Two Dimensional DCT-Based Channel Estimation for OFDM Systems with Virtual Subcarriers in Mobile Wireless Channels

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Abstract—In practical orthogonal frequency division multiplexing (OFDM) systems, some virtual subcarriers are reserved for easing the requirements on the filter. In this case, the conventional discrete Fourier transform (DFT)-based filtering in frequency-domain suffers from a spectral leakage, which results in an irreducible error floor. On the other hand, the DFT-based filtering or interpolation in time-domain also has performance degradation for system with high Doppler frequency. In order to solve these two problems, we propose a two dimensional discrete cosine transform (2-D DCT)-based channel estimator for OFDM systems with virtual subcarriers and high Doppler frequency. Simulation results show that the performance of the proposed method can well approach the 2-D minimum mean square error (MMSE) channel estimation.

I. INTRODUCTION

High data rate wireless transmission is required to meet the rapidly increasing demands for mobile communications [1], [2]. Orthogonal frequency division multiplexing (OFDM) technique has attracted much attention and found many applications due to its simple implementation and robustness against frequency-selective fading channels [3], [4]. For coherent OFDM detection, accurate channel estimation plays an important role in improving the overall system performance, especially when the channels are fast fading [5]. For practical applications, pilot-aided channel estimation is more attractive due to its simplicity and reliability, compared with blind and decision-directed methods [6], [7]. In this paper, we focus on the pilot-aided channel estimation method.

The two dimensional (2-D) optimum Wiener filtering for pilot-aided OFDM channel estimation was first derived in [8]. Since the optimum Wiener filter require the channel statistics, a robust interpolator was proposed in [9], which is a diamond shape lowpass filter implemented in the transform domain by 2-D fast Fourier transform (FFT) and inverse FFT (IFFT). However, in practical OFDM systems, some virtual subcarriers must be reserved for easing the requirements on the filter. In this case, the conventional discrete Fourier transforms (DFT)-based filtering in frequency-domain suffers from a spectral leakage, which results in an irreducible error floor. On the other hand, the DFT-based filtering or interpolation in time-domain also has the performance degradation for system with high Doppler frequency. In order to solve these two problems, we propose a 2-D discrete cosine transform (DCT)-based channel estimator for OFDM systems with both virtual subcarriers and high Doppler frequency.

The paper is organized as follows. Section II briefly reviews the statistics of the mobile wireless channel and establishes the signal model for channel estimation. Then, Section III derives the MMSE channel estimator for pilot subcarriers and data subcarriers, respectively, and proposes the 2-D DCT-based channel estimator. Next, Section IV presents simulation results of the proposed method. This paper is concluded in Section V.

Notations: Matrices are denoted by upper case boldface (e.g. \( \mathbf{A} \)) and column vectors are denoted by lower case bold face (e.g. \( \mathbf{x} \)). The \((p,q)\)-th entry of a matrix such as \( \mathbf{A} \) is denoted \([\mathbf{A}]_{p,q}\). An identity matrix is denoted as \( \mathbf{I}_N \), and an all-zero matrix is denoted as \( \mathbf{0} \). The superscripts \( T \), \( H \) and \( \dagger \) stand for transpose, conjugate transpose and pseudo inverse, respectively. We will reserve \( \otimes \) for Kronecker product, \( E\{\cdot\} \) for expectation with respect to all random variables within the brackets, and \( \text{diag}\{\cdot\} \) for a diagonal matrix with \( \cdot \) on its main diagonal.

II. SYSTEM MODEL

A. Channel Statistics

The baseband representation of the time domain channel impulse response (CIR) is characterized as

\[
h(t, \tau) = \sum_{l=0}^{L-1} \alpha_l(t)c(\tau - \tau_l),
\]

(1)

where \( \alpha_l(t) \) is the time-varying complex amplitude of the \( l \)-th path and \( \tau_l \) is the corresponding path delay, while \( c(\tau) \) is the aggregate impulse response of the transmitter-receiver pair, which is usually with a square-root raised-cosine spectrum. Parameter \( L \) is the total number of resolvable multi-paths of the channel. The channel is assumed to be time-invariant within one OFDM block with length \( T_f \) but changes from block to block. The time-domain correlation of each path is determined by normalized maximum Doppler frequency \( f_D T_f \). For Jakes’ model, the autocorrelation function of the \( l \)-th path can be expressed as

\[
r_l(\Delta t) \triangleq E\{\alpha_l(t+\Delta t)\alpha_l^*(t)\} = \sigma_l^2 J_0(2\pi f_D \Delta t) = \sigma_l^2 r_l(\Delta t),
\]

(2)
where $\sigma_l^2$ is the average power of the $l$-th path and $J_0(\cdot)$ is the zero-th order Bessel function of the first kind. Without loss of generality, we assume $\sum_{l=0}^{L-1} \sigma_l^2 = 1$.

Using (1), the channel frequency response (CFR) at time $t$ is given by
\[
H(t, f) = \int_{-\infty}^{\infty} h(t, \tau)e^{-j2\pi ft}\,d\tau = C(f)\sum_{l=0}^{L-1} \alpha_l(t)e^{-j2\pi lf\tau},
\]
where $C(f)$ is the Fourier transform pair of the transceiver’s impulse response $c(\tau)$. Sampling (3) at $t = nT_f$ and $f = k\Delta f$, where $\Delta f$ is the subcarrier spacing, the CFR at the $k$-th subcarrier of the $n$-th OFDM block is
\[
H(n, k) \triangleq H(nT_f, k\Delta f) = C(k\Delta f)\sum_{l=0}^{L-1} \alpha_l(nT_f)e^{-j2\pi k\Delta f\tau}. \tag{4}
\]

As it was shown in [5], the cross-correlation function $r_H(m, l)$, which characterized both the time- and frequency-domain correlation properties of $H(n, k)$ associated with different OFDM blocks and subcarriers can be expressed as
\[
r_H(m, l) \triangleq E\{H(n+m, k+l)H^*(n, k)\} = r_l(m)r_f(l), \tag{5}
\]
where $r_l(m) = J_0(2\pi m f_D T_f)$ is the time-domain correlation given in (2) and
\[
r_f(l) = |C(l\Delta f)|\sum_{l=0}^{L-1} \sigma_l^2 e^{-j2\pi k\Delta f\tau}. \tag{6}
\]
is the frequency-domain correlation. Equation (5) indicates that the correlation of the CFR at different times and frequencies can be separated into the multiplication of the time-domain correlation and the frequency-domain correlation [5].

B. 2-D Pilot-aided Channel Estimation

Fig. 1 shows the pilot pattern used in this work, where the known pilot symbols are inserted in every $D_t$ OFDM blocks and $D_f$ subcarriers. The number of pilot symbols in time- and frequency-domain is denoted by $L_T$ and $L_F$, respectively. Let $X(n_p, k_q)$ be the pilot symbol at the $k_q$-th subcarrier of the $n_p$-th OFDM block. Without loss of generality, we assume that the pilot symbols have unit magnitude, i.e., $|X(n_p, k_q)| = 1$.

For the rectangular grid that shown in Fig. 1, we have $n_p = pD_t + \alpha$ and $k_q = qD_f + \beta$, where $p = 0, 1, ..., L_T - 1$, $q = 0, 1, ..., L_F - 1$, $\alpha$ and $\beta$ are the initial position of pilot subcarrier in time- and frequency-domain, respectively. We can write the received pilot signal at the $k_q$-th subcarrier of the $n_p$-th OFDM block after the removal of the cyclic prefix and IFFT as
\[
Y(n_p, k_q) = X(n_p, k_q)H(n_p, k_q) + Z(n_p, k_q), \tag{7}
\]
where $Z(n_p, k_q)$ is the additive white Gaussian noise (AWGN) sample with zero mean and variance $\sigma_Z^2$.

Let $\tilde{X}(n_p) = [X(n_p, k_0), X(n_p, k_1), ..., X(n_p, k_{L_F-1})]^T$ be the block of $L_F$ pilot subcarriers of the $n_p$-th OFDM block, then we have
\[
\tilde{Y}(n_p) = \text{diag}(\tilde{X}(n_p))\tilde{H}(n_p) + \tilde{Z}(n_p), \tag{8}
\]
where the received pilot signal, the CFR, and the AWGN term at the $n_p$-th OFDM symbol are respectively given by
\[
\tilde{Y}(n_p) = [Y(n_p, k_0), Y(n_p, k_1), ..., Y(n_p, k_{L_F-1})]^T,
\]
\[
\tilde{H}(n_p) = [H(n_p, k_0), H(n_p, k_1), ..., H(n_p, k_{L_F-1})]^T,
\]
\[
\tilde{Z}(n_p) = [Z(n_p, k_0), Z(n_p, k_1), ..., Z(n_p, k_{L_F-1})]^T.
\]

Conventionally, the channel estimation can be performed to obtain initial channel estimates from (8) and enhanced ones by using the correlation in time domain. To achieve optimal channel estimation in the whole transmitted OFDM blocks, one needs to deal with the received pilot signals simultaneously. Let $\tilde{Y} = [\tilde{Y}^T(n_0), \tilde{Y}^T(n_1), ..., \tilde{Y}^T(n_{L_T-1})]^T$, then we have the following expression
\[
\tilde{Y} = \text{diag}(\tilde{X})\tilde{H} + \tilde{Z}, \tag{9}
\]
where the transmitted pilot signal, the CFR, and the AWGN term for all pilot subcarriers are respectively given by
\[
\tilde{X} = [\tilde{X}^T(n_0), \tilde{X}^T(n_1), ..., \tilde{X}^T(n_{L_T-1})]^T,
\]
\[
\tilde{H} = [\tilde{H}^T(n_0), \tilde{H}^T(n_1), ..., \tilde{H}^T(n_{L_T-1})]^T,
\]
\[
\tilde{Z} = [\tilde{Z}^T(n_0), \tilde{Z}^T(n_1), ..., \tilde{Z}^T(n_{L_T-1})]^T.
\]

III. CHANNEL ESTIMATION

A. Channel Estimation for Pilot Subcarriers

We start from the channel estimation for pilot subcarriers, of which the task is to recover $\tilde{H}$ from the observation $\tilde{Y}$ and the known $\tilde{X}$. The channel estimation will be extended to the data subcarriers in the subsequent subsection.

From (9), the linear MMSE solution of $\tilde{H}$ is [10]
\[
\hat{H} = \tilde{C}_{opt}\tilde{Y}, \tag{10}
\]

Fig. 1. Time-frequency grid in OFDM. Pilot subcarriers are marked in gray.
where $\tilde{C}_{\text{opt}}$ is an $L_TL_F \times L_TL_F$ matrix given by

$$
\tilde{C}_{\text{opt}} = \arg \min_C E \{ \| \tilde{H} - \tilde{H}^C \|^2 \} \\
= R_{\tilde{H}} \text{diag}(\tilde{X})^H (\text{diag}(\tilde{X})R_{\tilde{H}} \text{diag}(\tilde{X})^H + \sigma^2 I_{L_TL_F})^{-1} \text{diag}(\tilde{X})^H, \\
$$

(11)

where $R_{\tilde{H}} = E(\tilde{H} \tilde{H}^H)$, and the noise correlation matrix is assumed to be $R_Z = \sigma_Z^2 I_{L_TL_F}$.

**B. Channel Estimation for Data Subcarriers**

With the channel estimates of all pilot subcarriers in a transmitted slot as shown in Fig. 1, one could perform 2-D interpolation to get channel estimation of data subcarriers. In this subsection, we deal with the channel estimation for all the data subcarriers.

Let $H(n) = [H(n,0), \ldots, H(n,K-1)]^T$ be the CFRs at the $n$-th OFDM block and $H = [H^T(0), \ldots, H^T(N-1)]^T$ be the total CFRs for one transmitted slot, where $K$ and $N$ is the FFT size and block size respectively. Then, we have the following expression

$$
\tilde{H} = (B \otimes A)H, 
$$

(12)

where $A$ is composed of the $k_q$-th rows of $I_K$ with $q = 0, 1, \ldots, L_F-1$, and $B$ is composed of the $n_p$-th rows of $I_N$ with $p = 0, 1, \ldots, L_T-1$. Substituting (12) into (9) yields the following formula

$$
\tilde{Y} = \text{diag}(\tilde{X})(B \otimes A)H + \tilde{Z}. 
$$

(13)

Now the task of channel estimation is to recover $H$ from the observation $\tilde{Y}$ and the known $\tilde{X}$.

From (13), the linear MMSE solution of $H$ is

$$
\hat{H} = C_{\text{opt}} \tilde{Y}, 
$$

(14)

where $C_{\text{opt}} = R_{\tilde{H}}(B \otimes A)^H (R_{\tilde{H}} + \sigma^2 I_{L_TL_F})^{-1} \text{diag}(\tilde{X})^H$ and $R_{\tilde{H}} = E(\tilde{H} \tilde{H}^H)$.

**C. Channel Estimation based on Spectrum Decomposition**

From (11) and (14), we can see that the optimal filtering matrices are related to the channel statistics, it is valuable to have an insight into the matrix $R_{\tilde{H}}$ and $R_{\tilde{H}}$. With the separation property of channel correlation as shown in (5), we have

$$
R_{\tilde{H}} = R_t \otimes R_f, 
$$

(15)

where $R_t$ is the time-domain correlation matrix with $[R_t]_{n_1,n_2} = r_1(n_2 - n_1)$, and $R_f$ is the frequency-domain correlation matrix with $[R_f]_{k_1,k_2} = r_f(k_2 - k_1)$. Assume that the spectrum decompositions of Hermitian matrices $R_t$ and $R_f$ can be expressed as $R_t = U_t \Lambda_t U_t^H$ and $R_f = U_f \Lambda_f U_f^H$ respectively, where $\Lambda_t = \text{diag}(\lambda_t,0,,\lambda_{t,K-1})$ and $\Lambda_f = \text{diag}(\lambda_f,0,,\lambda_{f,K-1})$. Then the spectrum decomposition of $R_{\tilde{H}}$ is

$$
R_{\tilde{H}} = U \Lambda U^H, 
$$

(16)

where $U = U_t \otimes U_f$ and $\Lambda = \Lambda_t \otimes \Lambda_f$.

From (12), the correlation matrix of CFRs at pilot subcarriers can be expressed as

$$
R_{\tilde{H}} = R_t \otimes R_f, 
$$

(17)

where $\tilde{R}_t = BR_tB^H$ and $\tilde{R}_f = AR_fA^H$. Let $\tilde{R}_t = \tilde{U}_t \tilde{\Lambda}_t \tilde{U}_t^H$ and $\tilde{R}_f = \tilde{U}_f \tilde{\Lambda}_f \tilde{U}_f^H$, where $\tilde{U}_t$ and $\tilde{U}_f$ are the eigen-matrices of $\tilde{R}_t$ and $\tilde{R}_f$, $\tilde{\Lambda}_t = \text{diag}(\lambda_t,0,,\lambda_{t,L_t-1})$ and $\tilde{\Lambda}_f = \text{diag}(\lambda_f,0,,\lambda_{f,L_f-1})$ are the corresponding eigen-values. Then we have the following spectrum decomposition of $R_{\tilde{H}}$

$$
R_{\tilde{H}} = \tilde{U} \tilde{\Lambda} \tilde{U}^H, 
$$

(18)

where $\tilde{U} = \tilde{U}_t \otimes \tilde{U}_f$ and $\tilde{\Lambda} = \tilde{\Lambda}_t \otimes \tilde{\Lambda}_f$.

With the relations (16) and (18), the linear MMSE channel estimation as shown in (10) and (14) can be rewritten as

$$
\hat{H} = \tilde{U} \Gamma \tilde{U}^H \hat{H}_{\text{ini}}, 
$$

(19)

$$
\tilde{H} = U_E \Gamma \tilde{U}^H \hat{H}_{\text{ini}}, 
$$

(20)

where

$$
\hat{H}_{\text{ini}} = \text{diag}(\tilde{X})^H \tilde{Y}, 
$$

(21)

$U_E = UAU^H(B \otimes A)^H \tilde{U} \tilde{\Lambda}^+$ and $\Gamma = \tilde{\Lambda}(\tilde{\Lambda} + \sigma_e^2 I_{L_TL_F})^{-1}$. It can be proved that the matrix $U_E$ is independent of the eigenvalues of channel correlation matrices. Due to the space limitation, we omit the details of the proof. The above property of $U_E$ will be employed for developing extended DCT-based channel estimation in the subsequent subsection. It is worthy to note that $\hat{H}_{\text{ini}}$ is also the least square (LS) estimation of $\tilde{H}$. Equations (19) and (20) imply that the MMSE channel estimation can be performed by initial channel estimation followed by the time-frequency filtering in the transform domain.

**D. Two Dimensional DCT-based Channel Estimation**

The time-frequency filtering as in (19) for pilot subcarriers only and in (20) for all subcarriers are related to the spectrum decompositions of correlation matrices $R_{\tilde{H}}$ and $R_{\tilde{H}}$. This means that online estimation of the correlation matrices and online spectrum decompositions should be performed, which are all computational complicated. In this subsection, we discuss the fast implementation of the time-frequency filtering.

The time-frequency filtering can be expressed as a three-step procedure. First, the initial channel estimate $\hat{H}_{\text{ini}}$ is optimally decorrelated with the eigenmatrix $\tilde{U}$. Due to the constant magnitude property of pilot symbols, $\hat{H}_{\text{ini}}$ can be expressed as $\hat{H}_{\text{ini}} = \tilde{H} + \tilde{Z}_w$, where $\tilde{Z}_w$ is an AWGN vector with zero mean and covariance matrix $\sigma_e^2 I_{L_TL_F}$. The correlation matrix of $\tilde{U} \tilde{H} \tilde{U}_{\text{ini}}$ can be expressed as $\tilde{\Lambda} + \sigma_e^2 I_{L_TL_F}$. Second, the optimal filtering in the MMSE sense is performed on each element of the decorrelated vector with the eigenvalues of the correlation matrix $R_{\tilde{H}}$. Finally, the MMSE channel estimate $\hat{H}$ and $\tilde{H}$ are reconstructed with the eigen-matrices $\hat{U}$ and $U_E$ respectively. The $\hat{H}$ and $\tilde{H}$ are linear combination of the column vectors of $\hat{U}$ and $U_E$, respectively.
To simplify the implementation, the optimal decorrelation transform of a correlated signal is often replaced by a fast one in many applications such as image coding. The DCT is one of the popular fast transforms. The basic set of DCT provides a good approximation to the eigenvectors of the class of Toeplitz matrices that constitute the correlation matrices of finite order stationary Markov processes [11]. In other words, the correlation matrix of the correlated signal vector can be well diagonalized by the DCT matrix. We fix the matrix $\hat{U}_t$ and $\hat{U}_f$ to be the transpose of the $L_T$-point and $L_F$-point type II DCT matrix respectively. The $(k,l)$-th entry of the $L_T$-point type II DCT matrix $C_{L_T}^H$ and the $L_F$-point type II DCT matrix $C_{L_F}^H$ can be expressed as [12]

$$[C_{L_T}^H]_{k,l} = \kappa_k \cos \frac{\pi k(l + 0.5)}{L_T},$$  

$$[C_{L_F}^H]_{k,l} = \mu_k \cos \frac{\pi k(l + 0.5)}{L_F},$$

where

$$\kappa_k = \begin{cases} \sqrt{2}/\sqrt{L_T}, & k = 0, \\ \sqrt{2}/\sqrt{L_T}, & k \neq 0. \end{cases}$$  

$$\mu_k = \begin{cases} \sqrt{2}/\sqrt{L_F}, & k = 0, \\ \sqrt{2}/\sqrt{L_F}, & k \neq 0. \end{cases}$$

From the property of $U_E$, the matrices $U_{E,t}$ and $U_{E,f}$ can be approximated with matrices that are not related to the eigenvalues of the channel correlation matrices. We propose to replace $U_{E,t}$ and $U_{E,f}$ by the transpose of extended DCT matrices $C_{L_T}^H$ and $C_{L_F}^H$. The $C_{L_T}^H$ is defined as

$$[C_{L_T}^H]_{k,l} = \kappa_k \cos \frac{\pi k(l + 0.5)}{L_T},$$

where $0 \leq k \leq L_T - 1$ and $0 \leq l \leq N - 1$. And, the $C_{L_F}^H$ is defined as

$$[C_{L_F}^H]_{k,l} = \mu_k \cos \frac{\pi k(l + 0.5)}{L_F},$$

where $0 \leq k \leq L_F - 1$ and $0 \leq l \leq K - 1$. It can be verified that the following relations hold

$$C_{L_T}^H B^H = C_{L_T}^H,$$  

$$C_{L_F}^H B^H = C_{L_F}^H.$$

It can also be verified that the extended DCT matrix $C_{L_T}^H$ is the sub-matrix of the $(L_TD_t+D_t+1)$-point type I DCT matrix $C_{D_t+1}^H$ for even $D_t$ and the $(L_TD_t+D_t)$-point type II DCT matrix $C_{D_t+1}^H$ for odd $D_t$. Similar conclusion also holds for the extended DCT matrix $C_{L_F}^H$. The definition of the type I DCT can be found, for example, in [12].

With $\hat{U}$ and $U_E$ replaced by $(C_{L_T}^H \otimes C_{L_F}^H)^T$ and $(C_{L_T}^H \otimes C_{L_F}^H)^T$ respectively, (19) and (20) become

$$\hat{H} = (C_{L_F}^H \otimes C_{L_F}^H)^T \hat{d},$$  

$$\hat{H} = (C_{L_T}^H \otimes C_{L_T}^H)^T \hat{d},$$

where $d = (C_{L_T}^H \otimes C_{L_F}^H) \hat{H}_{ini}$. Let $d_{p,q}$ denote the $(pL_F+q)$-th element of $d$. Under the condition that the basis set of DCT approximates to eigenvectors, we have

$$E[|d_{p,q}|^2] \approx \tilde{\lambda}_t, \tilde{\lambda}_f, q + \sigma_z^2.$$

Thus, the $(pL_F + q)$-th diagonal entry of $\Gamma$ can be approximated by $E[|d_{p,q}|^2] - \sigma_z^2$.

From (30)-(33), we can see that the calculation of channel correlation matrix and its spectrum decomposition have been avoided. Thus, the time-frequency filtering can be implemented efficiently. First, the initial channel estimates $\hat{H}_{ini}$ are transformed to nearly decorrelated vectors $d$ via the 2-D DCT with size $L_T \times L_F$ as in (32). Then, from the resulted $d$, the diagonal matrix $\Gamma$ can be online estimated by using the relationship shown in (33). With the estimate of $\Gamma$, the filtering is performed on each element of $d$. Finally, the filtered vectors are transformed to $\hat{H}$ via the 2-D DCT as in (30) and $\hat{H}$ via the extended 2-D DCT as in (31).

In the implementation, the main computational load relies on that of the 2-D DCTs. The 2-D DCT can be implemented by applying 1-D DCTs to columns followed by 1-D DCTs to rows. When the number of pilot subcarriers in time-domain is a power of two, as well as the number of subcarriers in frequency-domain $L_T$, fast 1-D DCT algorithms are available, for example, in [12]. In this case, the number of real multiplications needed for implementation of (32) and (31) are $L_T L_F \left( \frac{3}{4} \log_2 L_T L_F - 2 \right) + 3(L_T + L_F)$ and $D_t (L_T + D_t + 1) (L_F + 1) \left( \frac{3}{4} \log_2 D_t (L_T + D_t + 1) (L_F + 1) - 2 \right) + 3(D_t (L_T + D_t + 1) + D_t (L_F + 1))$ respectively.

### IV. Simulations

To evaluate the performance of the 2-D DCT-based channel estimation for OFDM systems, simulations have been performed in Rayleigh fading channels. The OFDM system considered is with 256 subcarriers, a cyclic prefix of length 64 is used. The carrier frequency is 2.5GHz, and the bandwidth is...
The channel estimation for pilot subcarriers only and for all the data subcarriers are simulated separately. Normalized MSE (NMSE) performances of the channel estimation are evaluated. The NMSE (in dB) is defined as $10 \log(E\{||\tilde{H} - \hat{H}||^2\}/E\{||H||^2\})$ for pilot subcarriers only and $10 \log(E\{||H - \hat{H}||^2\}/E\{||H||^2\})$ for all the data subcarriers.

For pilot subcarriers only, the following four channel estimation methods are compared via simulations.

1) **LS CHE**: Least square channel estimation as the same as the initial channel estimation in (21).

2) **2-D MMSE CHE**: MMSE channel estimation with the initial channel estimation as in (21) and the time-frequency filtering as in (19), where the $\hat{U}$ and $\hat{\Gamma}$ are

1.28MHz. The power spectral density satisfies Jakes’ model, and the power delay profile is assumed to be exponentially distributed. The channel time delay is uniformly distributed over the interval $0 \sim 3.90625 \mu s$ for $L = 6$. The additive channel noise is white Gaussian with zero mean and the variance determined by the SNR. The vehicle speed is set to be 3km/h and 120km/h. The parameters in the pilot pattern as shown in Fig. 1 are as follows: The block length $N = 29$, the pilot spacing in time-domain $D_t = 4$, the pilot spacing in frequency-domain $D_f = 8$. The parameters for simulation are summarized in Table I.

The channel estimation for pilot subcarriers only and for all the data subcarriers, where the velocity speed is set to be 3km/h.

Fig. 2. The NMSE performance of the channel estimation for the pilot subcarriers only, where the velocity speed is set to be 3km/h.

Fig. 3. The NMSE performance of the channel estimation for the pilot subcarriers only, where the velocity speed is set to be 120km/h.

Fig. 4. The NMSE performance of the channel estimation for all the data subcarriers, where the velocity speed is set to be 3km/h.

Fig. 5. The NMSE performance of the channel estimation for all the data subcarriers, where the velocity speed is set to be 120km/h.
obtained from known channel correlation matrix.

3) **2-D FFT CHE**: 2-D FFT-based channel estimation with the initial channel estimation as in (21) and the time-frequency filtering as in (20), where the $\hat{U}_t$ and $\hat{U}_f$ are replaced by $L_T$-point and $L_F$-point DFT matrix respectively, $\Gamma$ is obtained from the online estimates of channel correlation matrix. The channel correlation matrix is estimated from the samples of initial channel estimates.

4) **2-D DCT CHE**: 2-D DCT-based channel estimation with the initial channel estimation as in (21) and the time-frequency filtering as in (30), (32) and (33).

For all the data subcarriers, we compare the following three channel estimation methods.

1) **Extended 2-D MMSE CHE**: Extended 2-D MMSE estimation with the initial channel estimation as in (21) and the time-frequency filtering as in (20), where the $U_{E,t}$, $U_{E,f}$, $\tilde{U}_t$, $\tilde{U}_f$ and $\Gamma$ are obtained from known channel correlation matrix.

2) **Extended 2-D FFT CHE**: Extended 2-D FFT-based channel estimation with the initial channel estimation as in (21) and the time-frequency filtering as in (20), where the $U_t$, $U_f$, $\tilde{U}_t$, $\tilde{U}_f$ are replaced by $N$-point, $K$-point, $L_T$-point and $L_F$-point DFT matrix respectively, $\Gamma$ is obtained from known channel correlation matrix.

3) **Extended 2-D DCT CHE**: Extended 2-D DCT-based channel estimation with the initial channel estimation as in (21) and the time-frequency filtering as in (31), (32) and (33).

Fig.2 and Fig.3 show the NMSE performances of the channel estimation methods for the pilot subcarriers with velocities set to be 3km/h and 120km/h respectively. From the results, we can see that the 2-D DCT-based channel estimation can well approach the 2-D MMSE channel estimation both in high and low mobility, while the 2-D FFT-based channel estimation has a performance degradation in high mobility.

Fig.4 and Fig.5 show the NMSE performances of the channel estimation methods for all the data subcarriers with velocities set to be 3km/h and 120km/h respectively. From the results, it can be observed that the extended 2-D DCT-based channel estimation still well approximates the MMSE channel estimation. The extended 2-D DCT-based channel estimation outperforms the extended 2-D FFT-based channel estimation by 0.5dB to 8.1dB for the low mobility and 2.0dB to 15.5dB for the high mobility in the range of measured SNRs.

**V. Conclusion**

In this paper, we have proposed a 2-D DCT-based channel estimator for OFDM systems with virtual subcarriers in mobile wireless channels. It can well approximate the optimal MMSE channel estimation with the low-complexity implementation. Simulation results have validated the performances of the proposed channel estimation.

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