Cooperative Coding using Cyclic Delay for Multiuser OFDM Systems

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Abstract—In this paper, the cooperative coding using cyclic delay diversity (CDD) for multiuser OFDM systems is introduced. In proposed scheme, multiple terminals with a single antenna share their resources and exploit spatial diversity. Also, in order to increase frequency selectivity in the relay channel, different cyclic time delay is used at the relays. Analysis of frame error probability (FEP) for proposed cooperative coding using CDD is shown. Simulation results show the frame error rate (FER) for various consideration. The proposed scheme provides the diversity gain according to the number of relays and cyclic delay.

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is one of the most promising technologies for high data rate wireless communications. OFDM has been adopted in wireless standards such as digital audio/video broadcasting, IEEE 802.11a, IEEE 802.16, and so on. In OFDM systems, a high-rate serial data stream is split up into a set of low-rate substreams to increase a symbol duration of OFDM signal with the number of subcarriers. As a result, OFDM systems provide the robustness to frequency selective fading and high spectral efficiency.

The combination of OFDM and multiple-input multiple-output (MIMO) techniques provides the diversity gain and/or increased capacity. However, MIMO techniques are not available in many cases due to limitation of size and complexity. Alternatively, cooperative diversity can be an attractive candidate in conjunction with OFDM systems for wireless communications. In cooperative diversity, multiple terminals with a single antenna share their antennas and obtain the benefits of MIMO techniques [1]-[5].

In this paper, we apply the cooperative coding to multiuser OFDM systems. In proposed scheme, multiple terminals with a single antenna share their resources and exploit spatial diversity. To improve the beneficial effects of the relays’s cooperation, cyclic delay diversity (CDD) is adopted in cooperative transmission of relays. CDD in the time domain results in phase rotation in the frequency domain [6], [7]. For cooperative transmission based on OFDM systems with multiple relays, CDD increases frequency selectivity in the relay channel which can be exploited at the receiver. CDD for cooperative diversity can easily handle frequency-selective fading without an increase in receiver complexity. Also, CDD for cooperative diversity provides a very flexible space-time coding approach and modest implementation complexity. We provide performance analysis of the proposed scheme in terms of frame error probability (FEP). In particular, we study the achieved diversity order for various interuser channel qualities.

This paper is organized as follows. In Section II, we introduce the system model of the cooperative coding using cyclic delay for multiuser OFDM systems. In Section III, we analyze proposed cooperative coding in terms of FEP and average power of the channel. After shown simulation results in Section IV, we conclude in Section V.

II. SYSTEM MODEL

We consider a wireless network with $M+2$ terminals which have a single antenna. A wireless network system consists of a single source $s$, a single destination $d$, and $M$ relays $\mathbf{I} = \{0, 1, \cdots, M-1\}$, $i \in \mathbf{I}$. When cooperation is applied, the total channels, which consist of $N$ time slots, are divided into two equal orthogonal subchannels for the source and relays, respectively. However, when the terminals do not cooperate with other terminals, each terminal uses the total channels for its transmission.

For convenience, we focus on the transmission of single source using multiple relays. However, this scheme can be similarly applied for the other terminals. The total transmission power for proposed scheme is less than or equal to the total transmission power of noncooperative direct transmission. The source uses the half of the total transmission power of direct transmission and the relays use the remaining half of the total transmission power. We assume perfect channel state
information at all receivers. However, the transmitter only knows the statistics of fading, but not the current realization.

In the first subchannel, the source terminal broadcasts half of its coded symbol to the relays and destination. Then, the relays decode the information symbol for the source. If cyclic redundancy check (CRC) for error detection indicates successful decoding at more than one relay, in the second subchannel the relays which succeed in decoding transmit the remaining half of the coded symbols for the source to the destination after re-encoding. When the relays transmit the signal, cyclic delay is applied to the relays for improved performance. If CRCs at all relays indicate failure of decoding, the source continuously transmits the remaining half of the coded symbol to the destination by itself in second subchannel.

After receiving whole coded symbol for the source, the destination demultiplexes signals received during whole \( N \) time slots and decodes it.

When a length-\( N \) sequence \((X_0, \ldots, X_{N-1})\) is transmitted in parallel on \( N \) subcarriers, OFDM modulation can be expressed as an inverse discrete Fourier transform (IDFT)

\[
x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) e^{j(2\pi/N)kn}, \quad n = 0, 1, \ldots, N-1. \tag{1}
\]

A cyclic guard interval (also called cyclic prefix) is added to sequence \( \{x_n\} \)

\[
\hat{x}(n+G)_{N+G} = x(n)_N, \quad n = 0, 1, \ldots, N + G - 1 \tag{2}
\]

where \( G \) is the guard interval length and \((n)_N\) is the residue of \( n \) modulo \( N \).

In the first subchannel for the source, the received signals from the source to the destination and \( i \)-th relay are given by, respectively

\[
y_{sd}(n) = h_{sd}(n) \otimes x(n) + n_{sd}(n) \tag{3}
\]

\[
y_{si}(n) = h_{si}(n) \otimes x(n) + n_{si}(n) \tag{4}
\]

where \( \otimes \) indicates circular convolution, \( y_{pq}(n) \) is the received signal from terminal \( p \) to terminal \( q \), \( x(n) \) is the transmitted signal from the source, \( h_{pq}(n) \) is the fading channel coefficients between terminal \( p \) and terminal \( q \) for \( n \)-th path, and \( n_{pq}(n) \) is the additive noise between terminal \( p \) and terminal \( q \). Note that \( h_{pq}(n) = 0 \) for \( n = L_{pq}, \ldots, N - 1 \) and \( L_{pq} \) is the number of multipath from terminal \( p \) to terminal \( q \).

The channel coefficients \( h_{pq}(n) \) and the noise \( n_{pq}(n) \) are independent zero-mean circularly symmetric complex Gaussian random variables with variances one and \( N_0 \), respectively. We also consider quasi-static fading channel, that is, the fading coefficients are constant over one frame, but are independent from terminal to terminal.

Fig. 1 shows the transmitter structure of relays for proposed scheme. In the second subchannel, \( Q \) relays, which correctly decode the received signal from the source, are participate in cooperation and after re-encoding and OFDM processing simultaneously retransmit the re-encoded symbol to the destination. Then, the received signal from the relays is given by

\[
y_{rd}(n) = \alpha \sum_{i \in V} h_{sd}(n) \otimes \hat{x}(n - \tau_i)_N + n_{rd}(n) \tag{5}
\]

where \( \alpha \) is the source to relay transmission power ratio, \( \hat{x}(n) \) is re-encoded and transmitted signal from the relays, \( \tau_i \) is the cyclic delay for \( i \)-th relay, \( V \) is the set of relays which correctly decode the received signal from the source, and \( V \subset I \). The number of relays which participate in cooperation is \( Q \). All \( Q \) relays have different \( \tau_i \) and the delay interval of \( \tau_i \) is the same as the symbol interval of \( \{x(n)\} \). \( E_{pq} \) denotes the average transmission power from terminal \( p \) to terminal \( q \) and \( \alpha = E_{id}/E_{sd} \).

Simply, each relay can transmit with normalized power by the total number of relays in the second subchannel, i.e., \( \alpha = 1/M \). However, if the relays know the number of active relays which participate in cooperation, more efficient power allocation for the relays is available [8]. For \( \alpha = 1/Q \) case, each relay transmits with normalized power by the number of active relays in the second subchannel. This more efficient power allocation for the relays can provide improved performance.

In the first subchannel, the received signal from the source at destination in frequency domain is given through (3) and its discrete Fourier transform (DFT)

\[
Y_{sd}(k) = H_{sd}(k)X(k) + N_{sd}(k). \tag{6}
\]

If the signal from the source is received through \( L_{sd} \) multipath fading, \( H_{sd}(k) \) is given by

\[
H_{sd}(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{L_{sd}-1} h_{sd}(n)e^{-j(2\pi/N)kn}. \tag{7}
\]

If the signal from the source is received through flat fading, \( L_{sd} = 1 \) and \( H_{sd}(k) = h_{sd}(n)/\sqrt{N} \).

In the second subchannel, the received signal from the relays at destination in frequency domain is given through (5) and its DFT

\[
Y_{rd}(k) = \alpha \sum_{i \in V} H_{sd}(k)e^{-j(2\pi/N)k\tau_i}\hat{X}(k) + N_{rd}(k). \tag{8}
\]

If the signal from the source is received through \( L_{id} \) multipath fading, \( H_{id}(k) \) is given by

\[
H_{id}(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{L_{id}-1} h_{id}(n)e^{-j(2\pi/N)kn}. \tag{9}
\]
For the second subchannel, the effective channel as seen by the destination in the frequency domain is given by

\[ H_{rd}(k) = \frac{\alpha}{\sqrt{N}} \sum_{i \in V} \left( \sum_{n=0}^{L_{id}-1} h_{id}(n) e^{-j(2\pi/N)kn} \right) e^{-j(2\pi/N)k\tau_i}. \]

(10)

From (10), in the second subchannel received signal from the relays to the destination can be rewritten as

\[ Y_{rd}(k) = H_{rd}(k) \tilde{X}(k) + N_{rd}(k). \]

(11)

After receiving whole coded symbol of (6) and (11), the destination demultiplexes whole coded symbol and decodes.

### III. Performance Analysis

In this section, we provide performance analysis of the proposed scheme. In particular, we study the achieved diversity order for various inter-user channel qualities because the inter-user channel quality on cooperative diversity has important meaning. When the inter-user channel has a good quality, the relays can participate in cooperation. On the contrary, when the inter-user channel has a poor quality, the relays cannot participate in cooperation.

Let \( P_f^C \) denote the FEP of the cooperative coding using cyclic delay for multiuser OFDM system. Each frame consists of \( N \) coded symbols. Then \( P_f^C \) can be given by [3, 5]

\[ P_f^C = (1 - P_f^{in}) P_f^{BF} + P_f^{in} P_f^{QS} \leq P_f^{BF} + P_f^{in} P_f^{QS} \]

(12)

where \( P_f^{in} \) denote the FEP of the first segment over the inter-user channel for \( N \) coded symbols, \( P_f^{BF} \) denote the FEP over the block fading channel in the cooperative case, and \( P_f^{QS} \) denotes the FEP of the whole frame over the quasi-static channel in the direct transmission.

Utilizing the pairwise error probability (PEP) expressions for the block Rayleigh fading channel and the union upper bound on the FEP, the upper bound on the FEP is given in (13), shown at the top of the next page.

\[ \lambda_{j,pq} \text{ for } j = 1, \ldots, r_{pq} \text{ denote the nonzero eigenvalues of the product of the codeword-difference matrix and its respective conjugate transpose for the fading block from terminal } p \text{ to terminal } q \text{, } \lambda_{j,q_{ps}} \text{ for } j = 1, \ldots, r_{pq} \text{ denote the nonzero eigenvalues of the product of the codeword-difference matrix between the two entire codewords and its conjugate transpose, } r_{pq} \text{ denotes the rank of the codeword-difference matrices for the fading block from terminal } p \text{ to terminal } q \text{, and } r_{qs} \text{ denotes the rank of the codeword-difference matrices between the two entire codewords of the source.} \]

For simplicity, we assume \( E_{sd} = E_{si} = E_s \), and \( E_{id} = \alpha E_s \). In this case, \( P_f^C \) can be approximately upper bounded by

\[ P_f^C \leq \left( \frac{E_s}{4N_0} \right)^{-K_1} (\alpha)^{-\sum_{i \in V} L_{id}} \cdot \left( \sum_{c} \sum_{e \not\in c} \left( \prod_{i \in V} \left( \prod_{j=1}^{r_{id}} \lambda_{j,ind} \right) \left( \prod_{j=1}^{r_{id}} \lambda_{j,sd} \right) \right) \right)^{-1} + \left( \frac{E_s}{4N_0} \right)^{-K_2} \]

(14)

where \( K_1 = (L_{sd} + \sum_{i \in V} L_{id}) \) and \( K_2 = (L_{sd} + \sum_{i \in V} L_{ls}) \).

For \( \alpha = 1/Q \) case, the degradation term of FEP performance, \( (\alpha)^{-\sum_{i \in V} L_{id}} \) can be reduced in (14) due to \( Q \leq M \). This more efficient power allocation for the relays can provide improved performance.

At high signal-to-noise ratio (SNR), FEP in (14) can be approximated as

\[ P_f^C \approx G_1 \left( \frac{E_s}{4N_0} \right)^{-\min(K_1,K_2)} \]

(15)

where \( G_1 \) represents the coding parameters. From (15), the proposed scheme can potentially provide a diversity order of \( \min(K_1,K_2) \).

1) Good Inter-User Channel: Assuming \( E_{sd} \approx E_{si} = E_s \), and \( E_{id} = \alpha E_s \), we can express FEP for very good inter-user channel quality as

\[ P_f^C \approx P_f^{BF} = \left( \frac{E_s}{4N_0} \right)^{-K_1} (\alpha)^{-\sum_{i \in V} L_{id}} \cdot \left( \sum_{c} \sum_{e \not\in c} \left( \prod_{i \in V} \left( \prod_{j=1}^{r_{id}} \lambda_{j,ind} \right) \left( \prod_{j=1}^{r_{id}} \lambda_{j,sd} \right) \right) \right)^{-1}. \]

(16)

Similar to (15), we can approximate FEP for good inter-user channel quality of (16) with the most dominant term \( G_2 \) as

\[ P_f^C \approx G_2 \left( \frac{E_s}{4N_0} \right)^{-K_1}. \]

(17)

Then, the maximum diversity order of the proposed scheme can be \( K_1 \) for very good inter-user channel quality.

2) Poor Inter-User Channel: We can express FEP for very poor inter-user channel quality as

\[ P_f^C \approx P_f^{in} P_f^{QS} = \left( \frac{E_s}{4N_0} \right)^{-K_2} \cdot \left( \sum_{c} \sum_{e \not\in c} \left( \prod_{i \in V} \left( \prod_{j=1}^{r_{id}} \lambda_{j,ind} \right) \left( \prod_{j=1}^{r_{qs}} \lambda_{j,q_{ps}} \right) \right) \right)^{-1}. \]

(18)

In this case, the upper bound on \( P_f^C \) is dominated by the term \( P_f^{in} P_f^{QS} \). Then, FEP of (18) for poor inter-user channel quality can be approximated as

\[ P_f^C \approx G_3 \left( \frac{E_s}{4N_0} \right)^{-K_2}. \]

(19)
The proposed scheme provides the performance improvement over noncooperative direct transmission, the performance of proposed scheme with cyclic delay can obtain.

IV. SIMULATION RESULTS

In this section, we show the simulation results for the proposed cooperative diversity with cyclic delay for OFDM systems. We use constraint length $K = 6$, rate $1/4$ $(53,67,71,75)$ convolutional codes, and binary phase-shift keying (BPSK) modulation. The extensions to other rate $1/4$ convolutional codes and higher order modulations are also available. In the first subchannel, the source uses rate $1/2$ $(53,75)$ convolutional codes which are known as optimal for a given rate and constraint length, and in the second subchannel the relays use rate $1/2$ $(67,71)$ convolutional codes [3]. The frame size is 256 bits. We assume an ideal CRC code to determine the relays’ participation in cooperation. The channel model has three Rayleigh fading taps at delays of $0 \mu s$, $2.5 \mu s$, and $5 \mu s$ with relative powers of $0 \, dB$, $-5 \, dB$, and $-10 \, dB$, respectively. Hence, $L_{sd} = L_{si} = L_{id} = 3$ for $i \in V$. OFDM sampling period is chosen as $2.5 \mu s$. Optimal cyclic delay, which satisfies $\tau_i \geq L_{id} \lambda_i$ for $i \in V$, is applied for the relays [9].

A. The Number of Relays

Fig. 2 shows the frame error rate (FER) of proposed scheme for the different number of relays with $\alpha = 1/M$. Although the performance of proposed scheme with $M = 1$ is worse than the performance of noncooperative direct transmission, the performance gain increases as the number of relays increases. The proposed scheme provides the performance improvement of about $4.5 \, dB$ at FER of $10^{-1}$, $8.8 \, dB$ at FER of $10^{-2}$, and $13.1 \, dB$ at FER of $10^{-3}$ for $M = 5$ over noncooperative direct transmission.

Fig. 3 shows the cooperation probability for the FER of proposed scheme for the different number of relays. The cooperation probability means that at least one relay participates in cooperation. As shown in Fig. 3, the more increase the number of relays are, the faster the cooperation probability reaches the probability one. Hence, for the large number of relays, the cooperation probability reaches the probability one at the low SNR.

B. The Effect of Cyclic Delay

Fig. 4 shows the frame error rate (FER) of proposed scheme with and without cyclic delay. We assume $\alpha = 1/M$. Increasing the number of relays increases the performance difference between the proposed scheme with and without cyclic delay. The proposed scheme without cyclic delay cannot obtain the performance gain according as the number of relays increases, on the other hand, the proposed scheme with cyclic delay can obtain.

C. The Power Allocation

Fig. 5 shows the FER performance of proposed scheme for different relay transmission power allocation. As expected, proposed scheme with $\alpha = 1/Q$ have better performance than proposed scheme with $\alpha = 1/M$. This more efficient power allocation for the relays provides improved performance according as the number of relays increases. Although proposed scheme using relays which transmit with normalized power by the number of total relays has poor performance than $\alpha = 1/Q$ case, it has an advantage of transmission power saving for whole network. Because the relay transmission power of inactive relays which do not participate in cooperation is saved.

V. CONCLUSIONS

In this paper, we introduce cooperative diversity using CDD for OFDM systems. We adopted cyclic delay technique...
to cooperative diversity. We analyzed the proposed cooperative diversity scheme in terms of FEP based on inter-user channel quality. The performance of proposed schemes is compared with that of direct transmission and conventional cooperative diversity which don’t apply cyclic delay. The proposed schemes provide better performance compared to other schemes.

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