Market-Based Supply Chain Coordination
by Matching Suppliers’ Cost Structures
with Buyers’ Order Profiles

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ABSTRACT

We study a competitive supply marketplace where multiple suppliers offer a single non-differentiated product to multiple buyers. Within this multi-supplier, multi-buyer marketplace, each buyer chooses the supply partner that serves the buyer’s order profile, described by his order size and delivery frequency, with the best price. Suppliers differ from each other in terms of their logistic cost structures, as captured by relevant economies of scale in a “setup cost” component, and storage, distribution and working capital related costs in a “holding cost per unit”. A supplier’s offering price to a buyer depends on how well the supplier’s cost structure matches the buyer’s order profile, as well as the prices potentially offered by her competing suppliers.

In this paper, we argue that the matching between buyers’ order profiles and suppliers’ cost structures represents the main source of supply chain coordination benefits in the many-to-many supply chain with economies of scale. We show that the most cost effective matching can be naturally achieved through price competition among the suppliers. Our analysis identifies the segment of the order space each supplier could win by taking advantages of her logistic cost structure. At the equilibrium of price competition, the winning supplier does not offer the lowest price she can offer, but instead she offers the lowest price of her closest competitor. We perform market share sensitivity analysis when a supplier’s logistic cost structure changes or when a new supplier enters the market. Finally, we suggest to implement such market-based cost effective matching through either a reverse auction mechanism or a logistic intermediary.

Keywords: Two Echelon Supply Chain; Coordination; Supplier-Buyer Matching; Market Share; Pricing; Competition; Equilibrium; Reverse Auction; Inventory Cost; Economies of Scale
1 Introduction

Supply chain coordination has been the central theme for the majority of supply chain management research. Most existing research, whether in single supplier-single buyer setting or in single supplier-multiple buyers setting, assumes that buyers are assigned to the supplier exogenously. The subsequent supply chain coordination is often achieved through adjusting buyers’ order profiles through pricing and contracting. These approaches share several common drawbacks. First, the supplier is assumed to be a local monopolist, the only choice for the buyers exogenously assigned to him. In reality, suppliers often compete with each other to gain their customer base. Second, the assignment between the buyers and the supplier can be a mismatch. In this case, coordination through adjusting buyers’ order profile will help. But the benefit will be limited since the coordination will not be efficient. Third, coordination through adjusting buyers’ order profiles may be costly or even infeasible in reality. Our paper advocates that a good match between suppliers and buyers should be the first step to achieve supply chain coordination. It frequently constitutes the major source of cost savings for the supply chain, while subsequent lot-sizing adjustments at suppliers and buyers is frequently a second order factor in term of cost saving. In particular, a cost effective matching can be achieved naturally through a market-based competition and selection process, as described in this paper for a two-echelon supply chain.

We study a marketplace with multiple suppliers offering a single non-differentiated product to multiple buyers. A “supplier” can be either a manufacturer or a logistical intermediary with an appropriate interpretation specific to the application context of our results. Similarly, a “buyer” can be a pure form buyer or simply another logistical and/or production intermediary in a complex value chain that fed by an upstream supplier. To avoid any further complexity on the supply side of the model, we assume that all suppliers order their needed inputs from a common, or similar in nature, source in all relevant dimensions (price, quality, lead-time, etc.). This further upstream supply tier is assumed to have ample supply. Within this multi-supplier multi-buyer marketplace, suppliers compete for buyer’s market by offering competitive prices and buyers choose their supply partners by comparing the prices offered by all suppliers. A supplier determines her offering price to a buyer based on her own logistic cost structure, the buyer’s order profile, and the potential offering prices of other competing suppliers.

The logistic cost structure of a supplier is captured by relevant economies of scale in a fixed cost nature “setup cost” component, and physical storage, distribution and work-
ing capital related variable costs as depicted in a “holding cost per unit” component. In a
manufacturer-reseller setting, the setup costs may include not only production switch-over
costs but also fixed transportation and loading/unloading distribution costs as the prod-
uct gets moved from a production facility to temporary warehousing to finally reaching the
reseller’s distribution, cross-docking or retail location. In a logistical/distribution interme-
diary - buyer setting (e.g., global supply chains in textiles and toys, with Li & Fung as the
Asian based global sourcing and distribution intermediaries, and fashion manufacturers, as
Calvin Klein, Gymboree and Liz Klaiborue, as the buyers) setup costs are mostly of a logis-
tical nature. It may include inventory accumulation and warehousing activities for timing
coordination, shipment container - vessel size economies of scale etc. Similarly, significant
differences can often exist in holding costs among suppliers within the broad interpretation
of our modeling framework. Long lead-time foreign suppliers have often to worry about serv-
icing their working capital needs in their long pipelines and providing JIT supply service to
their far away customers. Efforts to address such needs often imply leasing and operating ex-
pensive storage facilities close to their customers (either individually owned or shared supply
hubs), a policy frequently used by their North-American based competitors and customarily
demanded by large customers (see Kopczak (1998)).

A buyer’s order profile is described by his order size and order frequency. While it is
adjustable in some cases when the buyer is sufficiently compensated, as suggested by many
supply chain coordination literature, it can be difficult to change in many other situations.
Quite frequently, the order profile of a particular product is a result of derivative (dependent)
demand calculations (i.e., “explosion” of requirements of master production schedules, or
“implosion” of distribution requirements in multi-tier distribution structures) of complex
production/selling plans of multi-item assembled final goods, with the sourced product just
being one of the many required items for meeting the customer order. This is the case
when our “buyer” is either a distributor of systems, with assembly capabilities needed, or a
manufacturer herself. Even in a pure retailing context, selling and promotional considerations
of buyers, often operating in limited retail space stores with diverse product lines of partially
substitutable items, make it hard, and potentially infeasible, to change order profiles of any
one of the products, without affecting the order profiles of the others which are potentially
sourced from other suppliers. For functional, divisional or brand management reasons, the
sourcing of different products is often a separate decision handled by different purchasing
managers, thus making order profile adjustments an organizational nightmare.

Under the assumption that all competing suppliers’ logistic cost structures (i.e. setup
and holding costs) are common knowledge within the market place, we find that at the equilibrium of price competition, the market shares of all suppliers are fully determined by their logistic cost structures. More specifically, each supplier captures the market of buyers whose order profiles (i.e., order size and frequency) matches her logistic cost structure the best, i.e., the market “sweet spots” where she can serve more effectively than her competing suppliers. Suppliers with relatively low setup costs and high inventory holding costs capture the market of buyers who order infrequently but in large quantities. Similarly, suppliers with high setup costs and low inventory holding costs are the most effective in serving buyers who order frequently but in small quantity. Furthermore, for a given set of suppliers and any buyer order profile, we are able to explicitly characterize the prices offered to the buyer. At the equilibrium, the winning supplier does not offer the lowest price she can offer, but instead she offers the lowest price of her closest competitor(s). In case the suppliers’ cost structures are not common knowledge, we propose to implement the above market equilibrium, which matches the suppliers and buyers effectively, through certain auction mechanism, such as “private independent value second-price sealed bid” auction.

The remainder of the paper is organized as follows. Section 2 reviews relevant literature on supply chain coordination. Section 3 describes the model in detail. Section 4 focuses on the market share analysis for two suppliers, i.e., in a price competitive non-differentiated product environment, which part of buyer order space can each supplier win. The results are generalized to multiple suppliers in Section 5. In particular, we characterize the equilibrium market share obtained by each supplier and the equilibrium price offered to each buyer. An efficient algorithm is developed to determine the market shares for all coexisting suppliers. Section 6 performs market share sensitivity analysis as a supplier’s logistic cost structure changes or as a new supplier enters the market. This analysis provides insights on ways for suppliers to position and defend their markets. Section 7 illustrates our market share analysis by two numerical examples. In particular, it is shown that the market-based matching between suppliers and buyers, as proposed in this paper, can lead to significant cost saving for the whole supplier-buyer system. Section 8 discusses in detail how to implement the market-based matching through reverse auctions or logistical intermediaries.

2 Literature Review

The vast Supply Chain Coordination literature, with its multiple incentive alignment schemes and types of contracts, is, in a very effective and tutorial way, summarized in Cachon (2002),
Lariviere (1999) and Tsay and Agrawal (2002). In the presence of economies of scale, the following two types of supply chains have been widely studied: two stage linear supply chains (i.e., single supplier setting to single manufacturer) and two-stage divergent distribution systems with single supplier and multiple buyers. The primary coordination mechanisms suggested for the supply chain are carefully structured quantity discounts. For comprehensive reviews of the role of quantity discounts in achieving supply chain coordination see Silver, Pyke and Peterson (1998) and Tsay and Agrawal (2002).

The single supplier-single buyer setting has been studied among others in Abad (1994), Corbett and DeGroote (2000), Weng (1995), and more recently in Karabati and Kouvelis (2003). The traditional view in the supply chain coordination via discounts literature can be easily discussed in the single supplier-single buyer setting. For a decentralized, non-coordinated system, the assumed protocol of action involves the buyer choosing order quantity, and corresponding delivery frequency to meet a constant annual demand in a way that minimizes his holding and setup costs. The supplier, provided with the buyer's order profile, chooses an appropriate production lot size to optimize her holding and setup costs. For a centralized decision making (frequently referred to as cooperative strategy), the single decision maker, the “coordinator”, with full information of all parameters of the problem, chooses the supplier and buyer order sizes, under the usual, and rather non-restrictive, assumption that the supplier order is an integer multiple of the buyer order size, in a way that optimizes the overall supply chain inventory holding and setup costs (see Silver et al (1998)). In the coordinated solution, although the overall cost of the supply chain is lower, the buyer’s total cost increases. Thus, in order to achieve implementation of the coordinated solution in an independent decentralized decision making setting, the buyer has to be offered appropriate quantity discounts in an effort to align her incentives with the overall supply chain and adjust her order profile according to the coordinated solution (see Weng (1995)).

The study of the single-supplier multi-buyer divergent distribution system followed a similar research path to the two party(stages) linear supply chain. Centralized optimal (or equivalently cooperative) strategies have been studied in Graves and Schwarz (1977). Such strategies entail the use of non-stationary replenishment intervals, which are often too complex for practical use. Roundy (1985, 1986) ingeniously introduced simple coordination strategies for the problem by using the so-called power-of-two policies. Such intervals that are power-of-two multiples of the buyers’ replenishment interval, are simpler to implement and can be proved to be within 2% of optimality. Implementation of such cooperative strategies for independent buyers fails in the absence of the right incentives, as the buyer’s cost
increases when the coordinating order quantity deviates from the economic order one. Chen, Federgruen and Zheng (2001) propose an incentive compatible scheme for the implementation of coordinating power-of-two policies in decentralized settings. The scheme involves two components: (a) a common price discount scheme offered to buyers in causing the placement of order in intervals that achieve perfect channel coordination; and (b) each buyer is required to pay the supplier a franchise fee, which allows for redistribution of the coordination benefit in a way that the supplier affords to offer the needed discounts. Near perfect supply chain coordination strategies for this setting are offered by Wang (2001) using order timing coordination as derived via a Stackelberg game in which the supplier acts as the leader and buyers act as followers.

Even though this decentralized coordination approach via appropriately determined quantity discounts is appealing in its system optimizing logic, it has practical implementation flaws that could limit its effectiveness, even its feasibility, for certain supply chain environments, and in particular so for the non-differentiated product environment of our study. The approach starts by assuming the buyer has already picked his supplier, and then works within the confines of the supplier’s logistic cost structure, and via an assumed cooperative partnership, to improve the resulting supply chain. But this cooperative solution requires the buyer to change his order profile, which might not be desirable and/or even practically feasible. These implementation difficulties are easily overcome through our suggested “market based” supply chain coordination approach. From a conceptual viewpoint, our approach achieves overall supply chain efficiencies by creating a market mechanism that exploits the presence of multiple competitive suppliers with diverse logistical cost structures. The market mechanism leads to the most reasonable match between the buyer’s order with the supplier’s capabilities. In contrast to the quantity discount based coordination, where a pre-determined pair of supplier-buyer tries to move from the buyer’s desired order profile towards the profile that the supplier can support, a price directed market-based approach tries to select among a set of bidding suppliers the right supplier for the particular order profile, with the right supplier being defined as the one that has the logistical cost structure to profitably offer the most competitive price to the buyer. The market-based matching can be implemented via an appropriate auction mechanism even if the full knowledge of participating suppliers’ cost structures is not available.
3 Model Description

The two-echelon supply chain under consideration consists of $N$ suppliers and many buyers. A single product type with uniform quality is involved in the exchange between suppliers and buyers. The buyer’s order profile is characterized by his order quality $q$ and order frequency $\mu$. Both quantities are assumed to be fixed for each buyer. In reality, a buyer might have multiple orders with different profiles. In our exposition, we treat each order as a separate buyer. This model artifice has no implication for the results other than our expository convenience. In later sections of the paper, we use the ratio $\rho = \mu/q$ as an alternative characterization for a buyer. In either case, the buyer’s market can be described by a two dimensional space with parameters $q$ and $\mu$ or parameters $\rho$ and $\mu$. In the $q$-$\mu$ space, a point $(q, \mu)$ represents a buyer that orders $q$ products each time and $\mu$ times a year with an annual demand of $d = \mu q$. In the $\rho$-$\mu$ space, a point $(\rho, \mu)$ represents a buyer that orders $\mu/\rho$ products each time and $\mu$ times a year with an annual demand of $d = \mu^2/\rho$.

To fulfill buyers’ orders, all suppliers order from a common source with ample supply and they pay the same unit cost $C_0$. We assume that suppliers do not combine orders from different buyers in our model. The suppliers differ from each other in terms of their inventory holding costs $H_i$, $i = 1, \ldots, N$, and setup costs $S_i$, $i = 1, \ldots, N$. Without loss of generality, the indices of the suppliers are ordered according to their setup costs:

$$S_i < S_{i+1} \quad \forall i = 1, \ldots, N - 1.$$  

We assume

$$S_i \neq S_j \quad \text{and} \quad S_i/H_i \neq S_j/H_j \quad \forall i, j = 1, \ldots, N, i \neq j. \quad (1)$$

Indeed, if $S_i = S_j$ then either these two suppliers are identical (when their inventory holding costs are the same) or the supplier with a lower inventory cost dominates the other supplier. If $S_i/H_i = S_j/H_j$ then the supplier with a lower setup cost (and thus inventory holding cost) dominates the other supplier. These observations will be clearer in the later sections of the paper. The information about $H_i$ and $S_i$ for all suppliers is assumed to be a common knowledge among the suppliers. We will discuss the implementation issues in Section 8 when these information are not available.

We assume that there is no channel coordination or cooperation between any supplier and any buyer in this paper. A buyer chooses his supplier based on the price offered. The suppliers compete for buyer’s market by determining their inventory replenishment policy and pricing strategy. More specifically, we assume that all suppliers have the flexibility of
offering different prices to different buyers based on buyers’ order profile, i.e., their order quantities and order frequencies. Notice that this pricing strategy can be implemented through a price menu approach to avoid antitrust litigation under the Robinson-Patman Act.

Clearly, the lowest price a supplier can offer to a buyer depends on the supplier’s product cost and inventory cost. To minimize her inventory cost, it is optimal for each supplier to follow the “EOQ model with lumpy demand” for her inventory policy (Munson and Rosenblatt(2001)). More specifically, suppose buyer \((q, \mu)\) (or \((\rho, \mu)\)) orders from supplier \(i\), the supplier should order \(nq\) products each time and \(\mu/n\) times a year. The optimal multiple \(n\) is given by

\[
    n^* = \left\lfloor \frac{1}{2} \left( 1 + \sqrt{1 + \frac{8S_i\rho}{H_i q^2}} \right) \right\rfloor = \left\lfloor \frac{1}{2}(1 + \sqrt{1 + 8\rho S_i/H_i}) \right\rfloor,
\]

where \([x]\) represents the largest integer less than or equal to \(x\). The minimum annual inventory cost per product incurred to supplier \(i\) is given by:

\[
    C_i^* = \frac{S_i}{n^*q} + \frac{(n^* - 1)H_i}{2\mu} = \frac{H_i}{\mu} \left( \frac{\rho S_i/H_i}{\lfloor(1 + \sqrt{1 + 8\rho S_i/H_i})/2\rfloor} + \frac{1}{2} \left\lfloor \frac{4\rho S_i/H_i}{(1 + \sqrt{1 + 8\rho S_i/H_i})} \right\rfloor \right).
\]

It is well known that the inventory cost of most EOQ models are insensitive to the order quantity variations near the the optimal order quantity. In the remaining paper, we approximate the inventory cost \(C_i^*\) by ignoring the integer operation in (2). After some algebraic simplifications, we have:

\[
    C_i^* \approx \frac{4S_i\rho}{(1 + \sqrt{1 + 8\rho S_i/H_i})\mu}.
\]

Define

\[
    C_i(\rho) = \frac{4S_i\rho}{1 + \sqrt{1 + 8\rho S_i/H_i}}.
\]

It follows that the lowest price supplier \(i\) can offer to buyer \((\rho, \mu)\), while maintaining profitability, is given by:

\[
    P_i(\rho, \mu) = C_0 + C_i(\rho)/\mu.
\]
4 Market Segmentation and Pricing for Two Suppliers

To better understand how $N$ suppliers compete for buyers’ market share, we start by analyzing a special case where only two suppliers exist in the supply chain. Without loss of generality, let supplier 1 and supplier 2 (with $S_1 < S_2$) be the two suppliers. Since the buyers are price-takers by assumption, both suppliers will compete on the unit offering price to gain buyer’s market share. Therefore, for buyer $(q, \mu)$, the supplier with the smaller $P_i(\rho, \mu)$ will eventually win the price war. Define

$$R_{ij}(\rho) = \frac{C_i(\rho)}{C_j(\rho)} = \frac{S_i(1 + \sqrt{1 + 8\rho S_j/H_j})}{S_j(1 + \sqrt{1 + 8\rho S_i/H_i})}, \quad i = 1, j = 2.$$ 

Since the purchasing price $C_0$ is the same for both suppliers, $R_{12}(\rho) < 1$, or $C_1(\rho) < C_2(\rho)$, if and only if $P_1(\rho, \mu) < P_2(\rho, \mu)$. Notice that $R_{12}$ is a function of a single parameter $\rho = \mu/q$. This implies that supplier 1 and supplier 2’s market shares can be characterized by a single parameter $\rho$ instead of a pair of parameters $(q, \mu)$ or $(\rho, \mu)$. The boundary of the market shares between supplier 1 and supplier 2 can be determined by simply solving a nonlinear equation: $R_{12}(\rho) = 1$. To characterize the market share boundary, we need the following property about this equation.

**Proposition 1** Equation $R_{12}(\rho) = 1$ has at most one positive solution.

The proof of the proposition is provided in the appendix. With this result, we are able to provide a necessary and sufficient condition for supplier 1 and supplier 2 to share the market.

**Theorem 1** Suppose supplier 1 and supplier 2 (with $S_1 < S_2$) compete for buyer’s market share through pricing. Both suppliers have positive market shares (coexist) if and only if

$$S_1 H_1 > S_2 H_2.$$ \hspace{1cm} (3)

Otherwise, supplier 1 dominates supplier 2 by taking away all buyer’s market share.

**Proof:** Both Supplier 1 and supplier 2 will have positive market shares if and only if equation $R_{12}(\rho) = 1$ has a positive solution. By definition,

$$\lim_{\rho \to 0} R_{12}(\rho) = \frac{S_1}{S_2} < 1.$$
From Proposition 1, equation $R_{12}(\rho) = 1$ has at most one positive solution. It follows that a positive solution exists if and only if

$$\lim_{\rho \to \infty} R_{12}(\rho) = \sqrt{\frac{S_1 H_1}{S_2 H_2}} > 1.$$  

Clearly, this condition is equivalent to $S_1 H_1 > S_2 H_2$. On the other hand, if this condition is not satisfied, then $R_{12}(\rho) < 1$ for all $\rho > 0$ since equation $R_{12}(\rho) = 1$ has at most one solution. This implies that supplier 1 dominates supplier 2.

We call condition (3) the pairwise coexisting condition. The condition has the following implications. For a supplier to have a positive market share, she has to be competitive in either setup cost or inventory holding cost relative to her competitors. Furthermore, the setup cost plays a more critical role than the holding cost.

The pairwise coexisting condition is not transitional in general. I.e., even if two suppliers are able to coexist with a common supplier in their respective competitions, these two suppliers may not coexist when they compete directly. Consider the following three suppliers with $S_1 = 30$, $H_1 = 9$, $S_2 = 45$, $H_2 = 2$, and $S_3 = 60$, $H_3 = 3$. Clearly, $S_1 < S_2 < S_3$ and $S_1 H_1 > S_3 H_3 > S_2 H_2$. By Theorem 1, both supplier 2 and supplier 3 coexist with supplier 1. However, supplier 2 dominates supplier 3 when they compete.

Suppose now supplier 1 and supplier 2 are able to coexist. Let $M_1$ and $M_2$ be their respective equilibrium market shares as a result of their price competition. The next result determines the boundary of their market shares.

**Theorem 2** Suppose supplier 1 and supplier 2 satisfy the pairwise coexisting condition (3) and they compete for buyer’s market share through pricing. Let

$$\rho_{12} = \frac{H_1 H_2 (H_1 - H_2)(S_2 - S_1)}{2(S_1 H_1 - S_2 H_2)^2}. \quad (4)$$  

Then the equilibrium market shares of supplier 1 and supplier 2 are given by

$$M_1 = \{(\rho, \mu) > 0 | \rho < \rho_{12}\} \quad \text{and} \quad M_2 = \{(\rho, \mu) > 0 | \rho > \rho_{12}\},$$  

respectively.

**Proof:** Under condition (3), it is easy to verify that $\rho_{12} > 0$ is a positive solution of equation $R_{12}(\rho) = 1$. By Proposition 1, the solution is unique. From the proof of Theorem
1, \lim_{\rho \to 0} R_{12}(\rho) < 1. \text{ It follows that } R_{12}(\rho) < 1, C_1(\rho) < C_2(\rho), \text{ and } P_1(\rho, \mu) < P_2(\rho, \mu) \text{ for all } \rho < \rho_{12}. \text{ Therefore, all buyers with } \rho < \rho_{12} \text{ order from supplier 1 and } M_1 \text{ describes supplier 1’s market share correctly. By a similar argument, } M_2 \text{ correctly describes supplier 2’s market share.}

Figure 1 shows the market shares of both suppliers in both \(q\)-\(\mu\) and \(\rho\)-\(\mu\) spaces.

We have the following observations based on the above analysis:

- While the space of buyers is characterized by two parameters, \((q, \mu)\) or \((\rho, \mu)\), a supplier’s market share depends on only one parameter \(\rho = \mu/q\).

- Function \(C_i(\rho)\), which reflects a supplier’s inventory cost, plays a key role in the market share calculation. It is easy to verify that \(C_i(\rho)\) is a concave increasing function with respect to \(\rho\). Applying Theorem 2 to two pairwise coexisting suppliers, say supplier 1 and supplier 2, we have

\[
\begin{align*}
C_1(\rho) &< C_2(\rho) \quad \text{for all } 0 < \rho < \rho_{12} \\
C_1(\rho) &= C_2(\rho) \quad \text{if } \rho = \rho_{12} \\
C_1(\rho) &> C_2(\rho) \quad \text{for all } \rho > \rho_{12}.
\end{align*}
\]

Figure 2 relates the market shares of the two suppliers with cost functions \(C_i, \ i = 1, 2\). Clearly, the cost curves intersect at the two suppliers’ market share boundary. The supplier with a lower \(C_i(\rho)\) occupies the corresponding portion of the market share. In another word, the supplier wins the market share of buyers whose order profiles match her logistic cost structure better and this better match leads to a lower total inventory cost.

While the supplier with the lower inventory cost wins business from a given buyer, to maximizes her own profit, she does not have to offer the lowest price she can offer. Indeed, at the equilibrium of the price competition, she offers the buyer with the lowest price her competitor can offer. The result is formally stated in the following theorem.
Theorem 3 Under the assumption of Theorem 2, the equilibrium price $P(\rho, \mu)$ offered to buyer $(\rho, \mu)$ is given by

$$P(\rho, \mu) = \max\{P_1(\rho, \mu), P_2(\rho, \mu)\} = C_0 + \begin{cases} 
  C_2(\rho)/\mu & \text{if } (\rho, \mu) \in M_1 \\
  C_1(\rho)/\mu & \text{if } (\rho, \mu) \in M_2
\end{cases}$$

Proof: Consider a buyer $(\rho, \mu) \in M_1$. Based on the proof of Theorem 2,

$$P_1(\rho, \mu) = C_0 + C_1(\rho)/\mu < C_0 + C_2(\rho)/\mu = P_2(\rho, \mu).$$

As long as supplier 1’s offering price to buyer $(\rho, \mu)$ is under $P_2(q, \mu)$, supplier 1 will win the buyer’s business. On the other hand, if supplier 1’s offering price is strictly higher than $P_2(\rho, \mu)$, supplier 2 will be able to offer the buyer a price that is slightly below supplier 1’s price to win the buyer, but higher than $P_2(q, \mu)$ to make a profit. Since the objective of both suppliers is to gain market share and maintain profitability, at the equilibrium of price competition, supplier 1’s offering price to the buyer $(\rho, \mu)$ has to be $P_2(q, \mu)$. The same argument also holds for a buyer $(\rho, \mu) \in M_2$. □

5 Market Segmentation and Pricing for N Suppliers

We extend the market share analysis for 2 suppliers to $N$ suppliers in this section. Clearly, among the $N$ suppliers, if one of the suppliers is dominated by another, this supplier will not have any market share in the price competition. Such dominated suppliers can be easily identified by checking the pairwise coexisting condition (3) developed in the previous section. In this section, we assume all $N$ suppliers satisfy the pairwise coexisting condition and they are arranged according to the following sequence:

$$S_i < S_{i+1} \text{ and } S_iH_i > S_{i+1}H_{i+1} \quad \forall i = 1, \ldots, N - 1.$$  \hspace{1cm} (5)

In view of Theorem 1, condition (5) ensures that the $N$ suppliers do not dominate each other pairwise. This condition, however, does not guarantee that all $N$ suppliers will end up with positive market shares when they compete together. This is demonstrated by the following example. Consider three suppliers with $S_1 = 400, H_1 = 16, S_2 = 420, H_2 = 15$, and $S_3 = 440, H_3 = 14$. Clearly, $S_1 < S_2 < S_3$ and $S_1H_1 > S_2H_2 > S_3H_3$. The three suppliers satisfy pairwise coexisting condition (5) and will coexist when they compete pairwise. However,
applying (4) to calculate the boundaries of three suppliers’ market shares, we have \( \rho_{12} = 0.24 \), \( \rho_{23} = 0.156 \), and \( \rho_{13} = 0.107 \). By definition,

\[
C_2(\rho) > C_1(\rho) \quad \forall 0 < \rho < 10 \quad \text{and} \quad C_2(\rho) > C_3(\rho) \quad \forall \rho > 0.107.
\]

Therefore, when these three suppliers compete, supplier 2 does not have any market share. It is dominated by the presence of supplier 1 and supplier 3 together. The final price competition result is that supplier 1 and supplier 3 coexist with their market share boundary set at \( \rho_{13} = 0.107 \).

Before presenting a condition for \( N \) suppliers to coexist, we first study some order relationship among the market share boundaries when three suppliers compete for buyer’s market share.

**Proposition 2** Let 1, 2, and 3 be three suppliers competing for buyer’s market through pricing. Suppose they satisfy the pairwise coexisting condition (5). Then either

\[
\rho_{12} < \rho_{13} < \rho_{23},
\]

in which case the three suppliers share the market with

\[
M_1 = \{(\rho, \mu) > 0|\rho < \rho_{12}\}, \quad M_2 = \{(\rho, \mu) > 0|\rho_{12} < \rho < \rho_{23}\}, \quad M_3 = \{(\rho, \mu) > 0|\rho > \rho_{23}\},
\]

or

\[
\rho_{23} \leq \rho_{13} \leq \rho_{12},
\]

in which case supplier 2 is dominated by suppliers 1 and 3 and their market shares are given by

\[
M_1 = \{(\rho, \mu) > 0|\rho < \rho_{13}\}, \quad M_2 = \emptyset, \quad M_3 = \{(\rho, \mu) > 0|\rho > \rho_{13}\}.
\]

**Proof:** For three suppliers, we have either \( \rho_{12} < \rho_{13} \), or \( \rho_{12} > \rho_{13} \), or \( \rho_{12} = \rho_{13} \).

If \( \rho_{12} < \rho_{13} \), we show that \( \rho_{13} < \rho_{23} \). Suppose this is not true. Then there exists a buyer \((\rho, \mu)\) with \( \max\{\rho_{12}, \rho_{23}\} < \rho < \rho_{13} \). By definition of \( \rho_{ij} \) and the coexisting condition (5), we have the following inequalities:

\[
C_2(\rho) < C_1(\rho), \quad C_1(\rho) < C_3(\rho), \quad \text{and} \quad C_3(\rho) < C_2(\rho).
\]

However, these inequalities cannot hold simultaneously and this leads to a contradiction. Therefore, \( \rho_{12} < \rho_{13} < \rho_{23} \). To determine the market share for the suppliers, first consider
any buyer \((\rho, \mu)\) with \(0 < \rho < \rho_{12}\). Since \(\rho_{12} < \rho_{23}\), by definition of \(\rho_{ij}\) and condition (5), we have
\[
C_1(\rho) < C_2(\rho) < C_3(\rho)
\]
and supplier 1’s market share follows immediately. Supplier 3’s market share can be determined in a similar manner. For supplier 2, consider any buyer with \(\rho_{12} < \rho < \rho_{23}\). We have
\[
C_2(\rho) < C_1(\rho) \quad \text{and} \quad C_2(\rho) < C_3(\rho)
\]
and thus supplier 2’s market share.

In case \(\rho_{12} > \rho_{13}\), that \(\rho_{23} < \rho_{13}\) can be shown in a similar manner. Suppose this is not true, then there is a buyer whose \(\rho\) satisfies \(\rho_{13} < \rho < \max\{\rho_{12}, \rho_{23}\}\). Again, we have the following inequalities:
\[
C_3(\rho) < C_1(\rho), \quad C_1(\rho) < C_2(\rho), \quad \text{and} \quad C_2(\rho) < C_3(\rho).
\]
These inequalities cannot hold simultaneously and this leads to a contradiction. Therefore, \(\rho_{23} < \rho_{13} < \rho_{12}\). Since
\[
C_1(\rho) < C_2(\rho) \quad \forall \rho < \rho_{12}, \quad C_3(\rho) < C_2(\rho) \quad \forall \rho > \rho_{23}
\]
and \(\rho_{23} < \rho_{12}\), supplier 2 is dominated by the existence of supplier 1 and supplier 3 together. It follows that the market will be shared by only supplier 1 and supplier 3 with the boundary set at \(\rho_{13}\).

If \(\rho_{12} = \rho_{13}\), the problem is degenerate. We have \(\rho_{12} = \rho_{23} = \rho_{13}\) and supplier 2 is dominated. \(\blacksquare\)

We now generalize the market share analysis to the case of \(N\) suppliers. The following result provides a necessary and sufficient condition for \(N\) suppliers to share buyers’ market when they compete together.

Theorem 4 Suppose condition (5) holds for the \(N\) suppliers competing for buyer’s market share through pricing. All suppliers coexist if and only if
\[
\rho_{i-1,i} < \rho_{i,i+1} \quad \forall i = 2, \ldots, N - 1.
\] (6)

In addition, under condition (6), supplier \(i\)’s market share is given by
\[
M_i = \begin{cases} 
\{(\rho, \mu) > 0 | \rho < \rho_{12}\} & \text{if } i = 1, \\
\{(\rho, \mu) > 0 | \rho_{i-1,i} < \rho < \rho_{i,i+1}\} & \text{if } i = 2, \ldots, N - 1, \\
\{(\rho, \mu) > 0 | \rho > \rho_{N-1,N}\} & \text{if } i = N.
\end{cases}
\]
Proof: To show condition (6) is necessary, suppose \( \rho_{i-1,i} \geq \rho_{i,i+1} \) for some \( i = 2, \ldots, N - 1 \). Consider three suppliers \( i - 1, i, \) and \( i + 1 \). By Proposition 2, supplier \( i \) is dominated by supplier \( i - 1 \) and supplier \( i + 1 \) together. Thus, condition (6) is a necessary condition for \( N \) suppliers to coexist. We next show that condition (6) is also sufficient. For supplier 1, consider any buyer \((\rho, \mu)\) with \( \rho < \rho_{12} \). Under conditions (5) and (6), we have

\[
C_1(\rho) < C_2(\rho) < \ldots < C_N(\rho)
\] (7)

by definition of \( \rho_{ij} \). It follows that supplier 1 has a positive market share, which is given by \( M_1 \) in the theorem. The proof for supplier \( N \)'s market share is essentially the same. For supplier \( i \), consider any buyer \((\rho, \mu)\) with \( \rho_{i-1,i} < \rho < \rho_{i,i+1} \). Again by definition of \( \rho_{ij} \), we have

\[
C_1(\rho) > \ldots > C_{i-1}(\rho) > C_i(\rho) \quad \text{and} \quad C_i(\rho) < C_{i+1}(\rho) < \ldots < C_N(\rho).
\] (8)

Therefore, supplier \( i \) has a positive market share, which is given by \( M_i \) in the theorem. 

Figure 3 shows the market shares of \( N \) suppliers in both \( q-\mu \) and \( \rho-\mu \) spaces when they compete together.

**INSERT FIGURE 3 HERE**

We have the following observations about the market share characterization for \( N \) suppliers:

- When several suppliers compete for buyers through pricing, each supplier capture the market of buyers who they can serve most cost effectively. A supplier with a relatively low setup cost and a high inventory holding cost usually matches buyers that order in large quantities but infrequently. On the other hand, a supplier with a relatively low inventory holding cost and a high setup cost usually attracts buyers that order in small quantities but frequently.

- A supplier will have a positive market share as long as she is not dominated by her closest competitors. In our model, the closest competitors of a supplier are those suppliers that have a common market share boundary. We call these suppliers “neighbor suppliers”.

- For supplier 1 and supplier \( N \) to have a positive market share, condition (6) is not necessary. Condition (5) alone would be sufficient. Indeed, for supplier 1, if she
pairwisely coexists with all other suppliers, i.e., $\rho_{i,i} > 0$ for all $i = 2, \ldots, N$. Her market share is given by

$$M_1 = \{(\rho, \mu) > 0 | \rho < \min_{i=2,\ldots,N} \rho_{ii}\}.$$  

Similar result also holds for supplier $N$. The coexisting condition is weaker for suppliers 1 and $N$ because they have only one neighbor supplier. Hence, the competition comes from only one side.

Next, we study the equilibrium price offered to each buyer $(\rho, \mu)$. Similar to the two supplier situation studied in the previous section, for a given buyer, the supplier who can offer the lowest price will win the business as the result of price competition. However, at the equilibrium, the winner does not offer the lowest price she can offer. Instead, she offers the lowest price of her closest competitors, her “neighbor” suppliers in this case. The result is formally stated in the following theorem.

**Theorem 5** Suppose conditions (5) and (6) hold for the $N$ suppliers competing for buyers’ market share through pricing. The equilibrium price offered to buyer $(\rho, \mu)$ is given by

$$P(\rho, \mu) = \begin{cases}  
P_2(\rho, \mu) & \text{if } (\rho, \mu) \in M_1, \\
P_{i-1}(\rho, \mu) & \text{if } (\rho, \mu) \in M_i \text{ and } \rho < \rho_{i-1,i+1}, \\
P_{i+1}(\rho, \mu) & \text{if } (\rho, \mu) \in M_i \text{ and } \rho \geq \rho_{i-1,i+1}, \\
P_{N-1}(\rho, \mu) & \text{if } (\rho, \mu) \in M_N, \
\end{cases}$$

(9)

where $P_i(\rho, \mu) = C_0 + C_i(\rho)/\mu$.

**Proof:** In view of the proof for Theorem 3, it suffices to show that $P(\rho, \mu)$ is the second lowest price offered to buyer $(\rho, \mu)$. If $(\rho, \mu) \in M_1$, the result follows from (7). The same argument also holds for $(\rho, \mu) \in M_N$. If $(\rho, \mu) \in M_i$, by Proposition 2 and Theorem 4, we have $\rho_{i-1,i} < \rho_{i-1,i+1} < \rho_{i,i+1}$. From (8), either $P_{i-1}(\rho, \mu) = C_0 + C_{i-1}(\rho)/\mu$ or $P_{i+1}(\rho, \mu) = C_0 + C_{i+1}(\rho)/\mu$ will be the second lowest price offered to buyer $(\rho, \mu)$. If $\rho_{i-1,i} < \rho < \rho_{i-1,i+1}$, $C_{i-1}(\rho) < C_{i+1}(\rho)$ and $P_{i-1}(\rho, \mu)$ is the second lowest price. Otherwise, $P_{i+1}(\rho, \mu)$ is the second lowest price. This completes the proof.

We now present an algorithm to determine the market shares for all coexisting suppliers. Again we assume that all $N$ suppliers satisfy the pairwise coexisting condition (5) but not
necessarily condition (6). The algorithm is a built-on process. It determines the market share of supplier 1 by finding her “neighbor” supplier and locating the corresponding boundary. The dominated suppliers are eliminated in the mean time. Then the algorithm labels the neighbor supplier as the current supplier and continues to find the next neighbor supplier and the market share boundary. The process stops when the market share of supplier \( N \) is determined. In the following description of the algorithm, \( N' \) is the total number of coexisting suppliers, \( k(j) \) is the original index of the \( j \)th coexisting supplier, \( j = 1, \ldots, N' \), and \( M_{k(j)} \) is her market share.

Algorithm to Determine Market Shares for Multiple Suppliers

\begin{itemize}
  \item **Step 0** Set \( k(1) = 1, j = 1 \).
  \item **Step 1** Find
    \[ l = \arg \min_{t=k(j)+1,\ldots,N} \rho_{k(j),t}. \]
  \item **Step 2** Set
    \[ M_{k(j)} = \begin{cases} \{(\rho, \mu) > 0 | \rho < \rho_{k(j),l}\} & \text{if } j = 1, \\ \{(\rho, \mu) > 0 | \rho > \rho_{k(j),l}\} & \text{if } l = N, \\ \{(\rho, \mu) > 0 | \rho_{k(j-1),k(j)} < \rho < \rho_{k(j),l}\} & \text{otherwise.} \end{cases} \]
  \item **Step 3** Set \( j = j + 1, k(j) = l \). If \( l = N \), set \( N' = j \) and stop; otherwise, go to Step 1.
\end{itemize}

**Theorem 6** The above algorithm correctly determines the market share for each supplier.

**Proof:** Let \( \{k(1), \ldots, k(j), \ldots k(N')\} \) be the coexisting suppliers identified by the above algorithm. Notice that \( k(1) = 1 \) and \( k(N') = N \). The algorithm divides the buyer’s market, in terms of parameter \( \rho \), into the following segment:

\[(0, \rho_{1,k(2)}), \ldots, (\rho_{k(j-1),k(j)}, \rho_{k(j),k(j+1)}), \ldots, (\rho_{k(N'-1),N}, \infty).\]

To proof Theorem 6, it suffices to show

\[ \rho_{k(j-1),k(j)} < \rho_{k(j),k(j+1)} \text{ for all } j = 2, \ldots, N' - 1 \] (10)

and

\[ C_i(\rho) \geq \begin{cases} C_1(\rho) & \forall 0 < \rho < \rho_{1,k(2)}, \\ C_{k(j)}(\rho) & \forall \rho_{k(j-1),k(j)} < \rho < \rho_{k(j),k(j+1)}, \quad j = 2, \ldots, N' - 1, \\ C_N(\rho) & \forall \rho > \rho_{k(N'-1),N}. \end{cases} \] (11)
holds for all $i = 1, \ldots, N$. From Step 1 and Step 3 of the algorithm, we have

$$
\rho_{k(j-1),k(j)} < \rho_{k(j-1),k(j+1)} \quad \text{for all } j = 2, \ldots, N' - 1.
$$

By Proposition 2, we have

$$
\rho_{k(j-1),k(j)} < \rho_{k(j-1),k(j+1)} < \rho_{k(j),k(j+1)}
$$

and hence inequality (10). To prove (11), first observe that (11) is true for all suppliers $k(j)$, $j = 1, \ldots, N'$. This follows from (10), condition (5), and Theorem 4. It remains to show that the other suppliers are dominated by suppliers $k(j)$, $j = 1, \ldots, N'$. Indeed, consider any supplier $i$ such that $k(j) < i < k(j+1)$ for some $j = 1, \ldots, N' - 1$. From Step 1 and Step 3 of the algorithm, we have $\rho_{k(j),i} > \rho_{k(j),k(j+1)}$. By Proposition 2, supplier $i$ is dominated by suppliers $k(i)$ and $k(i + 1)$ together. \hfill \blacksquare

6 Market Segmentation Sensitivity Analysis

This section answers the following questions:

- If a supplier’s logistic cost structure (setup cost or inventory holding cost) changes, how will her market share and other suppliers’ market shares be affected?
- If a new supplier enters the market, will she acquire a positive market share? If so, how will other supplier’s market shares be affected? Will any existing supplier be eliminated because of the competition from the new entrant?

To answer the first question, let us study the two supplier situation first. Notice that the market share boundary $\rho_{12}$ is a function of the two suppliers’ inventory costs: $H_1$, $H_2$, $S_1$, and $S_2$.

$$
\rho_{12}(H_1, H_2, S_1, S_2) = \frac{H_1 H_2 (H_1 - H_2)(S_2 - S_1)}{2(S_1 H_1 - S_2 H_2)^2}.
$$

\textbf{Proposition 3} Suppose supplier 1 and 2 satisfy the pairwise coexisting condition (5) with $S_1 < S_2$. Then $\rho_{12}$ is a decreasing function of $H_1$ and $S_1$ and an increasing function of $H_2$ and $S_2$. 

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Proof: Since the two suppliers satisfy the pairwise coexisting condition, we have

\[ S_1 < S_2, \quad H_1 > H_2, \quad \text{and} \quad H_1 S_1 > H_2 S_2. \]

In view of expression (12), it is quite obvious that \( \rho_{12} \) is a decreasing function of \( S_1 \) and an increasing function of \( S_2 \). To see that \( \rho_{12} \) is a decreasing function of \( H_1 \), we rewrite the function as follows:

\[
\rho_{12}(H_1, H_2, S_1, S_2) = \frac{1}{1 - \frac{S_2}{S_1} \frac{H_2}{H_1}} \frac{1 - \frac{H_2}{H_1}}{1 - \frac{S_2}{S_1} \frac{H_2}{H_1}} \frac{H_2(S_2 - S_1)}{2S_1^2}.
\]

In the above expression, the third fraction is independent of \( H_1 \). The first fraction is a decreasing function of \( H_1 \) as long as \( H_1 S_1 > H_2 S_2 \). The second fraction is also a decreasing function of \( H_1 \) as long as \( H_1 S_1 > H_2 S_2 \) and \( S_2/S_1 > 1 \). This is because \((1 - x)/(1 - \alpha x)\) is an increasing function of \( x \) if \( \alpha > 1 \) and \( 1 - \alpha x > 0 \). Therefore, the whole function is a decreasing function of \( H_1 \). Following a similar approach, we can also show that \( \rho_{12} \) is an increasing function of \( H_2 \). \( \blacksquare \)

Now consider the situation when \( N \) suppliers share the buyer’s market. Let us apply Proposition 3 to supplier \( i \) whose market share is given by

\[ M_i = \{ (\rho, \mu) > 0 | \rho_{i-1,i} < \rho < \rho_{i,i+1} \}. \]

If supplier \( i \)'s setup cost \( S_i \) or inventory holding cost \( H_i \) decreases, \( \rho_{i-1,i} \) decreases and \( \rho_{i,i+1} \) increases. As a result, her market share expands in both directions at the expenses of her neighbor suppliers \( i - 1 \) and \( i + 1 \). Notice that other suppliers will not be impacted by supplier \( i \)'s market share expansion until one of her “neighbor” suppliers is eliminated. This happens when \( \rho_{i-1,i} \) reduces to \( \rho_{i-2,i-1} \) or \( \rho_{i,i+1} \) increases to \( \rho_{i+1,i+2} \), whichever happens first.

We have the following observations based on the above discussion. While the logistic cost structure (the relative positions of \( S_i \) and \( H_i \) in comparison with her competitors) of a supplier determines her market share position, the magnitude of her inventory costs determines the size of her market share. The lower the inventory holding and setup cost of a supplier relative to her competitors, the larger the supplier’s market share. In particular, the size of a supplier’s market share depends only on the relative inventory costs, holding and setup, between the supplier and her closest competitors. In our model, the closest competitors are the two “neighbor” suppliers with the similar logistic cost structure. Therefore, if a supplier seeks to gain market share, she has two options. The supplier may maintain her current logistic cost structure and improve on her inventory efficiency(i.e., total inventory costs). This
will help the supplier to gain market shares from her closest competitors. Alternatively, the supplier may change her logistic cost structure and position herself at a different segment of buyer’s market (described by buyer’s order behavior). This might require substantial re-engineering of supplier’s processes. This strategy will work if the competitors in the new market segment are less efficient in terms of their inventory costs.

Next we consider the situation when a new supplier $k$ enters the buyer’s market. Clearly, if supplier $k$ is dominated by any of the current suppliers, it will not have any market share. This happens when supplier $k$ fails to satisfy the pairwise coexisting condition (3) with the current $N$ suppliers. In the remaining section, we assume that supplier $k$ satisfies the following pairwise coexisting condition

$$S_i < S_k < S_{i+1}, \quad S_i H_i > S_k H_k > S_{i+1} H_{i+1}$$

(13)

for some $i = 1, \ldots, N$. As we discussed before, the above pairwise coexisting condition is necessary, but not sufficient, for supplier $k$ to coexist with the current $N$ suppliers. For supplier $k$ to have positive market share, she has to gain market shares from at least one of her neighbor suppliers $i$ or $i + 1$. Indeed, by applying Theorem 4 to the $N$ existing suppliers and the new supplier, we have the following results:

**Corollary 1** Let $1, 2, \ldots, i, \ldots, N$ be $N$ coexisting suppliers. The new supplier $k$ satisfying condition (13) will have a positive market share if and only if

$$\rho_{i,k} < \rho_{k,i+1}.$$  

Furthermore, it will coexist with all $N$ current suppliers if and only if

$$\rho_{i-1,i} < \rho_{i,k} < \rho_{k,i+1} < \rho_{i+1,i+2}.$$  

One the other hand, if $\rho_{i,k} < \rho_{i-1,i}$, or $\rho_{k,i+1} > \rho_{i+1,i+2}$, or both inequalities hold, then by Proposition 2, supplier $k$ takes away all market shares from her “neighbor” supplier $i$, or $i + 1$, or both. Once one of supplier $k$’s immediate “neighbor” suppliers is driven out of market, she will compete with her next “neighbor” suppliers and may take part or all of their market shares. More precisely, supplier $k$’s market share is determined by the following algorithm:

**Algorithm to Determine the Market Segment for a New Supplier**

**Step 0** Set $l_- = i$, $l_+ = i + 1$.  

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Step 1 Repeat \( l_- = l_- - 1 \) until \( \rho_{l_- - 1, l_-} < \rho_{l_-, k} \). Repeat \( l_+ = l_+ + 1 \) until \( \rho_{l_+, k} < \rho_{l_+, l_+ + 1} \).

Step 2 The market share of supplier \( k \) is given by \( M_k = \{(\rho, \mu) > 0 | \rho_{l_-, k} < \rho < \rho_{k, l_+}\} \).

The next theorem assures the validity of the above algorithm.

**Theorem 7** The above algorithm correctly determines the market share for the new entrant supplier \( k \).

**Proof:** Since \( \rho_{l_- - 1, l_-} < \rho_{l_-, k} \), by Proposition 2, supplier \( k \) coexists with supplier \( l_- \) and all suppliers \( l < l_- \). In case \( l_- < i \), we show that all suppliers with \( l_- < l \leq i \) are dominated by suppliers \( l_- \) and \( k \). Indeed, by the construction of the algorithm, we have \( \rho_{l_-, l_- + 1} \geq \rho_{l_-, k} \). On the other hand, since suppliers \( l_-, l_- + 1 \), and \( l \) coexist before \( k \) enters the market, we have \( \rho_{l_-, l} \geq \rho_{l_-, l_- + 1} \). Therefore, \( \rho_{l_-, l} > \rho_{l_-, k} \). By Proposition 2, supplier \( l \) is dominated by suppliers \( l_- \) and \( k \). Similarly, we can also show that \( k \) coexists with supplier \( l_+ \) and all suppliers \( l > l_+ \). In case \( l_+ > i + 1 \), all suppliers with \( i + 1 \leq l < i + 1 \) are dominated by suppliers \( l_+ \) and \( k \). Putting the above arguments together, the market share of supplier \( k \), is described by \( M_k \) in the above algorithm and all suppliers with \( l_- < l \leq l_+ \), if any, are by suppliers \( l_-, k \), and \( l_+ \). \[ \]

In summary, when a new supplier enters the market, it may be dominated by one of the existing suppliers or two of the existing suppliers together. In both cases, the new supplier will not survive. The new supplier may also share market with all \( N \) current suppliers. In this case, she takes some market share from one or two of her closest competitors (“neighbor” suppliers). Finally, if the new supplier is very competitive in inventory costs, she may eliminate her closest competitors that share the similar logistic cost structure. Our paper provides conditions for each cases and an algorithm to determine the market share for the new supplier.

7 Numerical Examples

In this section, we provide two examples to illustrate the market-based matching between suppliers and buyers proposed in this paper. The first example describes the matching processes and the resulting market shares for all suppliers. The second example demonstrates the benefit of using the market-based matching vs. the traditional lot-sizing-based coordination.
Consider a two-echelon supply chain with 8 suppliers. Their inventory costs are described in Table 1. By the pairwise coexisting condition (5), all suppliers, except supplier 8, coexist pairwisely. Supplier 8 is dominated by suppliers 5, 6, and 7, respectively. However, when these remaining 7 suppliers compete together, all suppliers, except supplier 1 and supplier 7, are not guaranteed to have positive market shares. To determine the market shares for each supplier, we apply the algorithm developed in Section 4. Starting from supplier 1, we find

\[ \rho_{12} = 0.24, \quad \rho_{13} = 0.16, \quad \rho_{14} = 0.26, \quad \rho_{15} = 0.24, \quad \rho_{16} = 0.29, \quad \rho_{17} = 0.36. \]

Since the value of \( \rho_{13} \) is the smallest, supplier 3 will be supplier 1’s neighbor supplier with their market shares separated at \( \rho_{13} \). Supplier 2 is dominated by the existence of supplier 1 and supplier 3 together. The algorithm then repeats with the following calculations starting from supplier 3

\[ \rho_{34} = 1.09, \quad \rho_{35} = 0.39, \quad \rho_{36} = 0.48, \quad \rho_{37} = 0.62. \]

Since \( \rho_{35} \) is the smallest, supplier 5 will coexist with supplier 1,3, and 7. Supplier 4 is dominated by supplier 3 and 5. Continuing this process, the algorithm finally determines that supplier 1,3,5,6, and 7 will coexist, and their market share are separated by

\[ \rho_{13} = 0.16, \quad \rho_{35} = 0.39, \quad \rho_{56} = 0.73, \quad \rho_{67} = 1.72. \]

Figure 4 describes the market shares of the coexisting suppliers in the \( q-\mu \) space.

Our paper determines the market share of a supplier by matching her logistic cost structure with buyers’ order profiles. In practice, the size of a supplier’s market share is often measured by a fraction of the total buyer demand she acquires. To make such transformation, we need the information on how the demand from all buyers is distributed across their order profiles (measured by \( \rho \) here). Suppose the total demand, after scaling, is equal to

Table 1: Setup and Inventory Holding Costs

<table>
<thead>
<tr>
<th>Supplier</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_i )</td>
<td>400</td>
<td>420</td>
<td>440</td>
<td>470</td>
<td>500</td>
<td>540</td>
<td>590</td>
<td>670</td>
</tr>
<tr>
<td>( H_i )</td>
<td>16</td>
<td>15</td>
<td>14</td>
<td>13</td>
<td>12</td>
<td>11</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>( S_iH_i )</td>
<td>6400</td>
<td>6300</td>
<td>6160</td>
<td>6110</td>
<td>6000</td>
<td>5940</td>
<td>5900</td>
<td>6030</td>
</tr>
</tbody>
</table>
to 1 and the demand follows a probability distribution \( f(\rho) \). Let suppliers 1, \ldots, \( N \) be the coexisting suppliers based on our market share analysis in Section 4 and their market shares are separated by \( \rho_{i,i+1}, i = 1, \ldots, N - 1 \). Then supplier \( i \)'s market share \( V_i \), measured in terms of the fraction of the total demand she acquires, is given by

\[
V_i = \begin{cases} 
\int_{0}^{\rho_{1,2}} f(\rho) \, d\rho & \text{if } i = 1, \\
\int_{\rho_{i,i+1}}^{\rho_{i+1,i}} f(\rho) \, d\rho & \text{if } i = 2, \ldots, N - 1, \\
\int_{\rho_{N,i}}^{\infty} f(\rho) \, d\rho & \text{if } i = N.
\end{cases}
\]

For numerical demonstration, let us assume that the demand follows a Gamma distribution \( GAM(\theta, 2) \), which has a mean of \( 2\theta \). Figure 5 shows the market shares of suppliers 1, 3, 5, 6 and 7, as a faction of total demand, under various values of \( \theta \). Clearly, as \( \theta \) increases, more demand is generated from buyers with larger \( \rho \). Those suppliers with larger setup costs and smaller holding costs will gain market shares due to their market share positions.

**INSERT FIGURE 5 HERE**

The second example demonstrates the benefit of cost effective matching between suppliers and buyers proposed in this paper. Consider a two-echelon supply chain with two suppliers, supplier 1 and supplier 2. The setup cost and inventory holding cost for the two suppliers are \( S_1 = 450, H_1 = 12 \), and \( S_2 = 800, H_2 = 6 \), respectively. By the pairwise coexisting condition, these two suppliers will coexist with the market share boundary at \( \rho_{12} = 0.105 \). Consider a buyer with annual demand \( D = 25000 \), setup cost \( s = 400 \), and inventory holding cost \( h = 13 \). If the buyer follow the EOQ model to place his order, he should order about \( q = 1240 \) units each time and about \( \mu = 20 \) times a year. Since \( \rho = \mu/q = 0.016 < \rho_{12} \), supplier 1 matches the buyer better than supplier 2. Once the buyer chooses a supplier, they also have an option of coordinating their inventory policies through lot-sizing. The following table shows the various total system inventory costs based on whether the buyer and the supplier matches or not and whether they coordinate or not. The numbers in the parenthesis indicate the savings from coordination, matching with the right supplier, and both. This example clearly shows that the saving from effective matching between suppliers and buyers can be significant and sometimes more significant than that from lot-sizing coordination.

<table>
<thead>
<tr>
<th></th>
<th>No Match</th>
<th>Match</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Coordination via Discount</td>
<td>$27908 (100%)</td>
<td>$25195 (90.3%)</td>
</tr>
<tr>
<td>Coordination via Discount</td>
<td>$27568 (98.8%)</td>
<td>$23505 (84.2%)</td>
</tr>
</tbody>
</table>
In other words, a mismatch between a supplier and a buyer can be very costly, even if a subsequent lot-sizing logistics coordination is implemented. See Chen and Kouvelis (2003) for further analysis on this issue.

8 Implementation of the Market-Based Matching

So far, our analysis of the market-based matching and suppliers' market share has been based on the assumption that all participating suppliers' logistic cost structures, or at least their cost curves $C_i(\rho)$ for a buyer order $(\rho, \mu)$, are common knowledge among all suppliers and buyers. In this section, we argue that this assumption is not necessary in implementing the equilibrium market-based matching and pricing, as stated in Theorem 4 and 5. In particular, we propose a reverse auction mechanism to achieve the equilibrium predicted by the paper. See McAfee and McMillan (1987), and Milgrom (1987) for excellent surveys on the literature and theory of auctions and Rasmussen (1989) for a test book level explanation.

Reverse auctions, in which a single buyer puts out a request for a quote on a specified purchase and multiple sellers bid is proved to be one of the most useful application of Business-to-Business (B2B) in supply chain coordination. Their application has been popularized by independent media like FreeMarkets, and can be often executed via Internet technologies either via the use of firm specific Internet portals (Cisco, Wal-Mart, GE, GM, Ford are among the many firms running their own private exchanges for sourcing purpose) or consortium exchange Internet platforms (like Covisint in the car industry, Exostar in the aerospace industry etc.). For details and many examples on the implementation aspects of online auctions, reverse auctions, and the various B2B exchanges in creating such markets see Kambil a VanHeck (2002) and Hall (2001).

The “second-price sealed-bid” auction, often referred to as Vickrey auction, will be the right mechanism to achieve the equilibrium stated in Theorem 4 and 5 of our paper in terms of the supplier-buyer matching and pricing. To perform the auction, each buyer requests a bid for his order $(\rho, \mu)$. Each supplier quotes a price through a sealed bid. The buyer with the lowest bid wins the order but pays the second lowest price among all participating suppliers. This auction mechanism dictates all suppliers to bid their prices according to their actual costs involved in executing the order. In particular, a supplier’s bid does not depend on his knowledge on other competing suppliers’ cost structures. Clearly, the supplier whose logistic cost structure matches the order profile best will be able to execute the buyer’s order.
most cost effectively. As a result, he will bid with the lowest price and therefore win the order. The winning price, according to the auction mechanism, will be the lowest price of his competitors, or the price offered by his closest competitor(s). Apparently, the above auction mechanism achieves the equilibrium predicted by our paper without requiring the common knowledge of suppliers’ cost structures.

Implementation of the second-price sealed bid auction has the obvious shortcoming that the suppliers end up revealing their cost \( C(\rho) \) for the particular retail order \((\rho, \mu)\). Therefore, the supplier runs into the risk that within a few rounds of reverse auction transactions with a particular buyer, his detailed logistic cost structure gets revealed. The revealed cost structure information may negatively affect the supplier’s future negotiation with the buyer. As is well known in the theory and practice of Vickrey auctions, they work only when trust prevails among the buyers and suppliers. That is why the use of reliable independent third parties, such as FreeMarkets, or using independent management boards to run the consortium exchanges, can facilitate the effective execution of such sourcing market mechanisms.

Recent global outsourcing practices suggest that global logistic intermediaries (e.g., Li & Fung) can perform in an equally effective way the “matchmaker” role between buyers and suppliers using their knowledge of the sourcing cost structures of supply networks in certain regions of the world. Li & Fung is a global supply chain integrator that provides high value front-and-back end services to mostly large buyers such as Gymboree or The Limited. On the front end a Li & Fung decision might be fully dedicated to serve the buying needs of a large customer. At the back end through a network of 20 or so branch offices, Li & Fung maintains unique knowledge and relationships with extensive supplier bases. Li & Fung has a network of 7,500 suppliers on more than 30 countries. Through this network, and the knowledge on its suppliers’s cost and other operating conditions, the company are able to provide its customers with fully customerized sourcing solutions. Li & Fung has been extremely successful, and its success exemplifies the value generated from supply chain coordination through matching customer orders with right suppliers. This example also proves that the market-based matching between suppliers and buyers can be effectively implemented through credible market intermediaries, such as Li & Fung, without the fear of information leakage.
9 Conclusions and Final Remarks

In this paper, we place emphasis on clearly identifying the matching of buyers’ order profile to suppliers’ logistic cost structures as the first-order effect and main source of supply chain coordination benefits for supply chains with significant economies of scale. Exploiting the presence of ample size, but diverse in logistics costs, supplier set, buyers can devise price directed market mechanisms to allocate different buyer orders to the supplier with the right cost structure without having to worry much about subsequent lot-sizing coordination with the chosen supplier. As we demonstrated in Chen and Kouvelis (2003), such lot-sizing coordination in our studied context has an almost second order effect (less than 15% in terms of cost-saving). Our analysis identifies the segment of the buyer order space a specific supplier could win by exploiting the advantage of her logistic cost structure as well as the price that the winning supplier ends up offering in this price competitive supply environment. As a byproduct of our analysis, each supplier can effectively estimate her “market segment” of buyer’s order. Finally, we show the implications of new supplier entries on both market segmentation and prices for the existing suppliers, and offer constructive suggestions on ways suppliers could compete in this competitive environment.

We also explain how buyers can implement this price directed matching of orders to suppliers with the use of appropriate from reverse auctions or through the series of logistics and global trading intermediaries such as FreeMarkets and Li & Fung. As we have argued, at the equilibrium of the market-based matching, the winning supplier will execute the buyer’s order by offering the lowest price (i.e., just covering the cost) of her closest competitor. By the nature of this result, the second-price sealed bid auction seems to be a natural and effective mechanism to implement the market-based matching. The usual fears of revelation of excessive cost information on the part of the winning supplier become less relevant in the business environment of today, where buyers, or their logistical intermediaries, such as FreeMarkets, inspect the supplier production process to verify feasibility and validity of submitted bid prior to awarding winning contracts after the reverse suction. And in some cases, global logistical intermediaries can be the perfect implementers of this competitive equilibrium due to their knowledge of supplier cost structures in certain sourcing regions of the world. Pursuing this line of thought, one can argue that our paper has provided the theoretical underpinnings behind the explanation of the value model that such third party global logistic intermediaries provide, and why they enjoy a much higher profit margin than their traditional competitors. (e.g., Li & Fung can charge up to 10% of certain outsourcing transaction when traditional trading companies with limited supplier sets only get 1 – 2%).
Reference


Appendix

Proof of Proposition 1

**Proof:** Suppose on the contrary that equation $R_{12}(\rho) = 1$ has two distinct solutions $\rho_1 > 0$ and $\rho_2 > 0$. For simplicity, denote

\[ a = 8\rho_1 S_1/H_1, \quad b = 8\rho_1 S_2/H_2, \quad c = 8\rho_2 S_1/H_1, \quad d = 8\rho_2 S_2/H_2. \]
By definition, we have $ad = bc$ and $a \neq b$, $c \neq d$ since $S_1/H_1 \neq S_2/H_2$ by assumption (1). Since $\rho_1$ and $\rho_2$ are both solutions of equation $R_{12}(\rho) = 1$, we have

$$
\frac{1 + \sqrt{1 + a}}{1 + \sqrt{1 + b}} = \frac{1 + \sqrt{1 + c}}{1 + \sqrt{1 + d}}.
$$

Using the fact that $ad = bc$, we can rewrite the above equality as

$$
\sqrt{1 + a} + \sqrt{1 + d} + \sqrt{1 + a + d + ad} = \sqrt{1 + b} + \sqrt{1 + c} + \sqrt{1 + b + c + bc}.
$$

(14)

We next lead to contradictions by considering the following three cases:

• If $a + d > b + c$, then

\begin{align*}
(\sqrt{1 + a} + \sqrt{1 + d})^2 &= 2 + a + d + 2\sqrt{1 + a + d + ad} \\
&> 2 + b + c + 2\sqrt{1 + b + c + bc} \\
&= (\sqrt{1 + b} + \sqrt{1 + c})^2.
\end{align*}

Since $ad = bc$, we have

$$
\sqrt{1 + a} + \sqrt{1 + d} + \sqrt{1 + a + d + ad} > \sqrt{1 + b} + \sqrt{1 + c} + \sqrt{1 + b + c + bc}.
$$

However, the above inequality contradicts (14).

• If $a + d < b + c$, the similar argument also applies.

• If $a + d = b + c$, we have

\begin{align*}
(a - c)(a - b) &= a^2 + bc - a(b + c) = a^2 + ad - a(b + c) = a(a + d - b - c) = 0.
\end{align*}

The second equality is true because $ad = bc$. Since $a \neq b$, $a = c$. This implies $\rho_1 = \rho_2$, which contradicts to our assumption that $\rho_1$ and $\rho_2$ are two distinct solutions to equation $R_{12}(\rho) = 1$. 

\[\blacksquare\]
Figure 1

Figure 2
Figure 3
The Market Shares for Suppliers

Figure 4
Market Share of Demand for Coexisting Suppliers

Figure 5