Distributed Nonlinear MPC Formation Control with Limited Bandwidth

Sami El-Ferik
Department of Systems Engineering, King Fahd University of Petroleum and Minerals, Dhahran, 31261 e-mail: selferik@kfupm.edu.sa.

Bilal A. Siddiqui
2

Frank L. Lewis
3
Research Institute, The University of Texas at Arlington, Texas 76118 email: lewis@uta.edu

Abstract—We address leader-follower formation control of autonomous vehicles in a non-ideal communication environment, e.g. underwater channel, where the bandwidth is limited and there are communication and computational delays. Moreover, the agents have both input and state constraints on their dynamics. A novel formulation of nonlinear model predictive control (NMPC) is presented, in which agents do not need to estimate neighbors’ dynamics and collision avoidance is guaranteed. Packet size is reduced considerably by data compression with neural networks. Moreover, this method allows the agents to be sampled at different rates, have different dynamics, constraints and prediction horizons, and be robust to propagation delays. Collision avoidance is achieved by means of a spatial filter based potential field. The sound analytical results are verified by simulations.

I. INTRODUCTION

In recent years, cooperative control of a team of autonomous agents has been extensively researched due to promising advantages of robustness, adaptivity, flexibility, and scalability. Cooperation means close relationship among agents in the team with information sharing playing an important role. One type of cooperative control is decentralized/distributed formation control, where a certain geometric pattern is formed and the leader is followed maintaining that reference. One of the most promising techniques for formation control is nonlinear model predictive control (NMPC) due to its inherent ability to handle constraints, incorporation of non-local information and reconfigurability. The main disadvantage is the heavy computational load for solving constrained nonlinear optimization problem at each sampling instant. Work on multi-agent formation control of autonomous vehicles using NMPC was pioneered by Dunbar at Caltech in 2001 [1] - [2]. A generalized framework for distributed NMPC for cooperative control of team of agents is proposed in [3] - [4]. However, effect of communication delay is not considered.

Pioneering theoretical work on extending distributed NMPC framework to a group of autonomous vehicles receiving delayed information (delay is fixed) from their neighbors was recently presented in [5]. Rigorous stability analysis is used to establish regional input-to-state (ISS), extending the work on NMPC of single systems in [6]. The delayed state information is projected in the prediction horizon using a forward forgetting-factor. In effect, it means that each agent assumes that the states of its neighbors will asymptotically go to zero (or equilibrium) and uses this assumed prediction to plan its own trajectory. Some other algorithms to deal with communication delay are presented in [7] and [8], but without strong theoretical framework.

In this paper, we address the problem of leader-follower formation control of constrained autonomous vehicles subject to propagation delays. Formation tracking is achieved through NMPC such that each agent performs local optimization based on planned trajectories received from its neighbors, without any need for agents to know or estimate their neighbors’ dynamics. The trajectory is compressed using neural networks, which is shown to reduce the packet size considerably. Moreover, the method allows the agents to be heterogeneous, multi-rate and have different horizons. Collision avoidance is achieved by formulating a novel spatial-filtered potential field. New theoretical results are presented along with verification with simulations.

In this paper, we extend the work in [5], and we contribute with the following original results:

- Asynchronous, multi-rate measurements are addressed.
- Robustness to inaccuracy in communicated trajectories is explicitly taken into account, resulting in practical stability instead of asymptotic stability, such as in [5].
- Collision avoidance is also explicitly catered for. A novel modification of potential field method is proposed.
- New input to state practical stability (ISpS) and generalized small gain conditions are derived, which show independence to network topology.

The paper begins with presenting some preliminary definitions, moving on to presentation of new theoretical results in ISpS and small gain theorems, its application to fleet of multi-agent systems and ends with conclusion and recommendation for future work.

A. Preliminaries

Let $\mathbb{R}$ and $\mathbb{Z}$ denote real numbers and integers, respectively. The $L_2$ Euclidean norm is denoted as $\| \cdot \|$ and $\| \cdot \|_\infty$ is the $L_1$ norm. For a set $A \subseteq \mathbb{R}^n$, the point to set distance from $\zeta \in \mathbb{R}^n$ to $A$ is denoted by $d(\zeta, A) \triangleq \inf \{ | \eta - \zeta |, \eta \in A \}$, and if $A$ is a closed set, its boundary is denoted by $\partial A$. The difference between two sets $A, B \subseteq \mathbb{R}^n$ is denoted by $A \setminus B \triangleq \{ x : x \in A \cap B \}$, and their distance is denoted by $\text{dist}(A, B) \triangleq \inf \{ d(\zeta, A), \zeta \in B \}$.

The indicator function of a subset $A$ of a set $X$ is a function $1_A : X \to \{0, 1\}$ defined as:

$$1_A(x) = \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{if } x \notin A. \end{cases}$$
A function $\alpha : [0, a) \to [0, \infty)$ is said to belong to class $\mathcal{K}$, if it is continuous, strictly increasing and $\alpha(0) = 0$. It belongs to class $\mathcal{K}_\infty$, if $\alpha(r) \to \infty$ as $r \to \infty$ and $a = \infty$. A function $\beta : [0, a) \times [0, \infty) \to [0, \infty)$ is said to belong to $\mathcal{KL}$, if for each fixed $s \geq 0$, the mapping $\beta(\cdot, s)$ belongs to class $\mathcal{K}$, and for each fixed $r \geq 0$, the mapping $\beta(r, \cdot)$ is decreasing and $\beta(r, s) \to 0$ as $s \to \infty$.

II. PROBLEM STATEMENT AND PROPOSED SOLUTION

In this section, we formulate the cooperative control problem using predictive control paradigm.

A. Regional Input-to-State practically Stable (ISpS)

Consider the discrete time nonlinear system:

$$x_{t+1} = f(x_t, w_t)$$

where $x_t \in \mathbb{R}^n$ and $w_t \in \mathbb{R}^r$ is the system state and input, respectively, with $f(0, 0) = 0$.

Definition 1: Regional Positive Invariant Set [5]: If the input is bounded $w_t \in W \subset \mathbb{R}^r$ and $\Xi$ is a set of $x_t$, $\forall w \in W$, such that if a solution of (1) starts in $\Xi$, it stays in $\Xi$ for all future time, then it is an RPI set of $x_t$.

Definition 2: Regional Input to State Practically Stable (ISpS): Given that $\Xi$ is compact, robust positively invariant, and contains the origin as an interior point, the system (1) is said to be regional ISpS in $X_t$ if $\forall t \geq 0, x_0 \in \Xi$ and $w \in W$, there exists a $\mathcal{KL}$-function $\beta$, a $\mathcal{K}$-function $\gamma$ and a constant $c > 0$ such that

$$|x(t, x_0, w)| \leq \beta(|x_0|, t) + \gamma(|w|_\infty) + c$$

We now state an important and new result in regional practical stability.

Definition 3: ISpS Lyapunov Function: A function $V : \mathbb{R}^n \times \mathbb{R}^r \to \mathbb{R}_{\geq 0}$ is an Input-to-State practically Stable (ISpS) Lyapunov function for (1) in $\Xi$, if the following hold:

1) $\Xi$ is compact, RPI having origin as its interior point, such that for a suitable $\mathcal{K}_\infty$-function $\alpha_1$:

$$\infty > V(x_t, w_t) \geq \alpha_1(|x_t|), \quad \forall x_t \in \Xi, w_t \in W$$

and for $\mathcal{K}$-function $\alpha_2$, $\mathcal{K}$-functions $\sigma_1$ and $\sigma_2$, and constant $\bar{c}$

$$V(f(x_t, w_t), w_{t+1}) - V(x_t, w_t) \leq -\alpha_2(|x_t|) + \sigma_1(|w_t|) + \sigma_2(|w_{t+1}|) + \bar{c},$$

$\forall x_t \in \Xi, \forall w_t, w_{t+1} \in W$

2) A set $\Omega \subset \Xi$ exists with origin as an interior-point, such that for suitable $\mathcal{K}_\infty$-functions $\alpha_3$ and $\sigma_3$ and constant $\bar{c}$ which satisfy:

$$V(x_t, w_t) \leq \alpha_3(|x_t|) + \sigma_3(|w_t|) + \bar{c},$$

$\forall x_t \in \Omega, w_t \in W$

3) Since, $V(x_t, w_t) < \infty, \forall x_t \in \Xi, w_t \in W$, then $\exists d > 0$ such that for $x_1 \in \Xi \setminus \Omega$ and $x_2 \in \Omega$, it holds that $|x_1| > |x_2|$ and

$$V(x_1, w_1) \leq V(x_2, w_2) + d$$

Theorem 1: If a system admits an ISpS-Lyapunov function in $\Xi$, then it is regional ISpS in $\Xi$, according to definition of (2) such that:

$$\beta(r, s) \triangleq \alpha_1^{-1}(3\beta(3\alpha_3(r), s))$$

$$\gamma(s) \triangleq \alpha_1^{-1}(3\beta(3\sum_{i=1}^{3} \sigma_i(s) + \beta(3\sigma_3(s), 0)))$$

$$c \triangleq \alpha_1^{-1}(3\beta(\beta(\sigma_1 + d), 0) + \alpha_1^{-1}\gamma(\mu(3\bar{c}))) + \alpha_1^{-1}\gamma(3\bar{c}))$$

where $\mu$ and $\gamma$ are suitable $\mathcal{K}_\infty$ functions, while $\beta$ is a $\mathcal{KL}$ function.

A proof is presented in [9]. This important result is particularized to the problem at hand in the following sections.

B. Distributed Multi Agent Nonlinear MPC

Consider a set of $N$ agents, denoted as $A^i$, each of which has nonlinear discrete-time dynamics:

$$x_{t+1}^i = f^i(x_t^i, u_t^i, w_t^i), \quad \forall t \geq 0, \quad i = 1, \ldots, N \quad (7)$$

The local states $x_t^i$, controls $u_t^i$ and external inputs $w_t^i$ belong to constrained sets:

$$x_t^i \in X^i \subset \mathbb{R}^{n^i}, \quad u_t^i \in U^i \subset \mathbb{R}^{m^i}, \quad w_t^i \in W^i \subset \mathbb{R}^{p^i} \quad (8)$$

One can observe that the agents’ dynamics (7) are coupled only in closed-loop under the control action which takes into account information about neighbors’ states. For each agent $A^i$, local states $x_t^i \in X^i$ and neighborhood intention matrix $W^i$ at sampling instant $t$, let the following finite-horizon cost:

$$J_t^t \left( x_t^i, \hat{x}_t^i, u_{t+N^p_c}^i, u_{t+N^c}^i, d^{h^i}, d^{l^i}, u_{t+t+N^p}^i, N^p_c, N^c \right) = \sum_{t=t}^{t+N^p-1} \left[ h^i \left( x_t^i, u_t^i, d^{h^i} \right) + q^i \left( x_t^i, \hat{x}_t^i, d^{l^i} \right) \right] \quad (9)$$

where $N_p$ and $N_c$ are the prediction and control horizons, respectively. Distributed cost function (9) consists of a local transition cost $h_t^i$, a local terminal cost $h_t^f$ and cooperation cost $q_t^i$. The alignment vectors $d^{h^i}$ and $d^{l^i}$ are used to define the cooperation among agents, e.g. the formation geometry. The control sequence $u_{t+t+N^p}^i$ is divided into two parts viz., $u_{t+t+N^p-1}^i$ and $u_{t+t+N^p-1}^i$. The first part is the control sequence which forms the decision variables of the optimization problem discussed later, and the latter part is generated by either a stabilizing auxiliary control law $u_t^i = k_t^i(x_t^i)$ for $t \geq N^c_t$. The distributed control law $u_{t+t+N^p}^i$ is obtained by solving the following optimization problem:

C. Finite Horizon Optimal Control Problem:

At every sampling time instant $t \geq 0$, given horizons $N_p$ and $N^c_t$, and auxiliary control $k_t^i$, find the optimal control sequence $\Theta^{i,*}(x_{t+t+N^p_c}, w_{t+t+N^p}^i) \triangleq u_{t+t+N^p-1}^i$, which minimizes distributed cost (9) for agent $A^i$, subject to the following constraints:
• Local dynamics (7), subject to input and state constraints (8), with initial local states $x_i^t$ and neighborhood intention matrix $\hat{W}^i$.

• Terminal state constraints $x_i^{t+N_p^i} \in X_i^t$.

Any control $u_i^{t+N_p^i-1}$, which is not optimal but does not violate the first constraint above is called an admissible control for the optimization problem. The closed-loop dynamics is thus given by

$$x_i^{t+1} = f^i (x_i^t, \Theta^{i,*} (x_i^{t+N_p^i}, w_i^{t+N_p^i})) = \tilde{f}^i (x_i^t, w_i^t) \quad (10)$$

D. Communication Model

To achieve cooperation, the agents transmit their planned state trajectories, $x_i^t \in X_i \subset \mathbb{R}^{n_i} \times N_p^i$ over the network. These communication packets are received at vehicles within the communication range $d_{\text{com}}$, where signal strength (SNR) is appreciable. This defines a neighborhood $G_i = A_i$, $\forall j \neq i, d(x_i, x_j) \leq d_{\text{com}}$ of the propagating agent $A_i$.

However, the reception occurs after some time delay $\Delta_{ji}$, not necessary an integer multiple of the sampling time $T_i$. If the entire trajectory is sent over the network, the packet would contain at least $n_i \times N_p^i$ floating point entries. To reduce the packet size, we propose to send an approximation of the trajectory in the form of neural network weights $(N_i^i \subset \mathbb{R}^q)$, a time-stamp $(T_i^i)$ and sampling time $T_i$. Hence, the communication packet from $A_i$ to $A_j$, denoted by $G_{t-\Delta_{ji}}^j$, would have the topology shown in Table II-D below.

<table>
<thead>
<tr>
<th>Communication Packet Topology</th>
<th>Data Register</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Agent Identity, $i$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Time Stamp, $T_i^i$</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Sampling Time, $T_i$</td>
<td></td>
</tr>
<tr>
<td>4 to $3+q$</td>
<td>Neural Net Weights, $N_i^i$</td>
<td></td>
</tr>
<tr>
<td>4+q onwards</td>
<td>Error &amp; Clock Sync Codes</td>
<td></td>
</tr>
<tr>
<td>Optional (leader)</td>
<td>Cooperation Goals</td>
<td></td>
</tr>
</tbody>
</table>

When the packet is received at $A_j$, the clock is first synchronized using the error control codes. This allows the communication delay $\Delta_{ji}$ to be estimated, by comparing the local clock time $t_j^i$ with the packet’s time-stamp $T_i^i$. The neural network at $A_i$ is trained using the state trajectory as output and the sampling instants as input. Using sampling rate $T_j^i$ and prediction horizon $N_j^i$ at $A_j$, the re-sampled trajectory $\tilde{w}_i^j \in X_i \subset \mathbb{R}^{n_i} \times N_j^i$ is generated using received neural network $N_i^i$. If the horizon is sufficiently long, states can be extrapolated with bounded error. If the transmitted packet is delayed more than a threshold $\Delta$, the packet is deemed to be lost. If sender is the leader, it also communicates the formation geometry to followers.

E. Collision Avoidance

We first define what constitutes a collision course for agents.

**Definition 4:** Collision Course: An agent $A_i$ is said to be on a collision course with at least one other agent if

$$\sum_{j \in G_i} 1_{(R_{\text{min}}^i - d_{ij}(k)) > 0, \forall k \leq (t+N_p^i)} > 0, \forall j \neq i \quad (11)$$

where $R_{\text{min}}^i$ defines the ‘zone of safety’ of an agent. To avoid collision, the repelling potential is formulated as:

$$\Phi_t^i = \sum_{j \in G_i} \frac{\lambda(R_{\text{min}}^i - d_{ij}(k))}{(t+N_p^i) - k + 1} \lambda(d_{ij}(k)), d_{ij}(k) \quad (12)$$

where $d_{ij}(k) = \sqrt{||x_i^t(k) - x_j^t(k)||^2 + ||y_i^t(k) - y_j^t(k)||^2}$ is the Euclidean distance between agent $A_i$ and $A_j$, $R_{\text{min}}^i$ is the distance when repulsive potential becomes active and repels the agent away from each other for avoiding collision. $\lambda(d_{ij})$ are the weights of a filter which are strictly decreasing in their argument, such that $\lambda \equiv \frac{t+N_p^i}{\sum_{k=t}^{t+N_p^i} \lambda(d_{ij}(k))}$. The cost (9) for agents on a possible collision course is then modified as:

$$J_t^i = J_t^i (1 + \Phi_t^i) \quad (13)$$

In other words, if at any instant $t \leq k \leq (t+N_p^i)$ in the prediction horizon, an agent $A_i$ has a feasible trajectory which falls within $R_{\text{min}}^i$ of agent $A_j$, the cost of taking such a course would be increased from (9) to (13). It should be noted that strength of potential field (12) is inversely proportional to the weighted average of the distance between the two agents $\sum_{k=t}^{t+N_p^j} \lambda(d_{ij}(k)), d_{ij}(k)/\lambda$. The weights $\lambda$, strictly decreasing with $d_{ij}(k)$, ensure that the smallest separation between two agents gets the highest weight. On the other hand, taking a simple average (i.e. $\lambda \equiv 1$) or temporal forgetting ($\lambda^k$), results in poor performance in collision avoidance, as trajectories which enter very late in zone $R_{\text{min}}^i$ (i.e. $R_{\text{min}}^i - d_{ij}(k) > 0, \forall k \to t+N_p^i$) have a small repelling potential (12), and hence not discouraged from very early on. This results in agents getting very close before they start repelling each other to avoid collision. However, with the cost in (13), trajectories are immediately penalized upon falling within $R_{\text{min}}^i$. Conditions for stability of this approach are shown in Section III-B.

F. Collision-Free Distributed NMPC Algorithm

Algorithm 1 describes the methodology adopted to solve the formation control problem in this paper.

III. STABILITY ANALYSIS OF PROPOSED ALGORITHM

The stability analysis will be carried out in two stages. First, the individual agents are shown to be ISpS in a subset of $X_i$, and robust to communication delays and trajectory approximations. Then, a generalized small gain condition is derived for ensuring stability of the team of agents.

A. Universal Function Approximation by Neural Networks

The basic universal approximation result says that any smooth function $w(t)$ can be approximated arbitrarily closely on a compact set using a two-layer NN with appropriate weights [10]. Let $w(t): \mathbb{R}^{n_i} \times N_p^i \to \mathbb{R}^{n_i}$ be a set of smooth functions, then we can assume that:

$$\tilde{w}(\tau) = w(\tau) + \xi, \forall \xi^i \leq \xi \quad (14)$$
Algorithm 1: CF-DNMPC Algorithm

1: Select $A_i^1$  \hspace{1em}  \triangleright  \; i = 1 \triangleq \text{Leader, } t = 0
2: Ensure $A_i^t \leftarrow x_i^t, d_i^t, d_i^{T}, g_i^t \; \triangleright \; t = 0$
3: Ensure $t^* \equiv t^J$ \hspace{1em}  \triangleright  \text{Sync. Clocks } \forall i \neq j$
4: procedure COLLISION FREE DISTRIBUTED NMPC
5:   Optimize cost (9) at $A_i^t$ for $u_{i,t+t+N_i^t+1}$
6:   Train NN Train Neural network for $x_{i,t+*}^t$
7:   Implement first element of block of $u_{i,t+t+N_i^t-1}$
8:   Transmit data packet \hspace{1em}  \triangleright  \text{See Table II-D}
9: procedure DELAY ESTIMATION & TRAJ. RECON.
10: if New Data Packet received then
11:    Estimate time delay $\Delta_{ij}$
12:    if Received packet older than previous then
13:       Reject received packet.
14:    Select last available packet.
15:    else if Delay greater than threshold then
16:       Reject received packet.
17:    Select last available packet.
18:    else
19:       Accept received packet.
20:    end if
21:    if $j \in G^t$ then
22:       Reconstruct $\hat{u}_i^{t+N_p}$ with selected NN \hspace{1em}  \triangleright \; j \in G^t
23:    end procedure \hspace{1em}  \triangleright \text{End Traj. Reconstruction}
24: procedure COLLISION AVOIDANCE
25:   if Agents are close, min($d(x_i^t, \hat{x}_i^t)) \leq R_{\min}$ then
26:      Modify Cost function as in (13)
27:   end if
28: end procedure \hspace{1em}  \triangleright \text{End Col. Av. procedure}
29: if Decide then target achieved
30:   Goto next Waypoint Or Terminate
31: else
32:   Goto Line 5
33: end if
34: Increment time by one sample \hspace{1em}  \triangleright \; t^* = t^J + T_s
35: end procedure \hspace{1em}  \triangleright \text{End CF-DNMPC Alg.}

where $\hat{u}(\tau)$ is the approximation of $u(\tau)$ by the NN, $\xi$ is called the NN function approximation error which decreases as the hidden-layer size $L$ increases. Also, $\xi$ will be affected by the delay in the information exchanged between agents due to extrapolation in predicting the tail of the trajectory from $t + N_p^t$ to $t + N_p^t + \Delta_{ij}$. Thus, for collision avoidance to be feasible, the following Lemma should be observed:

**Lemma 1:** Given that there is a neural network of $L^j \geq 2$ layers for approximating trajectory $w_i^{t+*+N_p^t}$, there is an upper bound on delay $\Delta_{ij}^j$, i.e $\Delta_{ij}^j \leq \hat{\Delta}$, in order to find feasible collision free solution to the finite horizon optimal control problem described in Section II-C.

A heuristic proof is provided in [9].

### B. Stability of Individual Agents without Collision Avoidance

One of the methods to ensure closed-loop stability of MPC is by specifying a terminal cost and terminal constraint set [11], such that a local linear control law (called terminal or auxiliary control) operates in the terminal constraint set. At this stage, the interconnection among agents’ decisions is ignored, and hence the exchanged information is considered as external input not resulting in any possible collision. Each agent has a terminal cost $h_i^T > 0$, a closed terminal constraint set $X_f \subset X, \forall X_f \in X_f$, and a terminal control law $k_i(x) \in U$, such that the following assumptions hold:

**Assumption 1:** $X_f$ and $k_i$ are such that:
(i) Terminal set, cost and control
(a) $|k_i^T(x)| \leq L^i_k |x|^l_i, L^i_k > 0, \forall x^i \in X_f^i.$
(b) $|f^i(x, k_i^T(x^i))| \leq L^i_f |x|^l_i, L^i_f > 0, \forall x^i \in X_f^i.$
(c) $X_f$ is RPL, i.e. $f^i(x^i, k_i^T(x^i)) \in X_f^i, \forall x^i \in X_f^i.$
(d) there exist suitable $K\infty$ functions $\alpha_1, f$ and $\alpha_2, f$, such that $\alpha_1, f(|x^i|) \leq h_i^T(x^i) \leq \alpha_2, f(|x^i|), \forall c^i \in X_f^i.$
(e) $w_{ii+1}^i = g(w_i^i), |g(w_i^i)| \leq L_{gw} |w_i^i|$
(f) $h_i^T(f^i(x^i, k_i^T(x^i))) + h_i^T(x^i, k_i^T(x^i)) + q_i^T(x^i, \tilde{w}^i) \leq \psi^T(w_i^i), \psi^T$ is a $K$-function and $\tilde{w}^i$ is the NN approximation of $w^i$.

(ii) Neural Network Inter/Extrapolation: NN received from the neighbors is used to reconstruct other agents’ intended trajectories with a bounded error given in (14), i.e. $\delta_x^i \leq \xi^i.$ Thus, $|\tilde{w}_i^i| \leq |w_i^i| + \xi^i.$

(iii) Transition Cost:
$v^i(|x^i|) \leq h_i^T(x^i, u^i, w^i) \leq L_h^i |x|^l_i + L_h^i |u|.$

(iv) Cooperative Cost:
$0 \leq q^i(x^i, w^i) \leq L_q^i |x|^l_i + L_q^i |w|.$

(v) ISpS Lyapunov Function: Let, $V(x_i^i, u_i^i, u_i^i) = J_i^f(x_i^i, u_i^i, u_i^i^{t+N_p^t})$, where $u_i^i^{t+N_p^t}$ is the solution minimizing (9). Define the following comparison functions (following the nomenclature of Theorem 1):
(a) $\alpha_1(s) = \alpha_2(s) = \chi$.
(b) $\alpha_3(s) = \alpha_2(s) = (L_s + L_h^i + L_q^i)(L_s^{-1}N_p^t-1) + \alpha_2(s)L_s N_p^t$.
(c) $\alpha_1(s) = \sigma_2(s) = \psi(s)$
(d) $\sigma_2(s) = s(L_q^i)(L_s^{-1}N_p^t-1)/(L_q^i - 1), \sigma_3(s) = s(L_q^i)(L_s^{-1}N_p^t-1)/(L_q^i - 1), \sigma_4(s) = \sigma_2(\xi)$
(e) $\bar{c} = \sigma_2(\xi)$

**Theorem 2:** Under assumptions 1, the individual agent $A_i^i$ under optimal $u^i$ and terminal $k_i^T(x^i)$ control laws, subject to constraints (7)-(8), is ISpS with robust output admissible set $X_{i,\text{MPC}}$, which is the set of initial states for which the solution of minimization problem of (9) exists, such that $X_{i,\text{MPC}} \subseteq X_i$.

Key to ensure stability is the design of the terminal control law $k_i^T(x^i)$, given below

**Lemma 2:** Lemma Let $h_i^T = x_i^{T}Q_i x_i + u_i^{T} R_i u_i, q_i^t \leq x_i^{T}S_i x_i + \Psi(|w_i^t|)$ and $h_i^T = x_i^{T}Q_i x_i$, where $Q_i, R_i$,
$Q_f$ and $\bar{S}^i$ are positive definite matrices ($Q_f$ is symmetric as well) and $\Psi(|w^i|)$ is a $K$-function. Suppose that there exists an auxiliary control $k^i_f(x_t) = K^i x_t^i$ such that $A^i = A^i + B^iK^i$ is stable, where $A^i$ and $B^i$ are Jacobians of $f(x,u)$ in (7), w.r.t. $x$ and $u$, respectively. Let $Q_f$ be the solution to the following Lyapunov inequality:

$$A^T e Q_f A^i - Q_f + Q_i + K^T R^i K^i + \bar{S}^i \leq 0 \quad (15)$$

Then, there exists a positive constant $\Gamma^i > 0$ and minimum horizon length $N_p$, such that for $N_p \geq N_p^i$, the terminal set is defined as $X_f^i = \{x_t \in X^i : \Gamma^i \geq x_t^T Q_f x_t^i\}$. To find the matrix $Q_f$, we pose (15) as a set of LMIs.

The proof of Theorem 2 and Lemma 2 is presented in [9], and omitted here in the interest of brevity.

C. Stability of Individual Agents with Collision Avoidance

To prove the stability of the agents under the collision avoidance scheme as described in (12), let $V(x_t, w_t^i) = J^i_t(x_t^i, w_t^i)$ be the local ISpS Lyapunov function for the $A^i$ without collision avoidance. Let $x_t^{i,*}_{t+N_p^i}$ be the optimal solution of the cost (9) and $\bar{x}^{i,*}_{t,t+N_p^i}$ be the optimal solution of the modified cost (13). We aim to prove that and $V(x_t^i, w_t^i) = J^i_t(x_t^i, w_t^i)$ is also an ISpS Lyapunov function obeying Theorem 1 and $\dot{V}(\bar{x}_t^i, w_t^i) \leq \bar{V}(x_t^i, w_t^i)$ and $V(x_t^i, w_t^i) \leq V(x_t^i, w_t^i)$, since $\bar{x}_t^{i,*}_{t,t+N_p^i}$ is admissible but suboptimal for the minimization of (9) (and $x_t^{i,*}_{t,t+N_p^i}$ is suboptimal).

Assumption 2: $\bar{x}^{i,*}$ and $\bar{V}_i^i$ are such that:

i) $d_{ij}(k) \neq 0$ for at least one instant $t \leq k \leq t + N_p^i$.

ii) $\bar{\lambda}(d_{ij}(k)) > 0$.

iii) $\bar{\lambda}(|\bar{x}^{i,*}_t|) \leq \pi^i|\bar{x}^{i,*}_t|$, for some $\bar{\lambda}^i, \pi^i > 0$.

iv) $\dot{V}(\bar{x}_t^i, w_t^i) - V(x_t^i, w_t^i) \leq \bar{z}^i$, for some constant $\bar{z}^i > 0$.

Theorem 3: Under Assumption 2, the agent $A^i$ (7) is ISpS and avoids collision with other agents if an optimal trajectory $\bar{x}^{i,*}_{t,t+N_p^i}$ can be found such that the distance between the agents increases on the average if it is on collision course:

$$\sum_{k=t}^{t+N_p^i} \lambda(d_{ij}(k)).d_{ij}(k) < \sum_{k=t+1}^{t+N_p^i+1} \lambda(d_{ij}(k)).d_{ij}(k) \quad (16)$$

Proof of Theorem 3 is given in [9].

D. Stability of Team of Agents under NMPC

In this section, we will establish a generalized small gain condition to prove the stability of the interconnected system. The result is general and is not limited by the number of subsystems and the way in which subsystem gains affect each other is arbitrary.

Theorem 4: Consider a team of heterogeneous agents $A^i$ (7), such that each agent is furnished with a local ISpS Lyapunov function $V(x_t^i, w_t^i)$. Let the ISpS Lyapunov gain from each agent $A^i$’s to another agent $A^j$ within its neighborhood (i.e. $j \in G^i$) be $\gamma_{ij}$, such that:

$$\gamma_{ij} \triangleq \alpha_{ij}^i \circ (\alpha_{ij}^j)^{-1} \circ \alpha_{ij}^i \circ (\alpha_{ij}^j)^{-1} \quad (17)$$

Then the team of agents is ISpS stable if the network is at least simply connected, as long as the following small gain condition is satisfied:

$$V(x_t^i, w_t^i) > \max_{j \neq i, j \in G^i} (\gamma_{ij}(V(x_t^i, w_t^i))) \quad (18)$$

where $\alpha_{ij}^i$ is an appropriately designed $K$-function, such that $V(x_t^i, w_t^i) - V(x_t^j, w_t^j) \leq \alpha_{ij}^i(|x_t^i - x_t^j|)$.

Proof of the above generalized small gain theorem is given in [9]. This generalized theorem is not limited to the number of agents in the network and is valid for both strongly connected (all agents communicate with all other agents) and weekly connected (not all agents can communicate with each other) networks.

IV. Simulation Results

In this section, simulation results are provided for a team of N=3 autonomous underwater vehicles (AUVs) moving in the horizontal plane, with the following continuous-time models:

$$m^i \ddot{x}_t^i = -\mu_i^i \dot{x}_t^i + (u_R^i + u_L^i) \cos \theta_i$$

$$m^i \ddot{y}_t^i = -\mu_i^i \dot{y}_t^i + (u_R^i + u_L^i) \sin \theta_i$$

$$J^i \ddot{\theta}_t^i = -\mu_i^i \dot{\theta}_t^i + (u_R^i + u_L^i) r_v$$

where $m, J, \mu_1, \mu_2$ and $r_v$ are the vehicle mass, inertia, linear and rotational damping coefficients, and the vehicle radius respectively. For simplicity they are considered to be the same for all agents, and are specified in [5]. The state vector is $x_t = [\theta_t, \dot{\theta}_t, x_t, \dot{x}_t, y_t, \dot{y}_t]^T$, while the control vector consists of $u_t = [\phi_t, \theta_t, r_t]^T$. The model (19) is discretized at $T=0.1s$ (assumed same for all vehicles). Inputs are constrained to $0 \leq |u_{R,L}^i| \leq 6$, and turn rate is to $|\dot{\theta}| \leq 57deg/s$. Communication delay is bounded by 0.1s = $\tau \leq \Delta_{ij} \leq 0.6s$. It is assumed that $\Delta_{ij}$ is uniformly distributed: $\Delta_{ij} = U(T, T)$, and that $\Delta_{ij} \neq \Delta_{ji}$.

Distributed cost function at each agent (leader $A^1$) is:

$$J_t^i = \sum_{t=\tau}^{t+N_p^i-1} \left( \|z_{t+1}^i - g_{t+1}^i + d_{t+1}^i\|_Q^2 + \|u_t^i\|_S^2 \right) + \sum_{j \in G^i} \|z_t^i - w_t^j + d_t^j\|_{S^{ij}}^2$$

where $d_{t}^j$ is the alignment vector between agents $A^i$ and $A^j$, and $\forall d_{t}^j = 0$. The goal $g_{t}^i$ is the target (waypoint, WP) for the leader, i.e. $g_{t}^1 = WP$ and for the followers it is the leader’s planned trajectory, i.e. $g_{t}^i = \tilde{w}_t^i, \forall i \neq 1$.

The optimization parameters for all agents are: $N_p = 50$, $N_c = 15$, $N_p = 30$, $Q = 0.1 diag(1,1,10,10,10,1)$. $R = 0.01 diag(1,1)$, $S^{ij} = 0.25Q$ and $S^{ij} = 0.2S^{ij}$, for $i = 1, 2, 3$ and $j \in G^i \setminus \{i\}$. Thus, the leader gives less weight to followers’ information, but followers give more weight to the leader’s.

For the neural network (NN), we use a network with 6 inputs, L=6 hidden layer neurons and 6 outputs. The NN is trained using Levenberg-Marquardt backpropagation. Thus there are a 84 NN weights and biases to be sent via data
packet. On the other hand, transmitting the full (planned) trajectory, we would have required 300 floating-point data. Thus, the packet size is reduced by about 72%.

Vehicles are requested to maintain a triangular formation, with the three vehicles at vertices of an equilateral triangle, oriented at 45° from the horizontal, at a distance of 15 m from each other. A minimum distance of 5 m should also be maintained to avoid collision. Waypoints in the leaders' database are: (100,100), (0,200), (-100,100) and (0,0), with required formation orientations at these waypoints 45°, 135°, −135° and −45°. The results are shown in Fig. 1 - Fig. 3. Spatial filter is defined as $\lambda(k) = 0.95^k$, where $k$ is the index of $d_{ij}$ arranged in ascending order. It is seen that the state (and input, not shown) constraints are not violated and despite the relatively large random delays, the proposed algorithm was able to perform well. As can be observed from Fig. 3, the collision avoidance algorithm repels agents 1 and 3, when they come too close.

V. CONCLUSION

The paper presented a novel NMPC framework for formation control of a team of constrained agents in the presence of communication constraints/limitations and propagation delays. By transmitting a neural network approximation of the planned trajectory, the packet size is reduced drastically (halved in some cases). The NN approximation as well as the time-stamps on packets allow for agents to be sampled at different rates and corrects for computation and communication time delays. Simulations showed the viability of the proposed scheme, though not all simulations are shown here for brevity. Future research directions include the need to cater for modeling errors, external disturbances, fault tolerance and asynchronous clocks at each vehicle.

ACKNOWLEDGMENT:

The author(s) would like to acknowledge the support provided by King Abdulaziz City for Science and Technology (KACST) through the Science and Technology Unit at King Fahd University of Petroleum and Minerals (KFUPM) for funding this work through project No. 09-SPA783-04 as part of the National Science, Technology and Innovation Plan.

REFERENCES