Abstract—New digital subscriber line (DSL) technologies are being developed to meet the ever-increasing bandwidth demand of the user. One very promising approach injects “common-mode” signals that superimpose a signal on two regular differential pairs. This technique requires new reliable multiconductor models for the telephony cables. Therefore, good approximations of the series impedance of the line are indispensable. In this paper, a model for this series impedance is theoretically derived and validated with measurements.

Index Terms—Cable series impedance, common-mode (CM) signal, quad cable, xDSL.

I. INTRODUCTION

THE DIGITAL subscriber line (DSL) technology provides broadband services over the twisted-pair copper wires of existing telephone networks. Current DSL systems only make use of differential-mode (DM) signals on a single pair. However, the telephone cable that enters a home often contains four conductors. Only two of them are used nowadays, but in the near future, the four wires can be exploited. In next-generation DSL technologies, three signal paths will be employed, namely, a DM signal on each of the two pairs and an extra common-mode (CM) signal between the pairs [1], [2]. Those three signals are labeled with a suffix 1, 2 (DM), or 3 (CM) in Fig. 1.

The DM signals 1 and 2 correspond to the voltage difference between two wires, i.e.,

\[
V_{p1+} - V_{p1-} = (V_3 + V_4) - (V_2 - V_1) = 2V_1 \tag{1}
\]

\[
V_{p2+} - V_{p2-} = (V_3 + V_2) - (V_5 - V_2) = 2V_2. \tag{2}
\]

It is clear that the DM signals are not influenced by the existence of the CM signal \(V_3\). This CM signal can also be considered as a differential signal using the four conductors, i.e.,

\[
(V_{p1+} + V_{p1-}) - (V_{p2+} + V_{p2-}) = [(V_3 + V_1) + (V_3 - V_1)] - [(-V_3 + V_2) + (-V_3 - V_2)] = -4V_3. \tag{3}
\]

From this perspective, it is clear why the CM signal remains insensitive to disturbing sources.

In the past, the CM signal was already used as a noise canceller [3]. Now, this signal will be used as an extra path together with the DM signal. Compared with the DM-only capacity, the capacity of the line can be increased three times [3]. To investigate the possibilities of this new DSL signal, we have to set up reliable transmission line models to describe the propagation of these CM signals. These models are derived based on the multiconductor transmission line theory [4]. As this theory is only valid for uniform line segments, we consider an infinitesimal section of such a line.

Fig. 2 shows the schematic diagram associated to the telegraph equation used to model the line. The representation contains a resistor \(R\Delta l\), an inductor \(L\Delta l\), a conductance \(G\Delta l\), and a capacitance \(C\Delta l\). For our model, we need accurate values for all of them.

If the dielectric permittivity and the loss factor of the dielectric that is used as an isolator in the cable are assumed to be frequency independent, it is easy to retrieve these elements using a quasi-static electromagnetic field calculation. \(G(f)\) and \(C(f)\) are then merely functions of the medium and the geometrical parameters, respectively. However, if an accurate model for the line behavior is needed, the frequency dependence of these quantities cannot be omitted.

In the past, models were developed for \(R\) and \(L\) of a single pair [5], but as the CM signal is defined between four wires in a medium, the values for \(L\) and \(R\) obtained using the two-wire...
model are not applicable. In addition to the induction effect, the skin and proximity effects also need to be taken into account to obtain a high level of accuracy. In this paper, it is shown how this impedance, called the “series impedance”, can be calculated for a quad (a cable with four conductors in a square formation, as shown in Fig. 3). Taking the real and imaginary parts of the series impedance, one obtains accurate values for $R$ and $L$, respectively, as a function of frequency. A first attempt to validate the model for the series impedance has been done in [6]. In this paper, the influence of the different geometric parameters in the model has been investigated. In addition, new measurements are shown, which correspond much better to the model.

II. MODELING

A. Modeling Setup

As shown in Fig. 3, we consider four wires with the same radius, which are symmetrically placed inside a tube. All conductors are assumed parallel, and the screen is supposed to be a perfectly conducting cylinder. The screen has the same permeability as the wires and the dielectric, and it carries no overall current.

B. Analytical Expression for the Series Impedance

To set up a model for the series impedance, the potential theory has been applied. In the past, the per-unit-length series impedance was derived by Belevitch [7].

Fig. 3. Geometry of the considered quad-line setup.

To obtain the analytical expression, the quasi-stationary approximation is used, also in the dielectric. This means that the displacement current inside the conductor is neglected and that the current in each wire is assumed constant.

The series impedance can be split up into three different contributions, i.e.,

$$Z_s = Z_{\text{ind}} + Z_{\text{sk}} + Z_{\text{prox}}$$  \hspace{1cm} (4)

where $Z_{\text{ind}}$ represents the inductive coupling between the four conductors, $Z_{\text{sk}}$ is the impedance as a result of the skin effect, and $Z_{\text{prox}}$ represents the impedance owing to the proximity effect.

The three contributions per-unit-length are formulated in (5), shown at the bottom of the page. The total impedance $Z_s$ is complex: It contains both a real and an imaginary term. The coefficients $A_m$ are the solutions of (6), shown at the bottom of the page. In theory, the index $m$ starts at 1 and goes up to infinity. For every $n$, going from 1 up to the end value of $m$, (6) has to be solved. We get a set of equations where the unknowns are the coefficients $A_m$. In practice, the index $m$ stops at 4 since the higher order coefficients are negligible, and thus, they do not influence the proximity effect anymore. Once we know the coefficients $A_m$, the impedance due to the proximity effect can be calculated, and as a consequence, the total series impedance can be found, adding $Z_{\text{ind}}$ and $Z_{\text{sk}}$.

The peculiar thing about the equations is the used dual-complex notation. The first complex notation is the classical temporal rotation, as is used to model impedances. The complex variable is denoted by $j$. A second complex notation is introduced to simplify the formulation of the geometrical dependence. There, $x$ and $y$ in Fig. 3 are considered to be the real and imaginary parts of a complex number, where $D = x + iy$. All the coefficients $D$ in (5) and (6) are complex numbers in this geometrical complex variable. The complex vector from the center of wire $s$ to the center of wire $t$ is labeled $D_{st}$, and the complex conjugate of $D_{st}$ is $D_{st}^\ast$. The product $i \cdot j$ is left undefined. In the end result, the variable $i$ will always disappear as each complex contribution in $i$ always has its complex conjugate counterpart.

The radius of each wire is labeled $r$, whereas $r_0$ is the distance between the center and the shield. The current in wire $s$
is \( I_s \), \( T_{nm} \), \( \lambda_n \), and \( F_{nm}(u) \) are notations used by Belevitch to simplify (6), and they are defined as follows:

\[
T_{nm} = \frac{(m + n - 1)!}{(n-1)!(m-1)!} \quad (7)
\]

\[
\lambda_n = -\frac{nJ_{n-1}(kr)}{J_{n+1}(kr)}. \quad (8)
\]

\( J_{n-1} \) represents the Bessel function of order \( n - 1 \) [8], and \( k = -j\omega\mu\sigma \). In addition

\[
F_{nm}(u) = nm \sum_{t=0}^{\min(n,m)} \frac{(m + n - t - 1)!}{(n-t)!(m-t)!t!} \left( \frac{u}{1-u} \right)^{m+n-t}. \quad (9)
\]

Once we know the series impedance, it is easy to find accurate values for \( R \) and \( L \) of the considered quad, i.e.,

\[
R = \text{real}(Z_s) \quad (10)
\]

\[
L = \text{imag}(Z_s). \quad (11)
\]

The quad can be used in two different modes: 1) the phantom mode and 2) the parallel mode. In the phantom mode, the normalized currents in the conductors are given by (Fig. 4)

\[
I_1 = I_3 = 1 \quad I_2 = I_4 = -1 \quad (12)
\]

whereas the currents for the parallel mode are

\[
I_1 = I_4 = 1 \quad I_2 = I_3 = -1. \quad (13)
\]

Equations (5) and (6) are valid for a quad configured in the phantom mode.

\[
\sum_{m=1}^{\infty} r^{m+n} T_{nm} \left\{ \frac{(-1)^m}{D_{12}^m D_{14}^m} - \frac{1}{D_{11}^m D_{14}^n} \right\} A_m + \left\{ \frac{(-1)^m A_{20}^m}{D_{31}^m D_{13}^n} \right\} + \lambda_n A_n
\]

\[
- \sum_{m=1}^{\infty} \frac{r^{m+n}}{D_{01}^m} \left\{ \frac{1}{D_{01}^n} F_{nm} \left( \frac{D_{01}^n D_{02}}{r_0^2} \right) + \frac{1}{-D_{03}^m} F_{nm} \left( \frac{D_{01}^n D_{03}}{r_0^2} \right) \right\} A_m
\]

\[
- \left\{ \frac{1}{D_{02}^m} F_{nm} \left( \frac{D_{01}^n D_{02}}{r_0^2} \right) + \frac{1}{D_{04}^m} F_{nm} \left( \frac{D_{03}^n D_{04}}{r_0^2} \right) \right\} A_m
\]

\[
= r^n \left[ -\frac{1}{D_{12}^n} + \frac{1}{D_{13}^n} - \frac{1}{D_{14}^n} \right] - \left[ \frac{D_{01}^n}{r_0^2 - D_{01}^n D_{02}} \right] A_0 + \left[ \frac{D_{02}^n}{r_0^2 - D_{02}^n D_{03}} \right] A_1 - \left[ \frac{D_{03}^n}{r_0^2 - D_{03}^n D_{01}} \right] A_2 + \left[ \frac{D_{04}^n}{r_0^2 - D_{04}^n D_{01}} \right] A_3 \quad (6)
\]

C. Mathematical Interpretation

The resistance \( R \) is mainly determined by the skin effect and increases with increasing frequency. The inductance \( L \) particularly depends on the inductive term and very slightly changes with frequency. Above 2 MHz, the inductive part of the skin effect is negligible (\(< 2\% \) of the total inductance). The proximity effect causes a small negative contribution to the inductance. As the word indicates, the inductive term does not contribute to the resistance. This is also visible in Fig. 5, where a simulation is shown of the different resistance contributions in function of the ratio between the wire radius and the diagonal distance between the conductors \( r/D \) at 10 MHz. As a consequence, this ratio is dimensionless and does not exceed 0.5. If this ratio increases, the resistance due to the skin effect decreases, and the proximity effect becomes more important. This is easy to explain. The rise of \( r/D \) can be accomplished in two ways: 1) the wire radius has increased, or 2) the insulation thickness has decreased.

In both situations, the conductors lie closer to each other. As a consequence, the proximity effect becomes stronger. As can be noticed from (5), the skin effect is independent on the isolation. The radius of the screen \( r_0 \) does not affect the skin effect either. Once this radius exceeds 2 cm, both the real and imaginary parts of the series impedance do not change anymore.
since the last two terms of the inductive part and the proximity
effect, which depend on the screen radius, vanish above 2 cm.

III. VALIDATION

A. Measurement Setup

This model has been validated using a setup that does not compromise the assumptions. This means that the conductors should be parallel to each other and surrounded by a homogeneous medium, e.g., air. Such a setup was constructed, as depicted in Fig. 6.

The length of the multiconductor transmission line is rather limited (2 m) due to practical mechanical problems. Four copper wires with a diameter of 0.85 mm are tied and stretched between two diagonal conductors. To be sure that the conductors stay parallel, spacers are used at regular distances. The distance $D$ between two screwed constructions is 6 mm. Since there is no shielding, $r_0 = \infty$. This implies that the contribution of the proximity effect will be very small in our measurements.

The physical quantities are given as follows:

$$\begin{align*}
\alpha &= 5.8 \times 10^7 \text{ S/m} \quad \text{(Wires are made of copper.)} \\
\varepsilon &\approx \frac{1}{36\pi 10^9} \text{ F/m} \\
\mu &= 4\pi 10^{-7} \text{ H/m.} \quad \text{(The surrounding medium is air.)}
\end{align*}$$

The values for $R$ and $L$, as obtained from the implementation of the analytical model, will be compared with measurements. We decided to test the previous formulas that are derived for the phantom mode. As can be seen in Fig. 7, conductors 1 and 3 and wires 2 and 4 are connected together for this test. The network analyzer (NA), namely, an HP3577B, which is remotely controlled by MATLAB, is used to measure the one-port scattering parameter $S_{11}$.

B. Mathematical Derivation for the Series Impedance

$S_{11}$ is defined with respect to the reference impedance $Z_{\text{cal}}$ at the input, i.e.,

$$S_{11}|_{Z_{\text{cal}}} = \frac{b}{a} \quad (14)$$

$S_{11}$ can also be defined as a function of the line length $l$, the complex propagation factor $\gamma$, and the reflection coefficients at the generator and load sides, i.e., $\rho_G$ and $\rho_L$, respectively [9]. Thus

$$S_{11} = \frac{\rho_G + \rho_L e^{-2\gamma l}}{1 + \rho_L \rho_G e^{-2\gamma l}} \quad (15)$$

with

$$\begin{align*}
\rho_G &= \frac{Z_C - Z_{\text{base}}}{Z_C + Z_{\text{base}}} \\
\rho_L &= \frac{Z_L - Z_C}{Z_L + Z_C}
\end{align*} \quad (16)$$

where $Z_C$ represents the characteristic impedance of the line, $Z_{\text{base}}$ is the impedance at the generator, and $Z_L$ is the impedance of the load.

To determine the propagation factor $\gamma$ and the characteristic impedance $Z_C$ of the line, two successive experiments are needed. In the first experiment, the line was measured with an open termination ($Z_L = \infty$) at the far end, whereas in the second experiment, it was terminated with a short ($Z_L = 0$) at the far end.

We obtain the following equations for an open and a short termination:

$$\begin{align*}
S_{11}^{\text{open}} &= \frac{\rho_G + e^{-2\gamma l}}{1 + \rho_G e^{-2\gamma l}} \quad (18) \\
S_{11}^{\text{short}} &= \frac{\rho_G - e^{-2\gamma l}}{1 - \rho_G e^{-2\gamma l}} \quad (19)
\end{align*}$$

By now, we have two equations with two unknowns. After some calculations [10], the propagation function $\gamma$ and the characteristic impedance $Z_C$ are found to be

$$\begin{align*}
\gamma &= -\ln(|X|) + j\angle X \\
Z_C &= \frac{Z_{\text{base}}(1 + Y)}{(1 - Y)}
\end{align*} \quad (20)$$

with

$$\begin{align*}
X &= \frac{S_{11}^{\text{open}} - S_{11}^{\text{short}}}{S_{11}^{\text{open}}} \quad (22) \\
Y &= \frac{X - S_{11}^{\text{open}}}{S_{11}^{\text{open}} - S_{11}^{\text{short}}}
\end{align*} \quad (23)$$
In (20), $|X|$ and $\angle X$ represent the magnitude and phase of $X$, respectively.

From the calculated quantities, the series impedance is easy to find, i.e.,

$$Z_S = Z_C \cdot \gamma.$$  \hspace{1cm} (24)

C. Experimental Results

As shown in Fig. 8, at low frequencies, the characteristic impedance decreases with increasing frequency, as expected [11]. Above 2 MHz, the impedance is nearly constant and is $\approx 115 \, \Omega$. The lowest test frequency of the NA is 100 kHz. Therefore, the measurement starts at this frequency.

Once $\gamma$ is known, the transfer function $e^{-\gamma l}$ can also be calculated. The magnitude of the transfer function (see Fig. 9) also decreases with increasing frequency. However, the decrease is rather small. The noise for both the characteristic impedance and the transfer function increases strongly for decreasing frequency, as the length of the line $l/\lambda$ becomes very small. As a result of the length constraint, the attenuation is also very small. The difference between the incident and the reflected signal comes close to the noise floor of the measurements. Because the final result for the series impedance is based on the difference between the measured value of the $S_{11}$ parameter and 0 dB, the system is very sensitive to noise at low frequencies.

D. Comparison Model–Measurement

As shown in Fig. 10, both the simulated series impedance and the impedance derived from the measurements increase very fast with increasing frequency. Because both curves lie almost on top of each other, the difference is plotted in dotted line. The norm of the residual measurement–model is more or less 20 dB below the value, which is a very acceptable result given the high noise sensitivity of these measurements.

E. Validation

To be able to determine whether there are remaining modeling errors, we have to investigate the variability of the obtained impedance based on the variability of the measured data. The $S_{11}$ measurements were repeated ten times, and the sample mean and the sample variance of the measurements were calculated. We assumed the noise on $S_{11}$ to be circular complex Gaussian noise. To determine the variance on the obtained impedance, we used the procedure as is described in the Guide to the Expression of Uncertainty in Measurement (GUM) and in its Supplement 1 [12], [13]. A Monte Carlo simulation was performed based on the mean measured data, perturbed by noise whose standard deviation is given by the sample standard deviation. Ten thousand realizations of this random process were then generated, and the impedance has been determined for each realization. Next, the probability density function has been identified. The histogram of the simulated impedance is shown in Fig. 11.

Seemingly, a Gaussian distribution is visible. However, to obtain a qualitative result, a Kolgomorov–Smirnov test has also...
been done. This test determines whether the distribution of our set of data points lies in the confidence region of a normal distribution. The test confirms that our sample set indeed belongs to a Gaussian distribution. A confidence interval for the series impedance can now safely be determined using the mean value and the standard deviation. In Fig. 12, the $2\sigma$ confidence interval is plotted.

The model lies outside this band. However, it is important to notice that the calibration uncertainty is not taken into account in the calculation of the confidence level. However, if this would be the case, the model would still not fit inside the uncertainty boundaries of the measurements. In general, there is a misfit between the model and the measurement of about $15\sigma$.

There are a lot of measurement challenges that could cause this deviation. First, because of some practical mechanical problems, the length of the multiconductor transmission line is limited. Because of this, the line attenuation is very small. This small difference is the key parameter in determining the line parameters. The baluns that are located inside the test set of the NA are a source of small nonlinearities. Finally, there is also coupling with the environment. All these phenomena could influence the measurements and cause a small difference between the measurement and the model. However, we may conclude that the model gives acceptable results, although small modeling errors remain present.

### IV. Example

Since we know that the model for the series impedance gives acceptable results, we can use the simulator to determine the values for the resistance $R$ and the inductance $L$ for different types of cables that are widely used. We consider two types of quads, which are used in practice for the last hundreds of meters between the central office and the customer. The first type is a 0.4-mm polyethylene France Telecom quad, and the second type is a 0.5-mm polyethylene Belgacom quad. Both cables were mechanically cut and measured to get the exact measures of the isolation thickness and the screen radius.

Filling in these parameters in our model, we get the series impedance of the quad. In Fig. 13, we see the resistance for both quads from dc up to 25 MHz. Fig. 14 shows the inductance in the same frequency band.

Care should be taken so that this model will not correspond to real measurements on the France Telecom cable or on the Belgacom cable. This because those wires are twisted and twisting is not yet incorporated into the model. This will be done in the future.

### V. Conclusion

A new model for the series impedance of a quad has been implemented and validated with measurements. In the past, only models for single pairs have been validated. In this paper, it has been verified, for the first time, for a quad. The model gives acceptable values for $R$ and $L$. These parameters are necessary to set up good transmission line models, which also take the CM signals into account.
Fig. 14. Inductance of a Belgacom quad and a France Telecom quad.

REFERENCES


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