

The method presented computes and simulates interactively the ship motions induced by a random sea surface based on a real wave spectrum. As the surface may be deformed by thousands of wave trains with different (small) amplitudes, frequencies and directions of propagation, the method uses the Inverse Discrete Fast Fourier Transform (IDFFT) algorithm to compute the final ship motion, which is given by the sum of all the singular motions induced by each wave component. As a result, the method allows simulating the ship motions in a virtual sea surface that can be deformed by a large number of waves increasing not only the visual realism but also the physical accuracy of the simulation. By establishing and maintaining the synchronism between the phases of wave trains and ship motions, the method allows simulating the motions for any ship position, speed and heading. In order to test the method, an ocean generator software system was developed, which creates virtual sea surfaces based on the random sea concept. Then, transfer functions of a LNG ship, which describe how the ship reacts to forces generated by wave trains with different properties, were used to apply the method for different sea states and manoeuvrability conditions. Good results were obtained from the performance and realism points of view in which the ship motions are coherent and realistic even for stormy seas composed by thousands of wave trains.

The remaining of the paper develops as follows: section 2 provides some background on the sea surface generation and on the consequently induced ship motions computation; section 3 describes the method used to compute and simulate the ship motions; section 4 presents implementation details of the method in a prototype software system; section 5 discusses the results; section 6 presents the methodology used to validate the results and finally the main conclusions are presented in section 7.

2 BACKGROUND

Ship motions are represented by six main components corresponding to the six degrees of
freedom in the 3D space, as presented in figure 1.

Figure 1: Ship motion is divided into six components in the six degrees of freedom.

Surge, sway and heave correspond to the translation motions along the $x$, $y$ and $z$ axis respectively, while roll, pitch and yaw are the rotation motions also in the $x$, $y$ and $z$ respectively.

Most of the ship motion simulators running on desktop computers do not actually compute the motions in real-time. Depending of the purpose of the simulator, ship motions may be computed previously, and the Virtual Environment (VE) works as a post-processing visualization and interaction tool for the simulation. Databases store the ship motions’ parameters for different wave patterns, and the data is then queried (in database) and applied in real-time. It is the case of Daqaq (2003), in which the ship motion must be realistic in order to test loading and unloading operations.

Simulation models that compute ship motions in real time such as Xiufeng et al. (2004) or Sutulo et al. (2002), normally do not compute wave induced motions and only the three degrees of freedom corresponding to the surge, sway and yaw in the manoeuvrability equations are considered. Such tools are used to simulate port operations where the motions induced by the sea waves (heave, roll and pitch) are negligible. Although three of the ship motions are not computed, these simulators include manoeuvrability models to calculate ship trajectories that consider the interaction between the water, the sea bottom (shallow waters) and coastal structures (walls, berths, etc.), which increases the computational work required, Sutulo et al. (2010).

In an attempt to compute the six motion components in real time, simplified models such as Ueng et al. (2008) have been developed. They use a moving grid attached to the ship generated by the vertical projection of the ship’s body into the horizontal plane. The bounding box of the projection is used and a uniform grid of cells is superposed on the bounding box. Then the height field of the sea surface is evaluated at each grid point and the average height field is multiplied by the area of the grid to compute the excitation force of water. For each ship motion component, resistance forces are estimated and the net force is obtained by subtraction. Accelerations are then calculated by applying the Newton’s second law.

Another approach to estimate the ship motions induced by sea waves is by computing commonly called Response Amplitude Operators (RAOs). In linear theory, ship motions are calculated as the finite sum of sinusoidal components with a random initial phase, Bhattacharyya (1978). The amplitude of each of the components is given by RAOs that define the relationship between ship motion and wave height versus regular wave frequency, Lewis (1988). RAOs may be obtained by wave tank experiments with scale models of the ships or may be computed by specialized software. The strip theory is the common approach to compute the RAOs of the ships for the six degrees of freedom, Salvesen et al. (1970). This theory assumes the ship as a slender body and includes ship motion coefficients such as added mass and damping in heave and pitch motions. Numerous comparison studies showed that strip theory generally gives good results for ship motions in low to moderate regular waves in which the influence of nonlinearities is still low. However, Fonseca and Guedes Soares (1998) presented a generalization of the theory to deal with large amplitude motions.

Pre-calculated RAOs, which are defined for each particular ship hull with specific cargo conditions, allows to compute the amplitudes of each ship motion component in frequency domain for predefined manoeuvrability conditions (ship speed and heading) and sea states (frequency and direction of propagation of the wave trains). The conversion of the ship motion component from frequency to time domain, which is the one of interest for the interactive simulation, takes into consideration the phase of each wave when it reaches the ship. In order to achieve a consistent and physically correct motion in time domain, the phase of each ship motion component and the phase of the corresponding wave train must maintain the relationship given by the RAO. The final motion is given by the sum of all the ship motions generated by all the wave trains.

Each ship motion component is given as a periodic function in time of the following type:

$$x_i = a_i \sin(\omega_i t + \phi_i)$$  \hspace{1cm} (1)

where $i$ corresponds to one of the motion types, $x_i$ is the ship motion of type $i$, $a_i$ is the amplitude of the motion derived by the RAO, $\omega_i$ is the frequency of
the motion, \( t \) is the time and \( \varphi \) is the phase difference between the ship motion and the wave train motion. This last value is also pre-computed by transfer functions that depend of the ship hydrodynamic properties. Naturally, these RAOs are only computed for a certain number of ship speeds and headings, and for a limited range of wave trains’ frequencies and directions of propagation. Hence, for intermediate values interpolations are normally applied. Examples of comparison of numerically simulated motions with measurements can be found in Fonseca and Guedes Soares (2002).

As the ship motions are mainly induced by ocean waves, the simulation of the sea state must be also realistic from the physical point of view. Currently, interactive simulations of ocean waves for Virtual Environments apply the concept of random seas based on wave spectra. This method allows creating a statistically valid irregular sea surface with very good visual appearance that may run at interactive frame rates with modern CPUs.

Random seas are composed by a large set of wave trains whose properties, namely the amplitude, frequency and direction of propagation, are taken from the discretization of a directional wave spectrum. Wave spectra are estimated from measurements of the surface elevation on ocean. The signal in time domain is converted to frequency domain by applying the Discrete Fourier Transforms (DFT). Therefore, in order to convert back the signals to time domain, Inverse Discrete Fourier Transforms (IDFT) are applied to the wave trains defined by the spectrum.

In random seas simulations, each wave train obtained from the spectrum discretization, imposes a small deformation into the sea surface. The final deformation is given by the sum of all the components. The main drawback of this procedure is that for every vertex of the surface the elevation of the sea must be evaluated by the sum of the deformations imposed by all the wave trains at that vertex. As the number of waves must be quite large in order to achieve a statistically valid and visually realistic simulation, the calculation easily becomes unsuitable for interactive simulations. The solution adopted currently is to use the IDFFT algorithm, which is an optimized form of the IDFT to compute the sum of all the deformations on every vertices of the sea surface. This solution imposes some restrictions to the spectrum discretization and to the distribution of the surface vertices in space. It is out of the scope of this work to go deeply on this issue, however, the relevant aspects that result from use of the IDFFT algorithm to generate and simulate the sea surface are the following:

- The sea surface is a regular grid with \( N \times M \) vertices (normally with \( N=M \), where \( N \) and \( M \) are power of 2.
- Wave vectors represented by the sea surface depend of the location of each vertex of the grid. Since the central vertex of the grid is located at \( x=(0,0) \), there are wave trains propagating in all the directions with different lengths. The wave number is given by the length of the wave vector and therefore there are also wave trains with different lengths. Consequently, by the relation of dispersion, wave trains also have different frequencies.
- From the previous point, the maximum number of represented wave trains is equal to \( N \times M-1 \). The only wave that cannot be represented is the one derived from the central vertex because its wave number is equal to zero.
- Although all the waves defined by the grid vertices may be represented, in most of the cases there is a large number in which the amplitude derived from the spectrum discretization is equal to zero.


### 3 COMPUTATION OF THE SHIP MOTIONS

#### 3.1 The RAO Operator

The computation of the ship motions in time domain is intrinsically related with the wave trains represented by the ocean grid in the sense that each wave train generates six components of the ship motion given by equation 1. The relation between the properties of each function is obtained by the RAOs of the ship.

Let \( h(\omega, \varphi, t) \) be the function describing the surface elevation at time instant \( t \) due to the wave train \( i \), and \( m(t) \) the value of the generated ship motion function also at instant time \( t \). Each component \( j \) of the ship motion \( m(t) \) is given by:
\[ m_j(t) = RAO_j[h_i(a_h, \omega_h, \phi_h, t)] \] (2)

where the operator \( RAO_j \) specifies the relation between the values of the amplitudes, \( a_h \), frequencies, \( \omega_h \), and initial phases, \( \phi_h \), of the wave train function and the homologous values of the component \( j \) of the generated ship motion.

In linear theory, the ship motion generated by each wave train is also described by a sinusoidal function with the same frequency but with different amplitude and phase as shown in figure 2.

![Figure 2: Relation between the ship motion component and the surface elevation for a single sinusoidal wave in linear theory.](image)

The frequency of the ship motion is the same as the frequency of the wave train, so equation 2 can then be written in its sinusoidal form:

\[ m_j(t) = A_j a_h \sin(\omega_h t + \phi_h + \Delta\phi_j) \] (3)

where \( A_j \) and \( \Delta\phi_j \) are the pre-calculated values specified by \( RAO_j \) for different ship speeds and angles, and for different frequencies of the wave trains.

For a specific ship heading, the encounter angle used to find the correct \( RAO_j \) must take into consideration the direction of propagation of the wave train as presented in figure 3, Guedes Soares (1995).

The encounter angle is given by:

\[ \gamma = \beta - \alpha \] (4)

where \( \alpha \) is the ship heading and \( \beta \) the wave angle.

Due to the longitudinal symmetry of the hull of almost all the existent vessels, encounter angles specified in the RAOs are only applied to portside and therefore they are defined in the interval \([0, \pi]\). On the other hand, wave angles and ship headings are defined in the interval \([0, 2\pi]\) and therefore if the encounter angle lies between \( \pi \) and \( 2\pi \), which is the example case of figure 3, it must be corrected by the expression:

\[ \gamma_c = \gamma - 2(\gamma - \pi) \] (5)

Having the current ship speed and the corrected encounter angle, the values of \( A_j \) and \( \Delta\phi_j \) can be easily derived from the RAOs tables.

![Figure 3: The encounter angle is given by the difference between the wave angle and the ship heading.](image)

### 3.2 Interpolations between RAO Values

As mentioned, RAO values are specified for specific vessel speeds, encounter angles and wave frequencies. However, in a generic simulation, the manoeuvrability conditions of the ship (speed and heading) or the sea state do not necessarily (and most of the time they do not) coincide with the specified RAO values. For such cases, interpolations must be performed in order to find the correct RAO values for the current ship and sea states.

Figure 4 represents schematically the data contained in the RAOs tables from which the simulation reads the values for the motion amplitudes and phases depending of the current status of the ship and the sea state.

When the manoeuvrability values are between the specified ones, linear interpolations are performed to get the real amplitudes and phase angles. In order to compute by interpolation the final values \( A_j \) and \( \Delta\phi_j \) for each motion \( mj \), the following arrays are stored in memory:

- The speed table, \( s_{Tb} \), contains \( N \) lines and two columns. Each line corresponds to a ship speed computed in pre-calculations. The first
column is filled with the speeds in which \( s_{\text{Tb}[0,0]} = 0.0 \) and \( s_{\text{Tb}[N,0]} \) is the maximum speed of the ship. The second column is associated to an encounter angle table for each ship speed.

- The encounter angle table, \( \gamma_{\text{Tb}} \), contains \( M \) lines and two columns. Each line corresponds to the encounter angles in which \( \gamma_{\text{Tb}[0,0]} = 0.0^\circ \) and \( \gamma_{\text{Tb}[M,0]} = 180.0^\circ \). The second column is associated to a wave frequencies table for each encounter frequency.

- The wave frequencies table, \( \omega_{\text{Tb}} \), contains \( P \) lines and two columns. Each line corresponds to the frequencies computed in pre-calculations. The first column is filled with the frequencies in which \( \omega_{\text{Tb}[0,0]} \) is the minimum wave frequency and \( \omega_{\text{Tb}[P,0]} \) is the maximum wave frequency computed. The second column is associated to the motions table for each frequency.

- Finally the motions table, \( m_{\text{Tb}} \) contains 6 lines and 2 columns. Each line corresponds to one of the six components of the motion. The first column stores the amplitude ratio, \( A_j \), and the second column the phase difference, \( \Delta \phi_j \), values for the motion.

Let \( s \) and \( \gamma \) be the ship’s speed and encounter angle respectively at time \( t \), in a wave train of frequency \( \omega \). The values of speeds, encounter angles and frequencies specified in the RAOs tables that will be of interest to perform the necessary interpolations to compute the final parameters \( A \) and \( \Delta \phi \) of the ship motion component are:

\[
 s_i = s_{\text{Tb}[n+i,0]} \quad i = 0,1
\]

where, \( n \) is the index corresponding to the speed immediately below \( s \) in the speed table.

\[
 \gamma_j = \gamma_{\text{Tb}[m+j,0]} \quad i = 0,1; j = 0,1
\]

where, \( m \) is the index corresponding to the encounter angle immediately below \( \gamma \) in the encounter angle tables and \( \gamma_{\text{Tb}[n+i,1]} \).

\[
 \omega_{jk} = \omega_{\text{Tb}[p+k,0]} \quad i = 0,1; j = 0,1; k = 0,1
\]

The speed factor \( s_{\text{fact}} \in [0,1] \) is computed by the expression:

\[
 s_{\text{fact}} = \frac{s - s_{\text{Tb}[n,0]}}{s_{\text{Tb}[n+1,0]} - s_{\text{Tb}[n,0]}} \tag{9}
\]

The heading factor \( \gamma_{\text{fact}} \in [0,\pi] \) is computed by:

\[
 \gamma_{\text{fact}} = \frac{\gamma - \gamma_{\text{Tb}[m,0]}}{\gamma_{\text{Tb}[m+1,0]} - \gamma_{\text{Tb}[m,0]}} \tag{10}
\]

The frequency factor \( \omega_{\text{fact}} \in [\omega_{\text{min}}, \omega_{\text{max}}] \) is computed by:

\[
 \omega_{\text{fact}} = \frac{\omega - \omega_{\text{Tb}[p,0]}}{\omega_{\text{Tb}[p+1,0]} - \omega_{\text{Tb}[p,0]}} \tag{11}
\]

Finally, being \( m \) the value of \( A \) or \( \Delta \phi \) specified in the motions table for a specific component the ship motion, \( m \) is calculated by the following expression:

\[
 m = m_0 + s_{\text{fact}}(m_1 - m_0) \tag{12}
\]

where the following equations apply:

\[
 m_j = m_0 + \gamma_{\text{fact}}(m_{1j} - m_0) \tag{13}
\]

\[
 m_j = m_0 + \omega_{\text{fact}}(m_{1j} - m_0) \tag{14}
\]
\[ m_{ijk} = \omega \cdot T b_{ij} [p + k, r], \quad r = 1, 2 \tag{15} \]

For \( r = 1 \) (second column) the corresponding \( A \) value is computed while for \( r = 2 \) (third column) the value of \( \Delta \phi \) is calculated.

As a simplification, the centre of rotation of the ship is the centre of buoyancy and remains unchanged during the simulation.

### 3.3 The FFT Algorithm

After computing the values of \( A \) and \( \Delta \phi \) for all the ship motions, evaluating at interactive frame rates the six components of the final ship motion at instant time \( t \) as the sum of all the motions generated by all the wave trains is only possible using the IDFFT algorithm. As an example for an ocean grid with 128x128 vertices and therefore 16383 wave trains (applying the random sea model mentioned in section 2), a total of \( 6 \times 16383 = 98304 \) ship motions may have to be evaluated every frame. Taking into consideration that each ship motion is described by a circular sinusoidal function, this sum can rapidly lead to unacceptable frame rates.

The IDFFT algorithm provides an efficient way to compute the following sum:

\[
\sum_{n=0}^{N-1} \sum_{m=0}^{M-1} F_{n,m} e^{i 2 \pi \frac{np}{N} \frac{mq}{M}} \tag{16}
\]

where \( F_{n,m} \) is a set of complex numbers uniformly distributed and \( p \in \{1, \ldots, N\} \) and \( q \in \{1, \ldots, M\} \).

From the application of the IDFFT algorithm to the random sea model, the elevation of the water surface as the sum of the elevations generated by all the wave trains at a grid point \( g_{p,q} = (p \cdot N/2, q \cdot M/2) \), is given by:

\[
h(g_{p,q}, t) = 2(-1)^{p+q} \text{Re} \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} a_n e^{i (\alpha_0 t + \omega_h t + \phi_h)} e^{\frac{2 \pi}{N} (np + mq)} \tag{17}
\]

where \( a_n \) and \( \phi_h \) are also evaluated for the wave train corresponding to the grid point \( g_{p,q} \).

If the position of the ship is added to equation 3, each ship motion is described in complex notation by the following expression:

\[
m_i(t) = A_i a_n e^{i (\alpha_0 t + \omega_h t + \phi_h + \Delta \phi_i)} \tag{18}
\]

where \( \phi_s \) is the phase term which depends on the ship position in the grid.

Therefore, from equation 17 and 18, the ship motion at the grid point \( x_{p,q} = (p \cdot N/2, q \cdot M/2) \), is given by:

\[
m_i(g_{p,q}, t) = 2(-1)^{p+q} \text{Re} \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} A_i a_n e^{i (\alpha_0 t + \omega_h t + \phi_h + \Delta \phi_i)} e^{\frac{2 \pi}{N} (np + mq)} \tag{19}
\]

Equation 19 has the form of equation 16 and therefore the IDFFT algorithm can be used to compute the sum of the ship motions at each grid vertex. In equation 19 the parameters \( A_i \) and \( \Delta \phi_i \) are computed from equation 12; \( a_n \), \( \omega_h \) and \( \phi_h \) are taken from the wave trains.

The value of \( \phi_s \) depends on the position and is given by:

\[
\phi_s = k_n \frac{||k||}{||k||} \cdot x_s \tag{20}
\]

where \( k_n \) is the wave number of the wave train, \( ||k|| \) is the normalized direction of propagation vector of the wave train and \( x_s \) is the ship position in the grid.

### 3.4 The Ship Position in the Grid

The term \( \phi_s \) appears because the ship most probably will not be positioned on the grid vertices which are the locations where the motions are computed. Therefore an additional phase term must be added. It depends on the ship position in the grid, \( x_s \), namely the position relatively to the vertex where the ship motion is computed.

Figure 5: The ship position in the grid \( x_s \) is required to compute the phase term \( \phi_s \) of the ship motion.
If the ship is in the position $p$, as shown in figure 5, then the ship motions are evaluated at vertex $v_{p,q}$ and the vector $x_s$ is used to compute the phase term $\phi_s$.

4 IMPLEMENTATION

A prototype system was developed to test the method described. The system generates a virtual random sea based on the parametrical wave spectrum of JONSWAP type. The user selects the size and the resolution of the ocean grid and then applies the spectrum to the grid generating a virtual ocean. The characteristics of the spectrum are also defined by the user, namely the wind speed, fetch and main direction of propagation. The first two parameters allow generating calm or rough seas with more or less predominance of swell waves.

After the ocean is created, the system loads a ship with a geometric definition and an associated physical behaviour described in ASCII files. The RAO arrays are created from the data contained in the files.

Figure 6 presents the UML class diagram of the ship. The class Ship is a SceneNode with a geometrical representation located and oriented in the tri-dimensional space. It points to a number of ship motions equal to the number of wave trains in the virtual ocean. Each ship motion has a pointer to its corresponding wave train and uses the information stored in the RAO tables, combined with the properties of the wave train, to compute its amplitude, frequency and initial phase. This last depends of the phase difference specified in the RAO tables and of the position of the ship in the grid.

As can be seen, the simulation uses only one time, which is stored by the ocean object. All the other objects use this simulation time to perform the calculations.

Figure 7 presents the UML Activity Diagram for a single frame.

The first task that must be performed before any other operation take place, is the update of the simulation time because its value is used on both the ocean and ship operations. After that, activities performed in the ocean and in the ship may run in parallel because they are independent.

The updated time allows the ocean to calculate for every wave train the elevation on each vertex of the grid. Then the FFT algorithm is used to compute the sum of all the elevations of every point of the grid and finally all the vertices are updated to new positions. Update the ship motions consists in performing the interpolations in the RAOs tables in order to compute the amplitude and phase difference for all the ship motions. Then, the ship is moved horizontally to its new position according to its speed and heading. The new location allows computing the ship position in the grid, which is necessary to calculate the value of $\phi_s$ in equation 20. With all the ship motions’ parameters calculated, the FFT algorithm is then applied to compute the sum of all the ship motions in the point where the ship stands. Finally the position and orientation of the ship are again updated to the final status.

The system was developed on the top of the OGRE3D (Object-oriented Graphics Rendering
Engine - http://www.ogre3d.org/) and the FFT calculations were performed using the FFTW library (Fastest Fourier Transform in the West - http://www.fftw.org/).

5 RESULTS

Simulation tests were conducted in a Pentium(R) Dual Core CPU E5200 @ 2.50GHz (2 CPUs) with 3070MB RAM. The graphics card was a NVIDIA GeForce 9600 GT with 1024.0 MB RAM.

Figure 8 presents screen captures of a simulation with the characteristics specified in table 1.

Table 1: Simulation parameters.

<table>
<thead>
<tr>
<th>Sea state</th>
<th>Beaufort wind scale</th>
<th>Significant wave height</th>
<th>Fetch</th>
<th>Peak frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>9</td>
<td>9.0 m</td>
<td>367.6 km</td>
<td>0.5 rad/s</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ocean grid</th>
<th>Resolution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>256x256</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Nº of waves (a&gt;0.0m)</th>
<th>18829</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Ship</th>
<th>Type</th>
<th>Length overall</th>
<th>Breadth</th>
<th>Draught</th>
<th>Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LNG</td>
<td>290.0 m</td>
<td>46.0 m</td>
<td>11.4 m</td>
<td>19.8 kn</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For the simulation presented in figure 8, waves are mainly coming from portside and stern. In figure 8a) it can be seen the irregular appearance of the sea surface due to the high number of represented waves. The graph of figure 8 represents the sea surface elevation and the three most visible motion components of the ship along time. Three screen captures are presented in different instants of the simulation in which one of the motion components reach high values. In figure 8b) the ship is trimmed with the stern raised and part of the rudder out of the water. Figure 8c) emphases the roll motion to portside and finally figure 8d) shows the difference between the heave and the surface elevation at amidships.

From the performance point of view, a frame rate of approximately 9 fps was achieved in the simulation of table 1. It is still below the necessary frame rate required for interactive applications. However, if the resolution of the ocean grid to is reduced to 128x128, the frame rate increases to 26 fps which is already an acceptable value. Therefore and as expected, the grid resolution is a determinant factor for the performance of the simulation. Figure 9 presents a screen capture of the same sea state but with an ocean resolution of 128x128 with the same side length.

Although it is clear that the grid resolution of 256x256 presents more realistic results of the ocean, because the number of waves is significantly higher,
the 128x128 grid does not stay far behind.

However, it is expectable that interactive frame rates can be achieved with 256x256 grid resolutions using a more powerful CPU already available at affordable prices.

All the calculations were preformed on the CPU. However, studies like Moreland and Angel (2003) allow computing the FFT on the GPU. As the sea surface height field and the ship motions numerical simulations are heavily based on the FFT, it is expected that the frame rate increases substantially if these calculations are performed on the GPU.

Using these features, several tests were performed with different wave trains, for specific components of the ship motion and for different speeds and encounter angles.

Although it is difficult to represent the validation tests (which are mainly based on animations) in the paper, we present screen captures and ship motions graphs in figures 10 and 11 for some of the simulations performed.

The wave train used for validation purposes had the lowest frequency of all the waves represented by the ocean grid. An amplitude of 10.0 meters was artificially assigned in order to increase and observe the ship motions clearly. For lower frequency wave trains, heave motions are almost synchronized with the surface deformation at amidships. This can be verified by the graph presented in figure 10.

The pitch motion has a phase difference of nearly 90°, which also validates the results. In fact when the surface elevation is minimum (at the wave trough), for instance at \( t=86 \text{s} \), the pitch angle is around 0.0°. This is the moment when the surface elevation is nearly the same at the stem and at the stern and that is why there is no pitch angle. When the surface elevation increases at amidships, the difference between elevations at the stem and stern also increases the pitch angle decreases leading the stem to rise (see figure 1). The minimum value of pitch occurs when the elevation is nearly zero at amidships, which is when the ship is half way between the trough and the crest of the wave.

With low frequency wave trains coming from starboard, the sway and roll motions present similar behaviour to the pitch motion as shown in figure 11.

As expected, both graphs in figures 10 and 11 have the same configuration of the graph presented in figure 2 in which the frequency of the motions and the surface elevation is the same, and there is a phase difference given by the RAOs of the ship.

6 VALIDATION

When the virtual sea state is composed by thousands of wave trains with different amplitudes, frequencies and directions of propagation, it is impossible to verify if the ship motions are absolutely coherent with the surface deformation.

Therefore some features were added to the system to artificially change the sea state and consequently the ship motions, but maintaining the same method of computation.

The first feature was to reset to 0.0 all the amplitudes of the wave trains with the exception of one whose frequency and direction of propagation was (obviously) known. Then a feature to assign specific amplitude to this wave train was added. By doing this, the ship motions are computed using the method described for all the wave trains, but only one wave actually generates the ship motions. In such situation it was possible to verify if the ship motion was coherent with the deformation of the surface. The possibility of only representing specified components of the ship motion (surge, heave, roll, etc.) was also added. Therefore the results of each separated component when subjected to a single wave train were analysed.
Figure 11: Ship subjected to a single wave train coming from starboard with amplitude of 10.0 m, frequency equal to 0.310 rad/s and wavelength of 640 m.

7 CONCLUSIONS

A method to simulate the ship motions in the six degrees of freedom at interactive frame rates in an irregular sea state based on a real wave spectrum was presented. In order to achieve interactive performance, the IDFFT algorithm must be used to compute the sum of all the ship motions generated by the wave trains. Even so, the maximum resolution allowed with the hardware available today is 256x256 if the calculations are performed in the CPU. The ship motions obtained with this method are visually coherent with the irregular sea state simulated; however, it was necessary to validate the results by applying the method for situations in which the ship is subjected to a single wave train of low frequency.

This method can be applied in simulation tools that require visual a physical realism of the sea and the ship motions at interactive frame rates such as ship simulators.

REFERENCES
