Recovering Haptic Performance by Relaxing Passivity Requirements

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ABSTRACT

Several causes of instabilities in the interaction of users with actively controlled haptic devices, notably sampling effects, sensor quantization, and hardware nonlinearities, have been identified and analyzed in prior work. However, certain instabilities may occur that cannot be attributed to these hardware limitations. The problem becomes acute when rendering passive virtual environments which require compensation of the intrinsic device dynamics. We show that passivity, the standard robust stability criteria for haptic interface systems, cannot be maintained in certain circumstances due to Bode gain-phase integral constraints. In an experimental study, we demonstrate that some performance can be recovered by relaxing passivity requirements while maintaining coupled stability. To test stability, we identify frequency domain models of the user interaction, and check the Nyquist stability criterion rather than passivity.

Keywords: coupled stability, passivity, Nyquist stability criterion, Bode gain-phase integral, impedance control, frequency-domain user modeling.

1 INTRODUCTION

Practical experience demonstrates that high-fidelity haptic rendering is frequently at odds with robust, stable feedback design. In demanding applications it is important to identify intrinsic performance limitations imposed by hardware properties and stability requirements. While a certain degree of conservativeness is desirable in stability requirements as it affords robustness, it invariably comes at some cost to performance. Through trial-and-error we can probe the nature and severity of the tradeoff; however analytic expressions are preferable as they provide firm limits satisfied by all control designs and can guide hardware selection prior to controller design. While analytic results have been derived to address intrinsic stability and performance tradeoffs in haptic rendering, our understanding is still incomplete.

Traditional frequency-domain techniques that assess stability robustness in terms of gain margin and phase margin are not sufficient to address human-in-the-loop stability problems. This problem is more appropriately treated in the framework of coupled stability [4]. An effective, commonly employed approach that ensures coupled stability is to design the controller such that the closed-loop dynamics rendered to the user through the haptic device remain passive for all user interactions. Then if the human user also remains passive, the coupled system is guaranteed to be stable.

Following this basic framework, prior analysis has identified ranges of feedback gains that result in a passive closed-loop dynamics. The limits on feedback gains depend on hardware choices including sampling frequency [1, 3, 5–7, 12, 13, 15, 17], quantization [1, 3, 6, 7, 13], hardware damping and friction [1, 3, 5–7, 13, 15, 17], and discontinuities [1, 3, 5–7, 12, 13, 17]. In practical terms, limits on feedback gains degrade the quality of hard contacts that may be rendered. On the other hand, violating the bounds on feedback gains tends to result in undesirable oscillations and chatter when the user contacts surfaces in the virtual environment.

A variety of theoretical techniques have been used to study the conflict between performance and stability in haptic rendering. A combination of energy methods and frequency analysis are applied in [5] to determine a condition for passivity within a sampled-data model of haptic interface systems. A general version of this result is obtained through direct frequency-domain analysis in [8]. Time-domain energy analysis has been applied to extend these results to include nonlinear and time-delay virtual environments [12, 14, 20]. Further time-domain energy analysis of sampled-data modes reveals non-passive behavior due to a combination of sampling and quantization effects [1, 7]. While passivity is commonly used as the stability criterion for haptic rendering, a less conservative stability analysis may also be performed within a linear systems framework. Closed-loop pole analysis of a continuous-time model [15] and a discrete-time model [9] generate conditions for stable controller parameters. A disadvantage of these linear system analyses is that they assume a model for the human user.

Additional factors exist that undermine passivity and coupled stability in a haptic rendering system—factors beyond the effects of sampling, quantization, feedback delays, and the discontinuities associated with rendering hard contact. These factors are pervasive as they are present even when rendering virtual environments that are linear and time-invariant. We have often observed unstable behavior when attempting only to render simple linear virtual environments (without discontinuity). In an experimental study presented here, we demonstrate that the existence and even the quality of this unstable behavior does not depend on sampling rate and quantization. In the absence of sampled-data effects or quantization issues, we choose to perform analysis on linear time-invariant, continuous models of the feedback system. This modeling framework makes available classical results of complex analysis including Bode’s gain-phase integral relationships [2, 16] and Nyquist stability [18].

This paper analyzes the tradeoff we observe in haptic interface systems between accurate rendering and coupled stability. We show that, for certain passive linear time-invariant virtual environments, there may exist no linear time-invariant controller that can both achieve the desired level of fidelity over a finite bandwidth and also maintain passive closed-loop dynamics. In contrast to prior results, the intrinsic limitation we show is not the result of sampling, sensor quantization, or nonlinearities, rather the result of bandwidth limitations. The result follows from Bode’s gain-phase integral relationships [2]. We further show that we may relax the phase requirements imposed by passivity at certain frequencies to recover performance while maintaining a measure of robust stability. To address coupled stability, we turn to the classical Nyquist stability criterion which requires a characterization of the frequency response of the user. We demonstrate experimentally the conflict between performance and passivity and the ability to recover performance through relaxation of passivity requirements.

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2 Coupled Stability in Haptic Rendering

In the laboratory we observe coupled instability when attempting to render simple passive linear dynamics. The traces in Fig. 1 depict the onset of coupled instability when a single-axis rotary haptic device is grasped. In this experiment, a feedback controller is attempting to render a mass-damper system. The device is initially at rest and the oscillations are not voluntary inputs from the user. The oscillations are not due to instability of the feedback loop between the device and the controller as the device will come to rest when the user lets go. The experimental results also reveal that the non-ideal behavior of the control system. To do so, we begin by modeling a haptic rendering and introducing appropriate frameworks for analyzing its stability.

![Figure 1: Experimentally observed instability induced though coupling with a user. Note that this instability is not remedied by increasing sampling frequency or encoder quantization.](image)

2.1 Modeling Haptic Rendering

A simplified schematic of a haptic interface control system is shown in Fig. 2. The measured signal is the handwheel position $y$, the control input is the motor torque $u$, and the user provides a torque input $f$. The schematic depicts an ideal impedance-type device in which the control input $u$ and the user’s input $f$ affect the output $y$ through the same dynamics. Using a linear model $P(s)$ to capture the dynamics of the motor and handwheel, we have

$$y = P(s)(f + u).$$

Further, let $C(s)$ describe the controller:

$$u = -C(s)y.$$  

The feedback interconnection of $P$ and $C$ is depicted by the block diagram in Fig. 3.

![Figure 2: Schematic of a rotary haptic device and controller.](image)

To analyze coupled stability, we include a user model in the feedback loop. As shown in Fig. 3, we let $H(s)$ describe linear relationship between $f$ and $y$ created by the user’s physical contact with the haptic device. A residual force $f^*$ may arise from volitional input and physical coupling not captured by the linear model. The user model is then given by

$$f = f^* - H(s)^{-1}y.$$  

For the sake of consistency, both $H(s)$ and $P(s)$ describe forward-dynamics (from force to motion). While describing the user’s interaction with the haptic device as a linear time-invariant system is a large simplification, as we will show later, the model nevertheless appears to be useful for the purposes of analysis, particularly at frequencies above 10 Hz where volitional feedback is negligible.

![Figure 3: Block diagram of the coupled haptic interface system and user.](image)

2.2 Closed-loop Control Objectives

A key transfer function in haptic rendering is the dynamic relationship presented to the user, or the rendered virtual environment. We denote this transfer function by

$$R(s) \triangleq \frac{P(s)}{1 + P(s)C(s)}.$$  

The desired closed-loop response, or simply the virtual environment, we denote by $R_d(s)$. Note that the controller is not synonymous with the virtual environment.

Performance, or the degree to which the rendered virtual environment $R(s)$ matches the virtual environment $R_d(s)$, is measured by distortion [11]. Denoted by $\Theta(s)$, it is defined as

$$\Theta(s) \triangleq \frac{R(s) - R_d(s)}{R_d(s)}.$$  

Accurate rendering is achieved by attenuating the magnitude frequency response of $\Theta(s)$. Due to inherent limitations such as actuator bandwidth and unmodeled high-frequency dynamics, this can
only be achieved over a finite bandwidth. A reasonable performance specification may take the form

$$|\Theta(j\omega)| \leq M_\Theta, \quad \text{for} \ 0 \leq \omega \leq \omega_c.$$  \hspace{1cm} (6)

In addition to performance, the feedback design must provide a degree of stability robustness and insensitivity to variation in hardware parameters. These goals are achieved by attenuation of the Bode sensitivity function [18]

$$S(s) \triangleq \frac{1}{1 + P(s)C(s)}. \hspace{1cm} (7)$$

Finally we note that there are identities that relate $R(s)$, $\Theta(s)$, and $S(s)$. It follows from $(4)$ and $(7)$ that

$$S(s) = \frac{R(s)}{P(s)}.$$  \hspace{1cm} (8)

Through this relationship, the sensitivity function is linked to the rendered virtual environment and a basic conflict is implied between performance and robustness goals.

### 2.3 Stability Criteria

A common and practical method to assure coupled stability of the human operator and haptic interface system is to study the dynamic response of the haptic interface system to be passive [4]. If both both the human operator and the actively controlled haptic device are passive, then the interconnection is stable. A necessary and sufficient condition for a linear time-invariant transfer function between a pair of power variables—such as force and velocity—to be passive is that its poles lie in the closed left-half plane and its Nyquist plot lies in the closed right-half plane [19]. Thus, for a stable feedback design, the user is presented with a passive dynamic response if and only if

$$\text{Real}[(j\omega)R(j\omega)] \geq 0 \ \forall \omega.$$  \hspace{1cm} (9)

In other words, the Nyquist plot of $R$ must lie entirely in the right-half plane, or equivalently, the positive-$\omega$ locus of $R$ must remain below the real-axis.

Coupled stability may also be determined by applying the Nyquist stability criterion to the feedback loop between the haptic interface and the user. The loop gain is given by $R(s)/H(s)$. Assuming for simplicity that $R(s)/H(s)$ is stable and minimum phase, the closed-loop system is stable if and only if the Nyquist plot of $R(s)/H(s)$ does not encircle the critical point.

The passivity criterion is advantageous because it does not assume a particular model of the user or even require that the user dynamics be linear or time-invariant. The drawback, however, is that the passivity-based approach provides a conservative stability result compared with the Nyquist stability criterion. Stability requirements will necessarily limit the range of virtual environments that can be rendered by a haptic device. By relaxing the passivity requirement, we may be able to recover some lost performance.

### 3 Limitations Imposed by Passivity

Let us re-visit the coupled instability problem captured in experimental traces of Fig. 1. The desired virtual environment in this example is a damped-mass system, but the rendered virtual environment is evidently not passive. It would be valuable to know whether there exists a controller that renders the virtual environment passively or whether there is an inherent conflict between maintaining passivity and accurate rendering of the desired dynamics. Interestingly, we now show that some passive virtual environments cannot be accurately rendered while maintaining passivity.

Proposition 1. Assume that $P(s)$ has relative degree two and has no open right-half plane poles. Given $0 < M_0 < 1$, a necessary condition for the existence of a proper, stabilizing controller $C(s)$ that meets the performance specification $(6)$ and passivity requirement $(9)$ is

$$\int_{0}^{\infty} \log \frac{|R(j\omega)|}{|P(j\omega)|} + \log |1 - M_0| \, d\omega \leq \int_{0}^{\infty} \arg P(j\omega) \, d\omega.$$  \hspace{1cm} (10)

for all $0 < \omega_0 \leq \omega_c$.

Proof. The following is a sketch. For a full proof, see [10].

As a stable, minimum phase transfer function with degree zero, the Bode sensitivity function $S$ satisfies (cf. [2, Eqn. 13-36])

$$\int_{0}^{\infty} \log |S(j\omega)| \frac{1}{\sqrt{1 - \omega^2/\omega_0^2}} \, d\omega = - \int_{0}^{\infty} \arg S(j\omega) \, d\omega.$$  \hspace{1cm} (11)

It follows from $(5)$, $(6)$ and $(8)$ that

$$\log |S(j\omega)| \geq \log \frac{|R(j\omega)|}{|P(j\omega)|} + \log |1 - M_0|.$$  \hspace{1cm} (12)

for $M_0 < 1$. From the identity $(8)$, we find that the phase of $S(s)$ is the difference of the phases of $R(s)$ and $P(s)$. Then, the phase requirement on $R(s)$ implied by $(9)$ becomes a phase requirement on $S(s)$:

$$-\arg S(s) \leq \pi + \arg P(j\omega).$$  \hspace{1cm} (13)

The inequalities $(12)$ and $(13)$ when applied to $(11)$ yield $(10)$.

We note that inequality $(10)$ does not depend on the controller and as such presents a fundamental design limitation. It is an existence condition based solely on the hardware dynamics, the virtual environment, and the performance specification. One important practical consequence of this integral condition is that compensation of hardware dynamics to render lighter, more responsive dynamics cannot be achieved without violating passivity at some frequencies. While $(10)$ predicts that some passive virtual environments cannot be rendered accurately without violating passivity, it does not prohibit non-passive feedback designs from achieving the performance requirements. For such feedback designs, coupled stability must be analyzed by less conservative criteria.

### 4 Example: Limitations due to Passivity and Performance Recovery

In this section we illustrate the limitation imposed by Proposition 1 and show that performance can be recovered if we permit passivity conditions to be violated at some frequencies. We treat coupled stability by identifying linear user models and applying the Nyquist stability criterion. For our example, we wish to render a mass-damper system whose frequency response is given in Fig. 4. An experimentally determined frequency response of the haptic device is shown alongside for comparison. The relative magnitudes indicate that the virtual environment has less inertia than is intrinsic to the hardware. The anti-resonance near 85 Hz and a resonance near 450 Hz are due to the location of the sensor on the motor rotor and the compliance between the handwheel and the motor rotor. As a practical matter, structural modes will always limit the useful bandwidth of the haptic device. For the purposes of controller design, the haptic device is captured by a pure inertia with a bandwidth limit.
of about 60Hz. Above this frequency, the impedance-type device model embodied in (1) no longer holds.

To generate feedback designs that approximate the desired closed-loop response using a stable, proper controller, we use controllers of the form

$$ C = \left( \frac{\tau s + 1}{R_d} - \frac{1}{\bar{P}} \right) \left( \frac{1}{\gamma s + 1} \right)^3. $$

(14)

The values of $\tau$ and $\gamma$ tune the controller bandwidth, and in this example, we compare three designs whose parameters are given in Table 4. We remark that there is nothing particularly special about (14). It is simply based on the fact that $C = R_d(s)^{-1} - P(s)^{-1}$ is the algebraic solution to obtain $R(s) = R_d(s)$.

<table>
<thead>
<tr>
<th>Design</th>
<th>$2\pi\tau^{-1}$ Hz</th>
<th>$2\pi\gamma^{-1}$ Hz</th>
</tr>
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<tbody>
<tr>
<td>A</td>
<td>15</td>
<td>25</td>
</tr>
<tr>
<td>B</td>
<td>0.8</td>
<td>5</td>
</tr>
<tr>
<td>C</td>
<td>0.09</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Table 1: Parameters $\tau$ and $\gamma$ of three feedback designs.

We begin by examining design A, which has the highest practical bandwidth for the available hardware. (Tuning the controller for higher bandwidths excited structural resonances of the device.)

The experimentally determined distortion achieved by design A is shown in Fig. 5. Using $M_d = 0.5$ as the desired distortion level, design A achieves a bandwidth of 0.7 Hz. As shown in Fig. 6, the phase of $R(s)$ drops below -180 degrees, indicating that the rendered dynamics are not passive. Indeed, design A results in the coupled instability shown in Fig. 1.

To remedy the coupled instability of design A, we seek a controller that maintains passivity. Perhaps by choosing a more sophisticated controller than (14), we can maintain the performance of A while recovering passivity. On the other hand, there may exist no controller that can achieve the performance of design A while maintaining passive dynamics. Figure 7 shows both sides of the inequality (10) in Proposition 1. The right-hand side of the integral is identically zero for a simple inertia model of the haptic device, and the left-hand side rises as the ratio of $|R_d(j\omega)|$ to $|P(j\omega)|$ increases. A critical point is reached at 0.7 Hz—above this frequency (10) is violated. It follows from Proposition 1 that the performance specification of $M_d = 0.5$ and $\omega_c > 2\pi(0.7)$ (rad/s) cannot be achieved while maintaining a passive closed-loop response. The only means to provide passivity is to tune the controller parameters to reduce the closed-loop bandwidth.
mentally verified by the phase of $R(s)$ shown in Fig. 6. However, performance, as measured by distortion in Fig. 5 is significantly degraded. Rather than accept this design, we may seek to recover some of the performance afforded by design A while avoiding coupled instability.

4.1 System Identification of the User

Analysis of coupled stability by the Nyquist criterion first requires identification of a set of user dynamics. The user model $H(s)$ may be obtained from the algebraic relationship

$$H(s) = \frac{1}{P_h(s)^{-1} - P(s)^{-1}},$$

(15)

where $P_h(s)$ is the directly measurable transfer function between $u$ and $y$, and is given by the coupled dynamics of $P(s)$ and $H(s)$:

$$P_h(s) \triangleq \frac{P(s)H(s)}{P(s) + H(s)}.$$  

(16)

The frequency responses shown in Fig. 8 capture several extreme grasps (in terms of magnitude) achieved by the authors. Fingertip grasp 1 has two fingers and the thumb lightly placed on the edge of the hand-wheel and fingertip grasp 2 includes three fingers and the thumb. In the palm grasp, the handwheel was firmly gripped with the entire hand. Photos of the fingertip and palm grasp are shown in Fig. 9.

![Figure 8: Experimentally determined frequency responses for several user grasps.](image)

![Figure 9: A light fingertip grasp (left) and a firm palm grasp (right).](image)

The quality of the user’s frequency response degrades significantly above 40 Hz, particularly for the lighter grasps. This is due in part to intrinsic limits of (15). We note that (15) assumes that the hardware behaves as an ideal impedance-type device, an assumption that breaks down at the anti-resonance of the hardware. Furthermore, identification of the user dynamics is intrinsically limited by the quality of the haptic device model $P(s)$ and the relative magnitudes of $P(s)$ and $H(s)$. Specifically, if $P_h(s)$ and $P(s)$ are close at a frequency, the user model is sensitive to small variations in $P(s)$. With some derivation, we can express this sensitivity precisely by

$$\frac{P(s) dH(s)}{H(s) dP(s)} = \frac{H(s)}{P(s)}.$$  

(17)

It follows that accurate user models are difficult to obtain from (15) at frequencies where $H(s)$ is very large compared with $P(s)$.

4.2 Nyquist Stability and Performance Recovery

Stability of the coupled human-in-the-loop feedback system may be checked for each user model identified. On a Bode diagram of the loop gain $R(s)/H(s)$, the loop gain must have positive phase margin at the gain cross-over frequency. The Bode diagram for design A (shown in Fig. 10) reveals that there is negative phase margin at the gain cross-over for both fingertip grasps. On the other hand, the model of the palm grasp has positive phase margin. These theoretical stability results, derived from linear time invariant models, match the experimentally observed behavior: a light fingertip grasp does induce coupled instability, but a more firm grasp does not. Furthermore, we expect from the Bode diagram that unstable modes will appear near 10 Hz. This also matches the experimental data shown in Fig. 1 which exhibit oscillations in the range of 7 to 10 Hz.

![Figure 10: Bode diagram of the loop gain $R(s)/H(s)$ for Design A. The Nyquist stability criterion predicts instability for the fingertip grasps.](image)

Comparing Fig. 10 with Fig. 6, we see that the phase of $R(s)/H(s)$ is roughly the same as the phase of $R(s)$ alone between 0 and 10Hz. The user models exhibit spring-like behavior in this frequency band and do not contribute significant positive phase until high frequencies. To avoid instability, the frequencies over which
$R(s)$ violates passivity should not be near the gain cross-over frequency of $R(s)/H(s)$.

Based on the intuition developed from Fig. 10, design B is allowed to violate the phase requirement of passivity between 0.5 and 5 Hz. As shown in Fig. 11, there is positive phase margin at the gain cross-over frequency for each user model. Experimentally, we indeed find no tendency to oscillation as we vary our grip on the gain cross-over frequency for each user model. Experimentally, and 5 Hz. As shown in Fig. 11, there is positive phase margin at lowed to violate the phase requirement of passivity between 0.5

5 Conclusion

This paper has identified a type of coupled instability between a user and a haptic interface system that has not been previously analyzed. It is shown that a fundamental conflict exists between passivity and high-fidelity rendering of certain passive virtual environments. This conflict cannot be circumvented through better encoders, faster sampling, reduced hardware nonlinearities, or more sophisticated controller design. Passivity, however, may be an overly restrictive condition. In the example, we have designed several controllers to render a mass-damper system on a single-axis rotary haptic device. The first design achieves the best performance, but violates passivity and results in coupled instability. By applying Proposition 1, we prove that no feedback design exists that can achieved the desired performance while maintaining passivity. In the second controller design, we sacrifice performance to recover passivity. This controller, however, has significantly degraded performance. We then demonstrate performance recovery by permiting some violation of the phase condition imposed by passivity. Using several experimentally identified user models, we are able check that the Nyquist stability criterion is satisfied.

References