Fair subcarrier and power allocation for multiuser orthogonal frequency-division multiple access cognitive radio networks using a Colonel Blotto game

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Abstract: The problem of subcarrier allocation (SA) and power allocation (PA) for both the downlink and uplink of cognitive radio networks (CRNs) is studied. Two joint SA and PA schemes based on Blotto games are presented for orthogonal frequency-division multiple access (OFDMA)-based CRNs. In this work, the authors consider a more practical scenario by taking into account the correlation between adjacent subcarriers. In the proposed games, secondary users (SUs) simultaneously compete for subcarriers using a limited budget. In order to win as many good subcarriers as possible, the SUs are required to wisely allocate their budget subject to the transmit power, budget and interference temperature constraints. Two PA and budget allocation strategies are derived to enable fair sharing of spectrum among the SUs. It is shown that by manipulating the total budget available for each SU, competitive fairness can be enforced. In addition, the conditions to ensure the existence and uniqueness of Nash equilibrium (NE) in the proposed methods are established and algorithms which ensure convergence to NE are proposed. Simulation results show that the proposed methods can converge rapidly and allocate resources fairly and efficiently in correlated fading OFDMA channels.

1 Introduction

Cognitive radio (CR) [1] is initiated by the apparent lack of spectrum because of the current rigid spectrum management policies. This promising technology can potentially alleviate spectrum scarcity in wireless communications by allowing secondary users (SUs) to opportunistically access the spectrum that is licensed to primary users (PUs). The main task of CR networks (CRNs) is to ensure that the SUs can maximise spectrum utilisation under the interference temperature constraints [2] of multiple PUs.

In order to efficiently utilise the valuable spectrum, an efficient subcarrier allocation (SA) scheme is required for orthogonal frequency-division multiple access (OFDMA)-based CRNs. Besides, power allocation (PA) can be used in CRNs to control the interference from SUs to PUs. Hence, SA and PA are two synergistic techniques to achieve efficient spectrum utilisation and guarantee protection for PUs. Some joint SA and PA techniques for OFDMA-based CRNs have been reported in [3–6]. These techniques focus on throughput maximisation without considering fairness. In these schemes, SUs with better channel conditions always dominate usage of spectrum, causing low throughput for SUs with poor channel conditions. To overcome this problem, the max–min fairness criterion [7] is used to maximise the throughput of SUs with poor channel conditions. However, this scheme results in low spectral efficiency. To resolve this conflict, proportional fairness [8] and Nash bargaining fairness [9] are applied where overall throughput is maximised subject to the condition that each user is guaranteed a portion of the resources.

According to [10], the aforementioned types of fairness indicators which are artificially decided by the system do not reflect fairness correctly from the user’s perspective. Hence, competitive fairness is introduced in [10] using an auction method where each user competes for the resources and is responsible for its own actions and resulting throughput. In [11], Han and Han proposed an enhanced auction-based SA method for OFDMA systems to achieve high spectrum utilisation and ensure competitive fairness. However, previous studies on fairness [7–11] do not take into account the correlation between adjacent subcarriers; that is it is always assumed that each subcarrier occupies the entire coherence bandwidth and hence the channel realisations experienced by each SU on different subcarriers are statistically independent. However, in practical OFDMA systems, a coherence bandwidth could be spanned by multiple subcarriers, thus giving rise to channel correlation among the subcarriers [12]. Furthermore, spectrum unfairness is more severe in correlated orthogonal frequency-division multiplexing (OFDM) channels because the subcarriers utilised by a SU may experience deep fades simultaneously and use of the techniques proposed in [7–11] may not achieve fairness. To date, this problem has received little attention and no effective SA method has been proposed to tackle this problem.

In this paper, we aim to develop a joint SA and PA technique which can achieve a good trade-off performance between fairness and efficiency in OFDMA-based CRNs. In
particular, we propose two joint SA and PA schemes based on Blotto games [13], for both the uplink and downlink. In game theory, a Blotto game [13] is a game where the players are tasked to simultaneously distribute their limited resources over several objects and the player allocating the most resources to an object wins the object. Intuitively, the goal of the players is to win the highest number of objects and their payoff is then equal to the total number of objects won. Apparently, the nature of Blotto game is similar to the environment of OFDMA-based CRNs in which SUs compete with each other to acquire a larger number of subcarriers with good channel conditions. Therefore it is suitable to model the SA and PA problems using Blotto games.

In this work, we consider a more practical scenario by taking into account the correlation between adjacent subcarriers. Unlike the conventional auction method, we model the SA and PA problems into a multi-dimensional auction where SUs simultaneously compete for subcarriers using a limited budget. The bidding process in this auction can be characterised using a Blotto game and Nash equilibrium (NE) of the Blotto game could be used to solve the auction problem. Subject to the power, budget and interference temperature constraints, the SUs need to wisely allocate their budget and power to win as many good subcarriers as possible. Two budget allocation (BA) and PA strategies are derived using a Lagrangian relaxation method and NE is shown to exist. We propose two algorithms which can guide the proposed games to achieve NE. It will be shown that the proposed algorithms can ensure competitive fairness among SUs while incurring negligible throughput degradation as compared to the existing techniques.

The rest of this paper is organised as follows. In Section 2, the model of a hybrid network and an interference temperature model are outlined. Section 3 addresses the problem and game formulation. This section also investigates the existence and uniqueness of NE in the proposed games. Section 4 presents algorithms to execute the proposed games. Simulation results and performance analysis are presented in Section 5. We end the paper with some concluding remarks in Section 6.

2 System model

2.1 Cognitive radio network model

We consider a hybrid network, consisting of a primary network (PRN) and a CRN as illustrated in Fig. 1. Both the PRN and CRN are single-cell OFDMA cellular networks that co-exist within the same geographical area. In the PRN, there are $M$ PUs communicating with a primary base station (PBS) in the downlink. At the same time, the CRN consists of a secondary base station (SBS) accommodating $N$ SUs where both the downlink and uplink are considered. As shown in Fig. 1, the channel gains of different links are defined as follows:

- $h_{n}^{k,U}$ (or $h_{n}^{k,D}$) denotes the channel gain of the communication link from the $n$th SU (SBS) to the PBS (SBS) on the $k$th subcarrier.
- $f_{n}^{k,L}$ (or $f_{n}^{k,R}$) denotes the channel gain of the interference link from the PBS to the SBS (PBS) on the $k$th subcarrier.
- $f_{n,m}^{k}$ (or $f_{n,m}^{k}$) denotes the channel gain of the interference link from the $n$th SU (SBS) to the $m$th SU (PBS) on the $k$th subcarrier.

The superscripts ‘$D$’ and ‘$U$’ of the channel gain notations indicate downlink and uplink, respectively. For simplicity, we will omit these superscripts in the following system model. Furthermore, we only model the uplink of the CRN as the downlink can be treated similarly.

In the hybrid network, the PRN is licensed to operate on a specific frequency band but the SUs are allowed to opportunistically access the licensed spectrum under the interference temperature constraints of PUs. The available spectrum is partitioned into $K$ subcarriers each with a bandwidth of $w$ which is much less than the coherence bandwidth of the channel such that each subcarrier experiences flat fading. Orthogonality between subcarriers is assumed to be preserved perfectly so that intercarrier interference does not exist. Besides, we assume a slow-fading channel such that the channel variations on each subcarrier are relatively slow as compared to the channel estimation rate performed by the PBS and SBS. With this assumption, the PBS and SBS can accurately track the channel state information (CSI) for all PUs and SUs on different subcarriers. In this context, we denote the PU, SU and subcarrier sets as $M = \{1, 2, \ldots, M\}$, $N = \{1, 2, \ldots, N\}$ and $K = \{1, 2, \ldots, K\}$, respectively.

The OFDM channel is assumed to exhibit frequency-selective Rayleigh fading where noticeable correlation between the channel gains of adjacent subcarriers exists. Using the Jakes model [14], the subcarrier correlation coefficient between the $j$th and the $k$th subcarriers for the $n$th SU is defined as

$$
\chi_{n,j,k} = \frac{1 + \delta_{n,j,k}}{2}\left[\frac{2\sqrt{\delta_{n,j,k}}}{(1 + \delta_{n,j,k})} - \frac{\pi}{2}\right]
$$

(1)

where $\hat{\mathbf{E}}$ is the complete elliptic integral of the second kind and $\delta_{n,j,k}$ is given as

$$
\delta_{n,j,k} = \frac{1}{\sqrt{1 + [2\pi(j - k)\sigma_{rms}^{\tau}]}}
$$

(2)

In (2), $\sigma_{rms}^{\tau}$ denotes the root-mean-square channel delay spread normalised by the number of subcarriers and is expressed in [9] as

$$
\sigma_{rms}^{\tau} = \sqrt{\sum_{i=1}^{L} A_i^2 \tau_i^2 - \sum_{i=1}^{L} A_i^2 \tau_i} / \sum_{i=1}^{L} A_i^2
$$

(3)

where $A_i$ and $\tau_i$ are the amplitude and time delay for the $i$th
ray, respectively, while \( L \) is the total number of rays in the Rayleigh model.

To model SA in the hybrid network, let \( c_n = (c_{n1}, c_{n2}, \ldots, c_{nm}) \), \( m \in M \) and \( d_n = (d_{n1}, d_{n2}, \ldots, d_{nk}) \), \( n \in N \) denote the SA binary decision vectors for the PUs and SUs, respectively, where

\[
\begin{align*}
\delta_m^k &= \begin{cases} 1, & \text{if the } k\text{th subcarrier is allocated to the } n\text{th PU} \\ 0, & \text{otherwise} \end{cases} \\
d_n^k &= \begin{cases} 1, & \text{if the } k\text{th subcarrier is allocated to the } n\text{th SU} \\ 0, & \text{otherwise} \end{cases}
\end{align*}
\]

(4) (5)

Since each subcarrier is exclusively utilised by only one PU and one SU at one time, we have \( (\sum_{m=1}^{M} \delta_m^k) \leq 1, \forall k \in K \), \( (\sum_{n=1}^{N} d_n^k) \leq 1, \forall k \in K \) and \( \sum_{k=1}^{K} \left( (\sum_{m=1}^{M} \delta_m^k) + (\sum_{n=1}^{N} d_n^k) \right) \leq 2K \).

In an OFDMA-based CRN, different subcarriers may experience different channel gains. If the transmit power of the \( n\)th SU on the \( k\)th subcarrier is \( p_n^k \), its received signal-to-interference-plus-noise ratio is given by

\[
\gamma_n^k = \frac{P_n^k d_n^k}{\sum_{m=1}^{M} c_m^k d_m^k h_n^k + \eta_n^k} = \frac{P_n^k d_n^k}{\bar{I}_n^k}
\]

(6)

where \( P_n^k, m \in M \) is the transmit power of the PBS to the \( n\)th PU on the \( k\)th subcarrier and \( \eta_n^k \) is the background noise power on the signal received by the SBS using the \( k\)th subcarrier. The maximal transmit power of the \( n\)th SU is denoted as \( P_{n,\text{max}}^k \geq (\sum_{k=1}^{K} d_n^k p_n^k) \). For the downlink, the maximal transmit power of the SBS is given by \( P_{\text{BS}}^\text{max} \geq \sum_{m=1}^{M} (\sum_{n=1}^{N} d_n^k p_n^k) \). We assume that \( M\)-ary quadrature amplitude modulation (MQAM) is adopted on the subcarriers. For a fixed desired bit-error rate (BER) performance, the achievable throughput for the \( n\)th SU on the \( k\)th subcarrier is given by \( R_n^k = w \log_2 \left( 1 + (\gamma_n^k/\Theta) \right) \) where \( \Theta = \ln(0.2/\text{BER})/1.5 \) [9, 15]. The total throughput of the \( n\)th SU is \( R_n = \sum_{k=1}^{K} d_n^k r_n^k \). Accordingly, the overall throughput \( R_F \) of all SUs over all subcarriers is given by

\[
R_F = w \sum_{n=1}^{N} \sum_{k=1}^{K} d_n^k \log_2 \left( 1 + \frac{P_n^k d_n^k}{\Theta (\sum_{m=1}^{M} c_m^k d_m^k h_n^k + \eta_n^k)} \right)
\]

(7)

To quantify spectrum fairness among SUs, we use the Jain’s fairness index [16], which is a widely used fairness indicator. In this context, the total throughput of each SU in the OFDMA CRN is used to compute the Jain’s fairness index, \( \rho \) as follows

\[
\rho = \frac{\left( \sum_{n=1}^{N} R_n \right)^2}{N \sum_{n=1}^{N} (R_n)^2}
\]

(8)

where \( \rho \) can range from \( 1/K \) (the most unfair case) to 1 (the fairest case).

### 2.2 Interference temperature model

The most popular criterion to quantify and manage interference in spectrum sharing between PRN and CRN is interference temperature [1]. According to [2], interference temperature is defined as the radio frequency (RF) power measured at the PU and is used to provide an accurate measurement of the acceptable RF interference in a frequency band. Any transmission from a SU in a frequency band is considered detrimental to the PU if the interference generated by the former exceeds the interference temperature threshold. In other words, the licensed frequency band could be made available to a SU provided the interference temperature limit is not exceeded. Let \( T_g \) denote the interference temperature of a channel with bandwidth \( w \) and central frequency \( f_c \), \( T_g \) is generally expressed as \( T_g(f_c,w) = \left[ P_{\text{BS}}(f_c,w) \right] \text{watts} \). \( P_{\text{BS}}(f_c,w) \) is the average interference power in watts centred at \( f_c \) covering bandwidth \( w \) and is the Boltzmann’s constant.

In this work, a generalised interference temperature model [2] is adopted where no prior information about the RF environment is available and hence a licensed signal cannot be identified in the presence of interference and noise. In this model, the interference temperature is measured at some point but not at the PUs. In other words, this model is frequency band-based and has the same interference temperature threshold on each frequency band. Under this generalised model, we have

\[
\frac{\left( \sum_{n=1}^{N} d_n^k p_n^k + \eta^k \right)}{\text{watts}} \leq T_g
\]

(9)

where \( T_g \) is the interference temperature threshold on each subcarrier with bandwidth \( w \) and \( \eta^k \) is the interference power sensed on the \( k\)th subcarrier at the measurement point. Note that the interference temperature constraints are per-subcarrier-based and each subcarrier can only be used by at most one SU at one time, hence (9) can be simplified as

\[
\frac{\left( \sum_{n=1}^{N} d_n^k p_n^k + \eta^k \right)}{\text{watts}} \leq T_g
\]

(10)

From (10), there are \( N \) active SUs, each of which interferes with the PUs. Therefore each SU needs to select appropriate subcarriers and transmit power to achieve the target quality of service without generating harmful interference to the PUs. Thus, it is assumed that every SU is endowed with the capability of adapting to its respective environment by making timely changes to certain operating parameters, that is, carrier frequency and transmit power.

### 3 Problem and game formulation

In this paper, we model the SA and PA problem using a Blotto game (SPBG) for the uplink and downlink of CRNs. The objective is to develop efficient techniques which can allocate power and subcarriers fairly to every SU under the interference temperature constraints.

#### 3.1 Blotto game formulation

In this work, we extend the two-person Blotto game into a stochastic \( N \)-player Blotto game [13] which is more appropriate to model the SA and PA problem in multiusers CRNs. In this game, each SU is allocated a total budget of \( B_n^\text{max} \) which is to be spent on \( K \) subcarriers subject to the constraint \( \sum_{k=1}^{K} b_n^k \leq B_n^\text{max} \), where \( b_n^k \) is the BA strategy of the \( n\)th SU on the \( k\)th subcarrier. The budget could be in
the form of fictitious credit [10] issued by the SBS for bidding purposes.

Let SPBG = \( \{ N, \{ P_n, B_n \}, \{ u_n(\bullet) \} \} \) denote the Blotto game where \( N \) is the index set for the bidders (SUs), \( P_n = [0, P_{n}^{\text{max}}] \) and \( B_n = [0, B_{n}^{\text{max}}] \) are the PA and BA strategy sets, respectively, while \( u_n(\bullet) \) is the utility function for the \( n \)th user. In this game, every SU tries to allocate their power and budget to maximise their utility functions (or win as many subcarriers as possible). To preserve high efficiency, the SUs should be encouraged to win the ‘good’ subcarriers (Note: ‘good’ subcarriers refer to the subcarriers with good channel conditions for a particular SU). In a fair auction, the SU who allocates the highest budget on a subcarrier wins that subcarrier. Therefore a rational SU will always allocate more budget to the good subcarriers. To preserve high efficiency, the SUs should be encouraged to win the ‘good’ subcarrier. From (11), \( \sum_{k=1}^{K} b_{n,k} \) is the probability of the \( n \)th user winning the \( k \)th subcarrier, and \( p_n = (p_n, p_{n,1}, \ldots, p_{n,K}) \) and \( p_n = (p_n, p_{n,1}, \ldots, p_{n,K}) \) are the budget and power vectors for the \( n \)th SU, respectively, where \( p_n \in P_n \) and \( b_{n,k} \in B_n \). The utility function in (11) combines BA and PA strategies for the DSPBG such that (11) can be maximised subject to the budget and interference temperature constraint of each PU. Unlike the conventional auctions, the proposed SPBG allows users to simultaneously bid for all subcarriers. Owing to limited budget, the SUs need to allocate their budget wisely in order to win as many good subcarriers as possible to maximise their own throughput. In the following, we propose two SPBG schemes for both the downlink and uplink.

### 3.2 DSPBG and USPBG

Since the SBS is a transmitter in the downlink which has high transmit power capability, we can assume its transmit power \( P_{\text{SBS}}^{\text{max}} \gg \sum_{n=1}^{N} \sum_{k=1}^{K} d_k^2 (p_n^k)^2 \) where \( p_n^k \) is the optimal transmit power for the \( n \)th SU on the \( k \)th subcarrier. The power constraint of the SBS is hence ignored in the formulation of the downlink SPBG (DSPBG) [4]. Optimal BA and PA strategies are required for the DSPBG such that (11) can be maximised subject to the budget and interference temperature constraints, that is

\[
(u_n(b_n, p_n), \text{s.t.}) \left\{ \begin{array}{l}
\sum_{k=1}^{K} b_{n,k} \leq B_{n}^{\text{max}} \\
\sum_{n} \sqrt{\sum_{k=1}^{K} r_{kn} b_{n,k}^2 + f_k^2} \leq T_{g}
\end{array} \right.
\]

where \( x^+ = \max(x, 0) \) and the downlink BA strategy is given by

\[
p_n^k = \left( \frac{\kappa W T_{g}}{f_{k,m}} - f_k^2 \right)^+ \text{, } \forall k \in K
\]

and the uplink BA strategy is obtained as in (14).

\[
(b_n)_{\text{max}} u_n(b_n, p_n) \text{, s.t.} \left\{ \begin{array}{l}
\sum_{k=1}^{K} b_{n,k} \leq B_{n}^{\text{max}} \\
\sum_{n} \sqrt{\sum_{k=1}^{K} r_{kn} b_{n,k}^2 + f_k^2} \leq T_{g}
\end{array} \right.
\]

The approach used in [13] to derive the equilibrium state in Blotto games is adopted in this paper to obtain the optimal BA and PA strategies.

**Proposition 1:** If all SUs maximise their utility function according to (12), then the downlink PA strategy is given by

\[
p_n^k = \left( \frac{\kappa W T_{g}}{f_{k,m}} - f_k^2 \right)^+ \text{, } \forall k \in K
\]

**Proof:** See Appendix.

Unlike the SBS that possesses high transmit power capability, each SU is normally equipped with a battery-operated, power-constrained transceiver. The transmit power constraint of such transceivers is the main concern in uplink PA and it also affects BA. Therefore the power constraint of each SU needs to be included in formulating the uplink SPBG (USPBG). In order to find the optimal solution for (15), we use the approach similar to Proposition 1 and obtain the following proposition.

**Proposition 2:** If all SUs maximise their utility function according to (15), the uplink PA strategy is given by

\[
p_n^k = \left( \frac{\kappa W T_{g}}{f_{k,m}} - f_k^2 \right)^+ \text{, } \forall k \in K
\]

and the uplink BA strategy is obtained as in (14).

**Proof:** See Appendix.

Propositions 1 and 2 demonstrate that the BA strategies for both the downlink and uplink are identical. However, there is an extra condition imposed on the uplink PA strategy because of the power constraint of SUs. If we let the maximum transmit power of the \( n \)th SU approaches infinity \( (P_{n}^{\text{max}} \to \infty) \) on the uplink, then \( \Delta_n \) in (16) will approach infinity and (16) can be simplified to (13). Besides, it is worth mentioning that the PA strategies given in (13) and (16) are found to correspond to the traditional waterfilling PA techniques which can provide high spectral efficiency [3]. Since the PA strategies are independent of the BA strategies, the transmit power can be waterfilled onto different subcarriers before implementing BA.

After BA, the SBS which acts as an auctioneer will assign the available subcarriers to the SUs based on their biddings. In general, SA is a combinatorial problem which requires complex algorithms to obtain the optimal solution [5, 9]. In this work, we modify the binary constraint in (5) into a probabilistic SA decision to facilitate design of a low-complexity SA algorithm. Thus, (5) becomes

\[
u_{n}^{k} = \begin{cases} 1, & \text{if } n = \arg \max_{i \in N} (b_{i,k}) \\ 0, & \text{otherwise} \end{cases}
\]
Next, we define the set $S_n$ to include all the subcarriers that have been allocated to the $r^{th}$ user, that is

$$S_n = \{k | d^k_n = 1, \; \forall k \in \mathcal{K}\} \quad (18)$$

By manipulating the amount of budget available to each SU, competitive fairness can be enforced in the CRN. The following proposition describes the condition to ensure fairness in the proposed schemes.

Proposition 3: Both DSPBG and USPBG can achieve competitive fairness if each SU is allocated an equal amount of budget.

Proof: Consider two SUs (SU 1 and SU 2) whose achievable throughputs on two subcarriers are shown in Fig. 2. If both SU 1 and SU 2 are allocated an equal amount of budget ($B_{\text{max}} = B_{\text{max}}^1 = B_{\text{max}}^2$), their BA strategies are $b_1 > b_2$, $b_1 > b_2$, $b_1 < b_2$ and $b_1^2 > b_2^2$ for both the downlink and uplink. We notice that the budget allocated by SU 2 for subcarrier 1 is greater than that of SU 1 although they achieve similar throughput difference between subcarriers 1 and 2 is negligible. Therefore we infer that SU 2 is willing to pay more for subcarrier 1 than SU 1. This concept is first introduced in [11] and results in achieving competitive fairness in the proposed games if each SU is allocated an equal amount of budget. \hfill $\square$

Note that the BA strategies proposed in Propositions 1 and 2 are not symmetrical, hence the symmetric monotonic Bayesian equilibrium in [13] is not applicable to the DSPBG and USPBG. In the next subsection, we will study the equilibrium of the DSPBG and USPBG.

3.3 Existence and uniqueness of NE in the DSPBG and USPBG

Since the proposed downlink and uplink PA strategies are independent of BA and do not require knowledge of other SUs’ action for implementation, the PA is an iterative waterfilling algorithm without divergence problem. However, the proposed BA strategies demonstrate some strategic interdependence among the SUs and consequently the BA strategy which optimises individual utility will depend on the BA strategies of other SUs in the system. Therefore it is necessary to formulate a set of BA strategies whereby all SUs are satisfied with the utility attained given the BA of other SUs. Such an equilibrium operating point is called a NE in game theory [17]. Next we investigate the existence of NE in the DSPBG and the USPBG.

Theorem 1: NE exists in the DSPBG and USPBG.

Proof: According to the Implicit Function Theorem [17], a Jacobian matrix must be non-singular at the point of existence. By using (14), we define

$$F_n = -b^k_n + \frac{B_{\text{max}}}{\sqrt{\sum_{i=1}^{K} |\sum_{j=1, j \neq n} b^i_j|^2}} B_k = 0,$$

\forall k \in \mathcal{K}, n \in \mathcal{N} \quad (19)$$

where $F_n$, $\forall n \in \mathcal{N}$ are differentiable functions. Taking the partial derivative ($\partial F_n / \partial b^k_n$), $\forall n \in \mathcal{N}$ results in $-1$ on the main diagonal of the Jacobian matrix while the terms outside the main diagonal can be expressed as

$$\frac{\partial F_n}{\partial b^k_n} = \frac{B_{\text{max}}}{\sqrt{\sum_{i=1}^{K} |\sum_{j=1, j \neq n} b^i_j|^2}} \left( \sum_{i=1}^{N} |\sum_{j=1, j \neq n} b^i_j|^2 \right)^{-1}$$

$$\forall i \in \mathcal{N}, i \neq n \quad (20)$$

Since $r^k_n \gg 1, \forall k \in \mathcal{K}, n \in \mathcal{N}$ (normally in Mbps), ($\partial F_n / \partial b^k_n$), $\forall i \in \mathcal{N}, i \neq n$ is always of the order of $10^{-3}$ or smaller. Therefore the values of the terms ($\partial F_n / \partial b^k_n$), $\forall i \in \mathcal{N}, i \neq n$ are extremely small and will only have a negligible impact on the non-singularity of the corresponding Jacobian matrix. Furthermore, since the Jacobian matrix is a continuous function with respect to $r^k_n$, solutions exist for the entire range of large values of $r^k_n$. In conclusion, by assuming that $r^k_n, \forall k \in \mathcal{K}, n \in \mathcal{N}$ are large enough, the solution for (18) exists and hence the existence of NE can be ensured. \hfill $\square$

We next prove the uniqueness of this NE, which ensures convergence of the algorithm to be proposed in Section 4. For this purpose, we use a discrete-time model where time is divided into iterations and we assume that all SUs only act once in one iteration and remain static during that iteration. Let $b^k_n(t + 1)$ and $b^k_n(t)$ be the BA strategies of the $r^{th}$ SU on the $k^{th}$ subcarrier at the next and current iterations, respectively, where $t$ is the iteration number. We can rewrite (14) as

$$f(b^k_n(t)) := b^k_n(t + 1)$$

$$= \frac{B_{\text{max}} \sqrt{\sum_{i=1, i \neq n}^{K} |\sum_{j=1, j \neq n} b^i_j(t)|^2}}{\sum_{i=1}^{K} \sqrt{|\sum_{j=1, j \neq n} b^i_j(t)|^2}} \quad \forall k \in \mathcal{K} \quad (21)$$

By using (21), the uniqueness of the NE which exists in the proposed games can be shown next.

Theorem 2: The DSPBG and USPBG have a unique NE.

Proof: In [17], it is shown that if a fixed point $b^k_n(t + 1) = f(b^k_n(t))$ exists and if the function $f$ satisfies three properties: positivity $f(b^k_n) > 0$, monotonicity $b^k_n > (b^k_n) \Rightarrow$
(14) as

\[ b^k_n(t + 1) = B^\text{max} \left( \frac{1}{\sum_{l=1}^{K} r^l_n(B^l(t) - h^l_n(t))} \right), \quad \forall k \in K \]  

From (24), the \( n \)th SU requires to know its achievable throughput on every subcarrier \( r^l_n(t), \forall k \) and the sum of all SUs’ allocated budget on every subcarrier \( B^k(t), \forall k \) to update its budget iteratively. In this work, we adopt the assumption made in [9] such that a reliable feedback channel from the SBS to every SU is available to share this information. By using (24), any cheating among the SUs can be avoided because each SU is unable to predict others’ strategies by just having the information of \( B^k, \forall k \).

The flow chart illustrating the DSPBG and USPBG algorithms is shown in Fig. 3. Initially, the SBS provides \( B^k, \forall k \) to all SUs via the feedback channels and broadcast \( B^k(0), \forall k \). Given all the necessary information, the SUs update their BA strategies as in (24). At each iteration, SUs need to submit their bidding to the SBS to update \( B^k(t), \forall k \). This can be done by allowing SUs asynchronously and iteratively broadcast a beacon at certain power level on a subcarrier which corresponds to their budget on that subcarrier. By using the available CSI, the SBS can estimate the budget allocated by each SU on each subcarrier. Furthermore, it is shown in Fig. 3 that the condition \( b^k_n(t) - h^l_n(t - 1) < \psi \) must be fulfilled by all SUs across all subcarriers before NE is declared where \( \psi \) is a convergence threshold. Upon reaching NE, the SBS

**4 Proposed algorithms for the DSPBG and USPBG**

**4.1 Algorithms for the DSPBG and USPBG**

It is noted from (14) that the BA strategies of other SUs are public information so that the proposed schemes can be analysed as a simultaneous-move game with complete information [13]. Given this information, the SUs can cheat in the games by purposely allocating budget slightly higher than the maximum one across all subcarriers in order to win all the subcarriers. To overcome this problem, we propose a distributed and iterative BA strategy by rewriting (14) as

\[ f(b^k_n) - f(e b^k_n) = B^\text{max} \left( \frac{\varepsilon}{\sum_{l=1}^{K} \sqrt{r^l_n(B^l(t) - h^l_n(t))}} \right) \]

**Fig. 3 Flow chart illustrating the DSPBG and USPBG algorithms**
allocates the subcarriers to the SUs paying the highest budget and the power can be waterfilled to the assigned subcarriers.

### 4.2 Signalling and computational complexity of the DSPBG and USPBG

While allocating budget in the DSPBG and USPBG algorithms, information exchange between the SBS and SUs is needed. The signalling complexity of the proposed schemes is similar to that of the iterative auction-based spectrum sharing [18], which has been proven to be feasible for practical implementation. In addition, our algorithms are more favourable than that of [18] because of rapid convergence (to be shown in Section 5).

In this work, the DSPBG and USPBG algorithms are proposed to overcome the complexity problem faced by the Nash bargaining solution (NBS) scheme. In [9], the NBS approach adopts cooperative game theory which requires users to negotiate and form coalitions to obtain the optimal solution. However, this results in high computational complexity. The Hungarian method is therefore introduced to the NBS scheme to reduce the overall complexity to $O((k^2 \log_2 N + k^4)T_{NBS})$ where $T_{NBS}$ is the number of

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Fig. 4 Convergence curves of the DSPBG and USPBG algorithms for different correlation coefficients

- a) DSPBG, $\chi = 0$
- b) USPBG, $\chi = 0$
- c) DSPBG, $\chi = 0.40$
- d) USPBG, $\chi = 0.40$
- e) DSPBG, $\chi = 0.70$
- f) USPBG, $\chi = 0.70$
- g) DSPBG, $\chi = 1.0$
- h) USPBG, $\chi = 1.0$

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iterations required for the NBS algorithm to converge. Nevertheless, the NBS approach is still not feasible for mobile applications particularly in networks with large numbers of SUs and subcarriers because its complexity is exponential to the number of SUs and subcarriers. Let $T_{WF}$ and $T_{BA}$ be the numbers of iterations required for the waterfilling PA and BA algorithms to converge, respectively. The overall complexities of the proposed algorithms are $O((T_{WF} + 2T_{BA} + 1)KN)$ and $O(2T_{WF} + 2T_{BA} + 1)KN$ for the downlink and uplink, respectively. Unlike the NBS scheme, we notice that the complexity of the proposed algorithm only increases linearly with the number of users and subcarriers.

5 Simulation results

We consider a PRN and a CRN each with the cell radius of 1 km. The PBS and SBS are placed at the centre of the cells where PUs and SUs are uniformly distributed around their respective base stations. The distance between the PBS and the SBS is chosen arbitrarily within (0, 1) km. To model an urban non-line-of-sight propagation environment, we use the path loss exponent $\nu \approx U(3, 5)$ and the shadowing effect is assumed to be $10 \log_{10} H_n \approx N(0, \sigma^2)$ where $\sigma^2 \approx U(4, 10)$ [17]. We simulate a four-ray ($L = 4$) Rayleigh model which has $\chi_k^{ij} = 0.8, \forall j, k \in K$ for a correlated OFDM channel (except for Fig. 6). The correlated channel is then partitioned into $K = 128$, each with a bandwidth of $w = 25$ kHz. Each SU deploys an isotropic transmitter with the same maximum power of $P_{\text{max}} = 50$ mW. In order for the SNR gap approximation to be valid, we consider a BER of $10^{-4}$ at the output of the 16-QAM demodulator [15]. The background noise is assumed to be additive white Gaussian noise of $n_n \sim N(0, 10^{-12})$. To enforce competitive fairness, we assume that the SBS always allocates equal amount of budget to every SU, $B_{\text{max}} = 1, \forall n \in N$ (except for Fig. 5). For convergence, $\psi = 10^{-5}$ is used.

We first show the convergence curves of the proposed DSPBG and USPBG algorithms in Fig. 4 where a CRN with $M = 20$ and $N = 20$ is considered. We simulate the DSPBG and USPBG algorithms in channels with correlation coefficients ranging from 0 (uncorrelated) to 1 (highly correlated). Each curve in these sub-figures corresponds to the aggregate budget allocated by all SUs on each subcarrier. In all cases, each SU first allocates an equal amount of budget to every subcarrier and the budget allocated on each subcarrier is seen to monotonically converge to a unique NE. The proposed algorithms are able to converge quickly (less than 10 iterations) regardless of the correlation coefficients. Interestingly, as evident by the less convergence curves observed in highly correlated scenarios (i.e. when $\chi$ increases), we also notice that the amount of budget spent on several subcarriers gradually becomes more similar. This phenomenon occurs because in highly correlated channels, the channel gains on different subcarriers experienced by a SU are almost similar and these encourage the SU to allocate budget more equally to every subcarrier to maximise its utility function.

Next, Fig. 5 illustrates the impact on spectrum fairness and efficiency because of asymmetrical BA by the SBS in a two-SU CRN. In both the downlink and uplink, it is depicted in Fig. 5a that the spectrum is dominated by SU 2 when SU 1’s available budget is less than that of SU 2. By gradually increasing the budget for SU 1, the throughput of SU 1 starts to increase as more budget is available for SU 1 to compete for the spectrum. SU 1 and SU 2 obtain almost identical throughput when they are allocated an equal amount of budget. A further increase in budget for SU 1...
will cause unfairness to SU 2 because the spectrum would then be dominated by SU 1. In Fig. 5, note that the highest total throughput can be acquired when the SBS allocates budget equally to the two SUs. This further encourages the SBS to implement symmetrical BA to every SU, not only for achieving competitive fairness, but also for attaining higher spectral efficiency.

Fig. 6 shows the relationship between spectrum fairness and subcarrier correlation for 16-QAM and 64-QAM. Performance comparison with the downlink NBS (DNBS) and uplink NBS (UNBS) schemes [19] is made. It is observed in Fig. 6 that systems adopting the DNBS and UNBS schemes suffer from more serious unfairness when $\chi$ increases for both 16-QAM and 64-QAM. This is because in highly correlated fading channels ($\chi > 0.5$), the subcarriers of a SU may experience deep fades simultaneously. In the DNBS and UNBS schemes, SUs with poor channel conditions are always ensured a small portion of the resources, but most of the resources will be assigned to SUs with good channel conditions to improve spectral efficiency. Unlike the DNBS and UNBS schemes, the proposed DSPBG and USPBG schemes achieve significant performance improvement to ensure fairness among SUs. In the DSPBG and USPBG schemes, if the SBS allocates an equal amount of budget to every SU, all SUs will have equal chance to compete for every subcarrier regardless of their channel conditions. However, a rational SU will always allocate more budget for strong subcarriers and this ensures high spectral efficiency. From Figs. 6a and b, we can conclude that by providing a fair competition to every SU, spectrum fairness can be enforced in CRNs in both uncorrelated and correlated channels.

Although the DSPBG and USPBG schemes can ensure fairness in CRNs, the spectral efficiency is sacrificed. Fig. 7 shows the total throughput and fairness index against the interference temperature for both downlink and uplink. The throughputs achieved by the downlink maximal-rate (DMR) and uplink maximal-rate (UMR) schemes are the maximum achievable throughputs. In term of fairness, the downlink max–min (DMM) and uplink max–min (UMM) schemes are the optimal SA solutions. It is observed in Figs. 7a and b that the performance of all SA schemes improves gradually when the interference temperature increases because the SUs are allowed to transmit at a higher power to achieve a higher throughput. As seen in Figs. 7a and b, the DSPBG and USPBG schemes can achieve near-optimal performance, which is $\sim 98\%$ of the maximal-rate throughput. Even though the DNBS and UNBS schemes attain a higher throughput than the proposed schemes, the performance gaps between the proposed schemes and the NBS schemes are negligible. The DMM and UMM schemes exhibit the worst throughput as they do not consider spectral efficiency. On the other hand, we notice from the Figs. 7c and d that the proposed schemes outperform the DNBS and UNBS schemes in terms of fairness. The fairness indices achieved by the DSPBG and USPBG schemes are $\sim 92\%$ of the max–min fairness indices for both the downlink and uplink. This demonstrates that the proposed schemes are efficient (from the system’s perspective) and fair (from the SU’s perspective).

6 Conclusion

In this paper, the SA and PA problems in OFDMA-based CRNs have been studied using a more realistic OFDMA channel model which considers the correlation between
adjacent subcarriers. It is shown that spectrum unfairness becomes more severe in correlated fading channels. To alleviate this problem, we have proposed the DSPBG and USPBG schemes for the downlink and uplink, respectively. By using a Lagrangian relaxation method, two PA and BA strategies have been derived for SUs to share the spectrum efficiently and fairly. If the SBS allocates an equal amount of budget to every SU, spectrum fairness can be enforced at a minimal loss of total throughput. The convergence of the proposed algorithms has been proven mathematically and via simulations. One of the key advantages of the proposed methods is that the algorithms can converge quickly even in large-scale CRNs. Numerical results show that the proposed methods are both efficient and fair.

7 References


8 Appendix

8.1 Proof of Proposition 1

By using Lagrangian relaxation, (12) can be rewritten as

$$\mathcal{L}(b_n, p_n, \alpha_n, \mu_n) = \sum_{k=1}^{K} \left[ \left( \frac{b_{k}^{n}}{\sum_{i=1}^{N} b_{i}^{n}} \right) w \log_2 \left( 1 + \frac{p_{k}^{n} b_{k}^{n}}{\Theta \Omega_{kn}} \right) \right] - \alpha_n \left( \sum_{k=1}^{K} b_{k}^{n} p_{k}^{n} \right) - \mu_n \left( \frac{f_{k}^{n} p_{k}^{n} + I_{k}}{\kappa} - T_g \right)$$

(25)

where $\alpha_n$ and $\mu_n$ are positive Lagrangian multipliers. Taking the partial derivatives of (25) with respect to $b_{k}^{n}$, $p_{k}^{n}$, $\alpha_n$, $\mu_n$, $\forall k \in K$ gives

$$\frac{\partial \mathcal{L}}{\partial b_n} = \left( \frac{\sum_{i=1}^{N} b_{i}^{n}}{\sum_{i=1}^{N} b_{i}^{n}} \right) w \log_2 \left( 1 + \frac{p_{k}^{n} b_{k}^{n}}{\Theta \Omega_{kn}} \right) - \alpha_n, \forall k \in K$$

(26)

$$\frac{\partial \mathcal{L}}{\partial p_n} = \left( \frac{b_{k}^{n}}{\sum_{i=1}^{N} b_{i}^{n}} \right) \frac{w \Theta_{kn}^{2}}{\kappa} b_{k}^{n} - \mu_n, \forall k \in K$$

(27)

$$\frac{\partial \mathcal{L}}{\partial \alpha_n} = B_{\max} - \sum_{k=1}^{K} b_{k}^{n}$$

(28)

$$\frac{\partial \mathcal{L}}{\partial \mu_n} = T_g - \frac{f_{k}^{n} p_{k}^{n} + I_{k}}{\kappa}$$

(29)

From (27) and (29), we can obtain $p_{k}^{n}$ as

$$p_{k}^{n} = \min \left\{ \frac{b_{k}^{n} \kappa w}{\mu_n f_{k}^{n} \left( \sum_{i=1}^{N} b_{i}^{n} \right)} + \frac{\kappa T_g}{f_{k}^{n} - f_{k}^{n}} \right\}$$

(30)

Since there is no power constraint for the downlink, the optimal power allocated to the $k$th subcarrier by the nth SU is only constrained by the interference temperature and can be simplified as

$$p_{k}^{n} = \left( \frac{\kappa T_g}{f_{k}^{n} - f_{k}^{n}} \right)^{+}$$

(31)

From (31), it is noticed that $p_{k}^{n}$ is independent of the BA and thus the BA strategy can be viewed as a one-dimensional maximisation problem with $p_{n}$ fixed. For simplicity, let $r_{n}^{k} = \log_2(1 + (p_{n}^{k} b_{k}^{n})/\Theta \Omega_{kn})$ and $(\partial \mathcal{L}/\partial b_{n}^{k}) = 0$, we have

$$\frac{\left( \sum_{i=1}^{N} b_{i}^{n} \right)}{\left( \sum_{i=1}^{N} b_{i}^{n} \right)^2} r_{n}^{k} = \frac{\left( \sum_{i=1}^{N} b_{i}^{n} \right)^2}{\left( \sum_{i=1}^{N} b_{i}^{n} \right)^2}$$

$$= \ldots = \frac{\left( \sum_{i=1}^{N} b_{i}^{n} \right)^K}{\left( \sum_{i=1}^{N} b_{i}^{n} \right)^K} r_{n}^{K} = \alpha_n$$

(32)

Without loss of generality, (32) can be rearranged and
represented using a general expression, that is

\[(B^k)^2 r^k_n \sum_{i=1, j \neq n}^N b^k_i = (B^k)^2 r^k_n \sum_{i=1, j \neq n}^N b^k_i, \forall k, l \in \mathcal{K} \quad (33)\]

where \(B^k = (\sum_{i=1}^N b^k_i)\) is the total budget allocated by all SUs on the \(k\)th subcarrier. By rearranging (33), \(B^k\) is denoted as

\[B^k = \sqrt{\frac{r^k_n \sum_{i=1, j \neq n}^N b^k_i}{r^k_n \sum_{i=1, j \neq n}^N b^k_i}} \forall k, l \in \mathcal{K} \quad (34)\]

Note that

\[B = \sum_{k=1}^K B^k = \sum_{i=1}^N B_i \quad (35)\]

where \(B_i\) is the total budget spent by the \(i\)th SU and \(B\) is the total budget spent by all SUs on all subcarriers. Substituting (34) into (35) yields

\[B = \sum_{i=1}^K B^k = B^k \sum_{i=1}^K \sqrt{\frac{r^k_n \sum_{i=1, j \neq n}^N b^k_i}{r^k_n \sum_{i=1, j \neq n}^N b^k_i}} \quad (36)\]

By rearranging (36), we have

\[B^k = \sqrt{\frac{r^k_n \sum_{i=1, j \neq n}^N b^k_i}{\sum_{i=1}^K r^k_n \sum_{i=1, j \neq n}^N b^k_i}} B = A^k_i B \quad (37)\]

The ratio of (32) for the \(l\)th SU over that of the \(n\)th SU gives

\[\frac{r^l_n (\sum_{i=1, j \neq h}^N b^l_i)}{r^l_n (\sum_{i=1, j \neq n}^N b^l_i)} = \frac{r^l_n (\sum_{i=1, j \neq h}^N b^l_i)}{r^l_n (\sum_{i=1, j \neq n}^N b^l_i)} = \ldots = \frac{r^l_n (\sum_{i=1, j \neq h}^N b^l_i)}{r^l_n (\sum_{i=1, j \neq n}^N b^l_i)} = \frac{\alpha_h}{\alpha_n} \quad (38)\]

Equation 38 can be rewritten in a general expression as follows

\[(B^k - b^k_h) b^k_n = \frac{\alpha_n}{\alpha_n} (B^k - b^k_h) b^k_n, \forall k \in \mathcal{K} \quad (39)\]

Note that

\[\sum_{i=1, j \neq h}^N B_i = \sum_{j=1}^K \sum_{i=1, j \neq h}^N b^l_i \frac{\alpha_h}{\alpha_n} r^l_n \sum_{i=1, j \neq n}^N B_i \quad (40)\]

Therefore

\[\frac{\alpha_h}{\alpha_n} = \frac{r^l_n (\sum_{i=1, j \neq h}^N B_i)}{r^l_n (\sum_{i=1, j \neq n}^N B_i)} = \frac{r^l_n (B - B_h)}{r^l_n (B - B_n)} \quad (41)\]

Substituting (37) and (41) into (39) produces

\[(A^k_i B - b^k_h) r^k_n = \frac{r^k_n (B - B_h)}{r^k_n (B - B_n)} (A^k_i B - b^k_h) r^k_n \quad (42)\]

Rearranging (42) gives

\[b^k_n = \frac{(B_h - B_n)}{(B - B_n)} A^k_i B + \frac{(B - B_h)}{(B - B_n)} b^k_h \quad (43)\]

Note that

\[B^k = \sum_{h=1}^N b^k_h = \sum_{h=1}^N \left( \frac{(B_h - B_n)}{(B - B_n)} A^k_i B + \frac{(B - B_h)}{(B - B_n)} b^k_h \right) \]

\[= \frac{1}{B - B_n} \left( A^k_i B \sum_{h=1}^N (B_h - B_n) + b^k_n B(N - 1) \right) \quad (44)\]

Rearranging (44), we obtain \(b^k_n\) as

\[b^k_n = \frac{B^k}{(N - 1)B} \left( (B - B_n) - \sum_{h=1}^N (B_h - B_n) \right) \]

\[= \frac{B^k B_n}{B} = A^k_i B_n \quad (45)\]

If we assume that the total available budget will be fully spent, then \(B_n = B_n^{\text{max}}\) and (45) becomes

\[b^k_n = \frac{B_n^{\text{max}}}{\sqrt{\sum_{i=1}^K \sum_{j=1, j \neq n}^N r^l_n (\sum_{i=1, j \neq h}^N b^l_i)}} \quad (46)\]

The proposed BA strategy is thus obtained. \(\square\)

### 8.2 Proof of Proposition 2

By using Lagrangian relaxation, (15) can be rewritten as

\[\mathcal{L}(b_n, p_n, \alpha_n, \mu_n) = \sum_{k=1}^K \left( \left( \frac{b^k_h}{\sum_{i=1}^N b^k_i} \right) w \log_2 \left( 1 + \frac{r^k_n b^k_h}{\Theta I^k_n} \right) \right) \]

\[- \alpha_n \left( \sum_{k=1}^K b^k_h - B_n^{\text{max}} \right) \]

\[- \mu_n \left( \sum_{k=1}^K r^k_n p^k_n + \frac{k}{KW} - T_g \right) \]

\[- \beta_n \left( \sum_{k=1}^K r^k_n p^k_n - P_n^{\text{max}} \right) \quad (47)\]

where \(\alpha_n, \mu_n\) and \(\beta_n\) are positive Lagrangian multipliers. Taking the partial derivative of (47) with respect to
Given that $b_1, b_2, \ldots, b_N$ are fixed, (50) can be rewritten as

$$p_n^k = \left( \frac{\Delta_n - \Theta k_n^{-1}}{g_n^k} \right)^+$$  (52)