Abstract—The insertion of atypical solutions (immigrants) in Evolutionary Algorithms populations is a well studied and successful strategy to cope with the difficulties of tracking optima in dynamic environments in single-objective optimization. This paper studies a probabilistic model, suggesting that centroid-based diversity measures can mislead the search towards optima, and presents an extended taxonomy of immigration schemes, from which three immigrants strategies are generalized and integrated into NSGA2 for Dynamic Multiobjective Optimization (DMO). The correlation between two diversity indicators and hypervolume is analyzed in order to assess the influence of the diversity generated by the immigration schemes in the evolution of non-dominated solutions sets on distinct continuous DMO problems under different levels of severity and periodicity of change. Furthermore, the proposed immigration schemes are ranked in terms of the observed offline hypervolume indicator.

Index Terms—Diversity generation, immigrants schemes, dynamic multiobjective optimization, evolutionary computation.

I. INTRODUCTION

Diversity loss not only can lead to premature convergence in the stationary phases of an optimization problem, but can also reduce the suitability of the candidate solutions population in dynamic environments, wherein the objective functions are subject to changes over time. In this context, the success of the recent advances in the design of Evolutionary Algorithms (EAs) for dynamic optimization has motivated the development of efficient diversity maintenance, generation, and control mechanisms e.g. [1], [2], [3].

In Multiobjective EAs (MOEAs), diversity maintenance promotes the exploration of different search paths leading to the trade-off surface, in which it is not possible to simultaneously improve all objective functions without deteriorating at least one value of those functions. Thus, the trade-off surface – the Pareto Front (PF) – should be approximated with the maximum possible diversity of points by a finite set of non-dominated solutions. Although the variation operators and the diversity maintenance strategies based on crowding distance and niching are commonly used to promote the dispersion of individuals in MOEAs, we argue that diversity loss cannot be avoided by such techniques. We then consider the insertion of atypical solutions (immigrants) [2] to improve the population adaptability to the changing environments in Dynamic Multiobjective Optimization (DMO). A generalization of the existing immigrants insertion strategies – originally defined for single-objective EAs in Cobb and Grefenstette [4] and applied in e.g. Yang [2] – is proposed, yielding three immigration schemes with respect to the dependence level with the evolving population: (i) uncorrelated; (ii) correlated; and (iii) a hybrid between the two schemes. The efficiency of each scheme on improving MOAEs performance on DMOs is assessed by experimentation in which the effects of the immigration rates and of the parameter controlling the balance between schemes (i) and (ii) in the hybrid strategy are investigated. For each scheme, a statistical correlation study between the various forms of measuring diversity and the quality of the non-dominated solutions set ($\mathcal{F}_1$) is carried out in order to determine the influence of the diversity generated by each immigration schemes on the evolution dynamics of $\mathcal{F}_1$.

The paper is organized as follows: Section II discusses a model proposed to characterize the role of immigration schemes, from which we may derive the expected genetic diversity, provided that the variances of the subpopulations are known. An extended taxonomy of immigrants insertion based on that model is presented in Sect. III, so is the related work about immigration schemes in single-objective EAs. In Section IV, we generalize the immigrants schemes for DMO and we explain the integration of the proposed diversity generators into the NSGA2 algorithm. Section V presents the DMO problems, performance indicators, and settings used in the experimental study, while Sections VI and VII discusses results and provides conclusions and future works, respectively.

II. IMMIGRATION SCHEMES

In biological populations, immigration schemes can increase genetic transmission, promoting alternative evolutionary paths. Biological immigration schemes are studied in e.g. Altrock et al. [5], in which diversity maintenance is demonstrated in terms of critical immigration rates. The purpose of im-
migration schemes in EAs is to allow the insertion of new information into the existing genetic pool of the evolving population. In this way, in face of changes in the fitness landscape, the presence of an immigrants subpopulation makes it more likely for the population to adapt itself to the new environment. One of the first EAs to employ such a scheme was the Random Immigrants Genetic Algorithm (RIGA) [4]. In RIGA, \( K = \alpha \times N \) immigrants (\( 0 \leq \alpha \leq 1 \)) is the immigration rate and \( N \) is the population size, are uniformly sampled from

\[
Y \sim \text{Uniform} \left[ (x_1^{\inf}, x_1^{\sup}) \times \cdots \times (x_m^{\inf}, x_m^{\sup}) \right], \tag{1}
\]

replacing the \( K \) worst individuals of the population.

A. A Probabilistic Model of Local Populations

Based on Altrock et al. model (Fig. 1), we formulate a probabilistic model with the purpose of characterizing the role of immigration schemes in EAs and of devising an extended immigrants insertion taxonomy. In Fig. 1, \( X \) and \( Y \) represent two local populations subjected to asymmetric (if \( \alpha_1 \neq \alpha_2 \)) immigration influxes, where \( \alpha_1, \alpha_2 \in [0, 1] \).

Without loss of generality, let \( Y \) and \( Z \) be two continuous random variables (r.v.) over the real line. Then, the proposed immigrants model is

\[
X = \begin{cases} 
Y, & \text{with probability } \alpha \\
Z, & \text{with probability } 1 - \alpha. 
\end{cases} \tag{2}
\]

Denote this model by \( X(\alpha) \), and let it represent the distribution of candidate solutions in a given generation. Then, \( Y \) is dubbed the immigrants subpopulation, \( Z \) is the current subpopulation (the evolving subpopulation), and \( \alpha \) is the immigration rate. In regard to the Altrock et al. model (Fig. 1), \( \alpha \equiv \alpha_2 \) and \( \alpha_1 = 0 \), i.e., we consider only the case of one-way immigration influxes. Among the descriptive advantages of the proposed model, we point out that it is possible to obtain the expected genetic diversity of \( X(\alpha) \) from Eq. 2, provided that the variances of both subpopulations are known. We consider the case where \( Y \) and \( Z \) are statistically independent.

In the computation of the centroid, \( C_X(\alpha) \), we have

\[
C_X(\alpha) = E \left[ X(\alpha) \right] = \sum_{i=1}^{2} P \{ X(\alpha) = x_i \} x_i = \alpha Y + (1-\alpha) Z. \tag{3}
\]

Hence, since \( C_X(\alpha) \) is a convex combination between the r.v.’s \( Y \) and \( Z \), it is also a r.v. itself. Calculating the expected value and the variance of \( C_X(\alpha) \), we obtain

\[
E \left[ C_X(\alpha) \right] = E [ \alpha Y + (1 - \alpha) Z ] = \alpha E[Y] + (1 - \alpha) E[Z], \tag{4}
\]

\[
Var \left[ C_X(\alpha) \right] = Var \left[ \alpha Y + (1 - \alpha) Z \right] = \alpha^2 Var[Y] + (1 - \alpha)^2 Var[Z]. \tag{5}
\]

The calculation of the variance of \( X(\alpha) \) yields

\[
Var \left[ X(\alpha) \right] = E \left[ \left( X(\alpha) - E \left[ X(\alpha) \right] \right)^2 \right] = \alpha Y^2 + (1 - \alpha) Z^2 - (\alpha Y + (1 - \alpha) Z)^2, \tag{6}
\]

which is also a function of \( Y \) and \( Z \). Moreover, since \( C_X(\alpha) = E \left[ X(\alpha) \right] \), the uncertainty of \( C_X(\alpha) \) must be accounted, and hence, the expected genetic diversity of \( X(\alpha) \), \( I_X(\alpha) \), is expressed as

\[
I_X(\alpha) = E \left[ \text{Var} \left[ X(\alpha) \right] \right] = \text{Var} \left[ C_X(\alpha) \right] \tag{7}
\]

in which it can be shown that

\[
E \left[ \text{Var} \left[ X(\alpha) \right] \right] = \alpha (1 - \alpha) E \left[ (Y - Z)^2 \right]. \tag{8}
\]

An implication from Eq. 6 is the fact of \( E \left[ \text{Var} \left[ X(\alpha) \right] \right] \) being identical for the models \( X(\alpha) \) and \( X(1 - \alpha) \), with fixed

\[\text{Equation 5 has been validated by Monte Carlo simulation for several uniformly distributed and distinct subpopulations, obtaining an absolute error of } \approx 10^{-5} \text{ between } I_X(\alpha) \text{ and the sample variance after 10,000 trials.}\]
\[ Y \text{ and } Z, \text{ Moreover, in this case, when the immigration rates are complementary, it is observed from Eq. 4, that} \]

\[ \text{Var}[Y] = \text{Var}[Z] \Rightarrow \text{Var}[C_{X(\alpha)}] = \text{Var}[C_{X(1-\alpha)}], \]

and, thus, a sufficient condition for the expected genetic diversity of two complementary immigration schemes \( X(\alpha) \) and \( X(1-\alpha) \) with fixed \( Y \) and \( Z \) be identical is the equality of the subpopulation variances (Fig. 2), i.e.,

\[ \text{Var}[Y'] = \text{Var}[Z] \Rightarrow I_{X(\alpha)} = I_{X(1-\alpha)}, \]

what demonstrates the difficulty of distinguishing distributional symmetries in the search space through the computation of centroid-based genetic diversity measures such as Morrison and De Jong’s moment of inertia [6] (see Fig. 3).

The most notably implication of this illustrative example to evolutionary optimization is that centroid-based diversity and fitness are not a one-to-one map: if one seeks to characterize the interplay between diversity and the average quality of the set of candidate solutions, due to the inability of centroid-based metrics to distinguish between distributional symmetries which may occur in local population models, one should keep in mind that there are infinitely many scenarios in which the average fitness will greatly differ under the same approximated diversity levels. Together with the ability of assessing the expected genetic diversity, this result may provide insights on the design of competent diversity-guided evolutionary search and diversity control in local populations schemes.

### III. A Taxonomy and Related Work

The model in Eq. 2 is used to characterize different possible schemes. According to the proposed extended taxonomy, the immigrants insertion is (i) uncorrelated when \( Y \) and \( Z \) are statistically independent; (ii) correlated when \( Y \) presents statistical dependencies with respect to \( Z \); and (iii) hybrid when \( K \) immigrants subpopulations \( (Y_2, \cdots, Y_{K+1}) \) are added into the model, each of them with an associated immigration rate \( \alpha_2, \cdots, \alpha_{K+1} \), such that \( \alpha_1 + \sum_{j=2}^{K+1} \alpha_j = 1 \). The hybrid scheme requires that at least two immigrants subpopulation differ with respect to the presence of statistical dependencies in relation to \( Z \). It is worth mentioning that only the cases of two immigrants subpopulations have been studied in the literature. Following our taxonomy, we present an overview of the recent advances in immigrants schemes for EAs.

#### A. Uncorrelated Immigration Schemes

Despite the model from Eq. 2 be general enough to accommodate arbitrary probability density functions (pdf) in the immigrants generation process, the current uncorrelated immigrants approaches heavily utilizes the uniform pdf. For instance, in the population-based incremental model of Yang and Yao [7], a percentage of the worst individuals is replaced by random immigrants. The authors report that the effects of the random immigrants insertion in the studied GAs for dynamic optimization is “problem dependent” and show that “intermediate” levels for \( \alpha \) (about 0.4) produces the best results with statistical significance. It was also observed that such approach degrades the performance of the studied GAs in the stationary stages of a dynamic process.

#### B. Correlated Immigration Schemes

We identified two approaches under the correlated category, namely the elitism-based [2] and the memory-based [2], [8]. The goal of such schemes is to generate diversity in the previously explored regions of the search space. Both techniques keep an external archive of the best visited points to be used as the basis to generate new immigrants [9]. Hence, diversity generation is biased towards the regions in the vicinity of the best stored solutions. Yang [2] compares the performance of GAs utilizing the two abovementioned schemes. In both strategies, the immigrants are generated by a series of mutations of previous solutions. In the elitism-based, the best individuals from the previous optimization state are used in the immigrants generation step, whereas in the memory-based one, older solutions can also be considered. In the proposed taxonomy, we consider the elitism-based scheme as a special case of the memory-based one, in which the memory is updated every generation and the solutions are retrieved from the memory without replacement, so that only the previous elite is effectively used. Yang argues that the search is more effective when (i) the environmental changes are small (elitism); or (ii) when the environments are correlated with previous states of the evolutionary process (memory).

#### C. Hybrid Immigration Schemes

Cheng and Yang [10] presented novel immigration schemes to cope with infeasible solutions in combinatorial problems. Three strategies are described, applied whenever an environmental change is detected: (1) substituting the infeasible individuals with random immigrants; (2) repairing them to obtain feasible solutions, and updating the elite; or (3) with \( \frac{3}{2} \) probability, utilize strategy (1); else utilize strategy (2). They reported that the hybrid strategy (3) is most promising in face of smooth environmental changes, while strategy (2) performs better in abruptly changing environments.

### IV. Proposed Multiobjective Immigration Schemes

These are the essential questions one must ask when designing diversity generators:

1) **How to generate diversity?** – In the light of the immigrants generation taxonomy devised in Sect. III, we present the generalized Immigrants-based Diversity Generators (IDG) for dynamic multiobjective optimization (Definition 1), what allows for three possible strategies (see subsequent Sections).

**Definition 1 (Immigrants-based Diversity Generator):** Any sampling process over the search space capable of generating potentially distinct solutions is prompted as an IDG.

2) **When diversity levels should be increased?** – For the sake of simplicity, we adopt a “diversity maintenance” strategy [11], in which diversity is generated throughout the dynamic optimization process (i.e., every generation).

3) **How many solutions should be generated/discarded?** – The answer seems to be problem-dependent (see Sect. III-A).
The Correlated IDG (cIDG) extends the Elitism-based Immigrants GA (EIGA) strategy, investigated in e.g. Yang [2], to multiobjective optimization by using the NSGA2 partial order, $\preceq_{\text{ND}}$, based on non-dominance sorting and crowding distance in the following way: suppose we aim at generating $K$ correlated immigrants. In Pseudocode 1, firstly, the population ($\Psi$) is sorted using $\preceq_{\text{ND}}$ (line 1). Then, in line 3, if $K \geq |\mathcal{F}_1|$, we take the Elite as the first $K$ solutions of the ordered population, in which $|\mathcal{F}_1|$ is the cardinality of the non-dominated solutions set. Else, if $K < |\mathcal{F}_1|$, all non-dominated solutions are taken to be part of the Elite (line 5). Finally, the immigrants are generated by mutating randomly selected Elite individuals (lines 9 to 11), in which each Elite member may serve as the basis for generating one or more immigrants.

B. Uncorrelated IDG (uIDG)

The Uncorrelated IDG (uIDG), on the other hand, extends the RIGA (Sect. II) strategy to multiobjective optimization, in which K immigrants are generated by sampling from a multidimensional uniform pdf (Eq. 1) over the decision space.

C. Generalized IDG (gIDG)

The Generalized IDG (gIDG) scheme employs both cIDG and uIDG for generating $K$ immigrants, in which $\beta \in [0, 1]$ defines the proportion of the uncorrelated immigrants within the generated ones, i.e., $\beta = 1$ means that 100% of the $K$ immigrants are generated by the uIDG strategy, and so on.

D. Incorporating gIDG into NSGA2

Defined all IDG variants, there still the problem of choosing which solutions shall be replaced by the generated immigrants. We use the most common replacement operator for single-optimization: the replace-the-worst one. Hence, we define the $K$ worst individuals as the last $K$ ones found in the ordered MOEA population, using the $\preceq_{\text{ND}}$ operator (Sect. IV-A).

The incorporation of IDG into NSGA2 is accomplished by replacing the $K$ worst solutions with the generated immigrants after the application of the variation operators and before the survival selection step, so that the immigrants do not interfere in the offspring generation process, but compete with the newly generated solutions for survival. The notation NSGA2$\text{gIDG}(\alpha, \beta)$ is used to indicate the IDG variant we are referring to. From Tab. I, it can be noted that the choice of $\alpha$ and $\beta$ yields six functional behaviors to NSGA2, ranging from the standard implementation, to a random search strategy. It is worth noting that when $\alpha = 1$, all individuals are replaced by either random ($\beta = 1$) or elite-based ($\beta = 0$) immigrants.

V. EXPERIMENTAL DESIGN

We consider three two-objective Dynamic Multiobjective Optimization Problems, namely, FDA1, FDA2 [13] (Fig. 4), and a modified version of Goh and Tan’s dMOP3 [14] under four combinations between severity ($n_g$) and periodicity ($t_g$), $n_g,t_g \in \{5,10\}$, according to the temporal dynamics

$$t = \frac{1}{n_g} \left\lfloor \frac{g}{t_g} \right\rfloor,$$

in which $g$ is the current generation and $t$ is the environment time index. It should be noted that both FDA1 and dMOP3 are Type I problems, i.e., while they present a stationary PF, their Pareto Set ($\Omega^*$) – the values assumed by the decision variables which are mapped to the PF – does change over time. FDA2, on the other hand, is a Type III problem, i.e., $\Omega^*$ is stationary, but the PF shape changes over time. The dimensionality of the decision space is $m = 20$ for FDA1, $m = 30$ for FDA2, and $m = 10$ for dMOP3. For FDA1, we have chosen $X_I = \{x_1, \ldots, x_{10}\} \in [0,1]$, and $X_{II} = \{x_{11}, \ldots, x_{20}\} \in [-1,1]$, in which the first objective function is computed as $f_1(x) = \sum_{m=1}^{10} x_m$ and the second objective is computed as in Farina et al. [13]. As for FDA2, we have taken $X_I = \{x_1, \ldots, x_{7}\} \in [0,1]$, $X_{II} = \{x_8, \ldots, x_{13}\} \in [-1,1]$ and $X_{III} = \{x_{16}, \ldots, x_{30}\} \in [-1,1]$. Still, the following modifications were made, according to Farina et al. [13] guidelines and following Sola [15]: the first objective is defined as $f_1(x) = 1 + \sum_{m=1}^{7} x_m$ and the auxiliary functions which generate the temporal dynamics in FDA2 were taken as $h(x_{II}, f_1, g) = 1 - \left(\frac{f_1}{g}\right)^{H(t)} \sum_{s=1}^{5}(x_{s} - H(t)/2)^2$ and $H(t) = 5\cos(\pi t/4)$.
was modified as $f_1(x_r) = x_r$, in which $r = v \cdot m \cdot (g/G) \mod m$ ($G = 200$ is the maximum number of generations and $m = 10$), i.e., the first objective is a function of the $r$-th decision variable, in which $v$ controls the periodicity with which $r$ changes over time. In this paper, $v = 3$, what leads to the periodic sequence $x_1, x_2, \cdots, x_m, x_1, x_2, \cdots x_m, \cdots$, with $r$ changing every 5 generations.

The Welch T-Test (which does not assume equal variance among treatments) is used to assess the statistical significance at the 0.05 level between the indicators means. The results are averaged over $E = 50$ trials for each parametric combination. Previous works on DMO have used a variety of indicators for assessing the quality over time of the evolved $F_i$ set. In this paper, we propose using for that task the offline hypervolume:

$$\overline{H}_{\beta} = \frac{1}{G} \sum_{i=1}^{G} \left( \frac{1}{E} \sum_{j=1}^{E} h(i,j) \right),$$

in which $G = 200$ is the number of generations, $E$ is the number of trials, and $h(i,j)$ represents the hypervolume [16] of the domination region bounded by reference point $z_{ref} = (11,11)$ and the solutions in $F_1$ of generation $i$ in trial $j$. We believe $\overline{H}$ can prove to be a robust performance indicator for reporting DMO results in the literature. Refer to e.g. [11] for discussion on performance metrics for dynamic environments.

Diversity in the decision space is measured by the moment of inertia, $I_M$ [6]. Also, in order to assess how evenly distributed the solutions are among the Fronts identified by NSGA2, we compute the normalized entropy [17]:

$$H_N = -\frac{1}{\log_2 |F|} \sum_{i=1}^{|F|} p_i \log_2 p_i, \quad 0 \leq H_N \leq 1,$$

in which $|F|$ is the number of Fronts and $p_i = P\{x \in F_i\}$ is estimated from the relative frequency of the number of solutions in the $i$-th Front, $F_i$. It should be noted that $H_N = 0$ if $|F_i| = N$, $|F_j| = N$ and $|F_j| = 0$, $j = 2, \cdots, N$,

$H_N = 1$ if $|F_i| = |F_2| = \cdots = |F_k|$, $k \leq N$,

where $N = 100$ is the population size used in all experiments.

In addition, the accumulated genetic diversity, $I_M(i)$, and the average Front diversity, $\frac{1}{k} \sum_{i=1}^{k} H_N(i)$, are computed across all generations. The Pearson correlation coefficients between those two quantities and $\frac{1}{G} \sum_{i=1}^{G} h(i)$ are also reported. Finally, both the immigration rate ($\alpha$) and the proportion of uncorrelated immigrants ($\beta$) take values in $\{0.0, 0.3, 0.5, 0.7, 1.0\}$, yielding 25 combinations between the values of those two parameters, five of which leading to identical functional behaviors (standard NSGA2, when $\alpha = 0$). We point out that exactly the same number of fitness evaluations are performed in each generation, for all implemented immigration scheme.

VI. RESULTS AND DISCUSSION

We experimentally assess the effects of (i) increasing $\alpha$ while keeping $\beta$ constant at levels zero and one (Sect. VI-A); and of (ii) increasing $\beta$ at fixed $\alpha \in \{0.3, 0.5, 0.7, 1.0\}$ (Sect.VI-B). Also, the amounts of accumulated and average diversity generated in each strategy are reported (Sect. VI-C) alongside the Pearson correlation coefficients between these quantities and the offline hypervolume ($\overline{H}$) over the evolved $F_1$ using NSGA2$_{dDG}(\alpha, \beta)$ (Sect. VI-D).

A. Increasing the Immigration Rate ($\alpha$)

The increasing of $\alpha$ leads to higher $\overline{H}$ values in the experiments with NSGA2$_{dDG}(\alpha, \beta)$ for all $\alpha \neq 0 \times \beta$ combinations, when compared to the results obtained with $\alpha = 0$, i.e., using the standard NSGA2 without diversity generation. This result is illustrated in Fig. 6 for FDA1, with $\beta = 0$ and $\beta = 1$. The only exception is the case NSGA2$_{dDG}(1, 1)$ in which it was observed that the overall quality of $F_1$ on the evolved populations over time is degraded. This particular case corresponds to replacing the whole population with uniformly generated random immigrants, what is indeed expected to be the worst possible optimization strategy. Also, it is observed a decreasing trend in performance for $\alpha > 0.3$ and $\beta = 1$, that is, while replacing 30% of the population with random immigrants is clearly better than inserting no immigrants, there is a critical level at $\alpha = 0.3$ after which $\overline{H}$ actually tends towards worse results than the standard NSGA2.

The NSGA2, on the other hand, seems to benefit from the increasing of $\alpha$ with a fixed $\beta = 0$, having achieved the best results in terms of $\overline{H}$ at $\alpha = 1$. The difference in behavior when increasing $\alpha$ for fixed $\beta = 0$ and for $\beta = 1$ may be due to the higher survival likelihoods of the elite-based immigrants (for $\beta = 0$) when compared to the random immigrants survivability in the population (for $\beta = 1$). Thus, for $\alpha = 1$, despite the whole population being replaced...
Thus, we conclude that the NSGA2 gIDG observed for $\beta$ between severity ($n$) optimization scenarios are considered – for all combinations (parts (b), (c) and (d) of the Figure). When the 12 possible levels to fall under those of when random immigrants ($\alpha$) over time for fixed genetic diversity ($I_M$) for all $\alpha \times \beta$ combinations on FDA1 with $n_g = t_g = 5$. It can be observed that a pure cIDG/NSGA2 strategy does not succeed in generating as much genetic diversity as all the other cases of mixtures between cIDG and uIDG. Surprisingly though, for $\alpha = 0.7$, a pure uIDG/NSGA2 strategy was not the one which generated the optimization stages wherein the severity of change seems to be higher. Conversely, when the severity of change seems to be smoother, the average hypervolume levels resulting from the NSGA2 gIDG($0,0.5$) variant clearly settles above those resulting from NSGA2 gIDG($0.5,1.0$), what suggests that not only a mixture between both cIDG and uIDG strategies may be desirable in certain dynamic MOPs, but also that an auto-adaptive strategy to adjust $\beta$, while keeping $\alpha$ constant, could possibly lead to dramatic performance improvements, provided that the most appropriated strategy is triggered under the corresponding best matching change scenario.

The effects of increasing $\beta$ under several values of $\alpha$ are shown in Fig. 7 (for $n_g = t_g = 5$, but the following discussion is valid to the other combinations). Clearly, for all $\alpha$ values except $\alpha = 1.0$, the hybridization between the cIDG and uIDG is desirable for improving NSGA2 performance on dynamic environments, leading to the best results in terms of offline hypervolume. In addition, it is observed from Fig. 7 that whenever $\beta = 1.0$, NSGA2 gIDG performance is degraded (or statistically equal to, at best) when compared to the values tested with $0 < \beta < 1$. When $\alpha = 1$, however, the best performing immigration scheme is indeed the cIDG ($\beta = 0$) for all the DMOPs we have tested.

C. Measuring the Amount of Diversity Generated by gIDG

This Section aims to investigate the underlying factors behind the differences in the average performance of NSGA2 gIDG by analyzing the diversity levels generated throughout the executions of the studied algorithms. Fig. 8 shows box-plots of accumulated genetic diversity ($I_M$) for all $\alpha \times \beta$ combinations on FDA1 with $n_g = t_g = 5$. It can be observed that a pure cIDG/NSGA2 strategy does not succeed in generating as much genetic diversity as all the other cases of mixtures between cIDG and uIDG. Surprisingly though, for $\alpha = 0.7$, a pure uIDG/NSGA2 strategy was not the one which generated the
highest accumulated $I_M$ levels, what raises the hypothesis that
the balance between immigrants generated by either cIDG or
uIDG affects in a non-intuitively way the survivability of the
immigrants subpopulation, which is expected to be reflected
on the net amount of diversity generated throughout evolution.

Interestingly, when the median values of average Front
diversity ($H_N$) in Fig. 9 are compared against the cor-
responding median values of accumulated $I_M$ (Fig. 8), it can
be noted that, in general, for fixed $\alpha$ and for $0 < \beta < 1$,
there is a clear decreasing trend on $H_N$, in which, for all
$\alpha \in \{0.3, 0.5, 0.7, 1.0\}$, there is at least one value of $\beta$ (except
for $\alpha = 1.0$) for which $H_N$ is less than the $H_N$ obtained for
$\beta = 0$ (using only cIDG). That may explain the reason why
for $0 < \beta < 1$, among the tested values, NSGA2gIDG presented
the best results in terms of offline hypervolume (Fig. 7): since
$I_M$ becomes higher and $H_N$ lower, the gIDG is actually generating more genetic diversity while also exploiting more
the regions in the decision space around the solutions in $F_1$,
what means that, somehow, gIDG is able to generate diversity
around the regions that may be closer to the basins of attraction
that leads to the optimal $PF(t)$.

**D. Correlation Between Diversity and Offline Hypervolume**

The results regarding the apparent relationship between
the diversity levels generated by the gIDG and the offline
hypervolume obtained with the insertion of immigrants into
the population evolved by NSGA2 have motivated us to
perform a statistical correlation study that aims to characterize
the influence of diversity on evolving high-quality $F_1$ sets
in dynamic environments utilizing the proposed gIDG. We
illustrate the case of FDA1 ($n_g = t_g = 5$) when $\beta$ is allowed
to vary from zero to one and $\alpha > 0$.

With regard to the Pearson correlation coefficients ($\rho$)
computed between average Front diversity [17], $\overline{H_N} = \frac{1}{G} \sum_{i=1}^{G} H_N(i)$, and average hypervolume, $\overline{\sum_{i=1}^{G} I_h(i)}$, it
is observed from Fig. 10 (a) that $\overline{H_N}$ most often significantly
negatively correlates with hypervolume, most notably when
$\beta = 0$ i.e. when all immigrants are generated by the cIDG.
This result suggests that, under elitist immigration schemes,
the more NSGA2 exploits the regions around the first Fronts
in the population (when $H_N \rightarrow 0$), the better is the quality
of the evolved $F_1$ for the tested DMO problems. However, as
uncorrelated immigrants are inserted into the population (as
$\beta \rightarrow 1$), the influence of $H_N$ on the quality of $F_1$ seems
to decrease, since the magnitudes of the sample $\rho$s decrease
(except for $\alpha = 1$), despite $\rho$ being significantly nonzero
in the majority of cases. For instance, when $\alpha = 0.7$ and
$\beta \in \{0.3, 0.5, 1.0\}$, $\rho$ lies within $[-0.278, 0.278]$, meaning
that the correlation measured is not statistically significantly
different from zero.

Finally, the Pearson coefficients shown in Fig. 10 (b) corrob-
orate to the conclusion that, especially when $\beta = 0$ (and only
cIDG is being effectively used), since $\rho$ is positive and signifi-
cantly different from zero, the best scenario is to generate more
genetic diversity while keeping more individuals distributed
over fewer Fronts in the population (i.e., keeping individuals
in the population close enough to $F_1$ in the objective space
but distant enough from the population center of mass in the decision space). Again, the relative importance of genetic diversity on the observed offline hypervolume values seems to decrease for $\beta \neq 0$ (despite few exceptions, e.g., when $\alpha = 1.0$ and $\beta = 0.5$). In fact, the two diversity indicators seem to be sometimes counter-correlated (most notably when $\beta = 0$), as shown in Fig. 10 (a)–(b), suggesting that, when used altogether, those indicators can play a valuable role on informed decisions for controlling the diversity levels generated by the generalized immigrants-based diversity generator on dynamic multiobjective optimization problems.

VII. CONCLUSION AND FUTURE WORKS

Dynamic Multiobjective Optimization (DMO) is a relatively recent research topic in which the role of diversity is crucial for the algorithmic performance on evolving high-quality non-dominated solutions sets ($F_1$). While a few specific Dynamic Multiobjective EAs (DMOEAs) does exist do cope with the difficulties of tracking the changing optimal Pareto Front over time ($PF(t)$), e.g. [14], we believe that a clear separation between diversity generation and optimization towards the $PF(t)$ is needed in order to further advance research in DMO. This paper has thus proposed and discussed the properties of a probabilistic diversity generation model for DMO from which an extended immigration scheme taxonomy was developed and an expression for expected genetic diversity was obtained. This taxonomy was shown to be flexible to allow many possible implementations of the proposed generalized Immigrants-based Diversity Generator (gIDG), by considering distinct sampling processes over the decision space.

The initial results regarding the incorporation of gIDG into NSGA2 [12] show that higher quality $F_1$ sets are kept in the population over time when a certain mixture degree between correlated (elite-based) and uncorrelated (random) immigrants are inserted into the NSGA2 population. Such hybridization has lead to statistically significantly better results in terms of offline hypervolume when compared to the standard NSGA2 (without diversity generation) or to the standalone correlated and uncorrelated schemes. Moreover, the study of the Front diversity indicator ($H_N$) [17] as well as of genetic diversity ($I_M$) [6] has revealed that diversity generation is more effective on improving NSGA2 performance on the studied DMO problems when fewer Fronts are kept in the population (i.e., lower $H_N$ values), while promoting genetic diversity in the decision space (i.e., higher $I_M$ values).

Future works should ideally explore the potential of the proposed extended taxonomy by either considering linkage learning in local population models, in such a way that the right profile immigrants are “accepted” into the current population according to a quasi-optimal dynamic immigration policy, or by simply considering auto-adapting both the immigration rate and the proportion of uncorrelated immigrants as a function of the perceived severity and periodicity of change. In this scenario, controlled mixtures between both cIDG and uIDG strategies could potentially lead to dramatic performance improvements. Also, the incorporation of gIDG into other state-of-the-art MOEAs should be an easily accomplishing research goal, wherein the different levels of synergy between the distinct diversity maintenance strategies employed in those MOEAs may alter the conclusions obtained in the proposed experimental study. We are currently considering the problem of how and where to generate diversity in a way to promote the exploration of promising regions in DMO problems, while improving the adaptability of the population on changing environments. Our approach considers mutlivariable order statistics to provide a suitable framework to generate individuals on regions with higher non-dominance probabilities, following a multiobjective estimation of distribution scheme.

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REFERENCES