Agent based e-commerce systems that react to buyers' feedbacks – A fuzzy approach

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Abstract

In this paper, we have introduced an agent based e-commerce system which recommends products to buyers as per their preferences. Initially, the agent collects the buyers' preferences in fuzzy or linguistically defined terms and based on this, presents them an ordered set of products. After obtaining the buyers' feedbacks when they actually come across the products, the seller's agent interacts with the buyer (buyer's agent), revises the products preferential order and recommends either the same set of products or a new set of similar products with the revised preferential order. The buyer's revised preferences are taken here as his/her feedback after he/she comes across with the actual products (presented products). Concepts of fuzzy logic and Fuzzy Linear Programming are used here to identify the buyer's feedbacks on the initial presentation of the products. Our methodology also measures the degree of customers' focus on the products which are finally recommended by the e-commerce agent. The product ranking obtained through buyers' initial preferences is considered here as his/her subjective information and the available information from the agents' presented products are taken as the objective information.

1. Introduction

In recent years the e-commerce sites increasingly use agent based systems for providing goods and services to their customers. In online business, with out seeing the actual products, normally the customers give their product preferences through products' attributes. Based on this, the agent based systems suggest the products to the buyers. Very often the buyers' usually change their preferences in the product attribute values when they actually come across them and have a real feeling of the products. This can happen in any market whether it is online or traditional. One of the possibly reason could be due to the buyer's or his/her agent's ambient intelligence and the surrounding effects. Most of the agent based systems do recommend the products using the traditional methods of customers' preferences and the historical data of market transactions. The existing e-commerce systems do not have any procedure to obtain the customers' feedbacks when the customers actually come across the products. To our knowledge, this changed attitude of the buyers' in the online business is not accounted in any of the existing e-commerce systems. Articulating the buyers' changed outlook in their preferences will not only make the online business more customer focused but also makes the business an extra successful. Further, the systems will behave as an added personalized recommender to the buyers' in their product choices. In the process, the buyers will have an impression that, as if the system is entirely for them, and thereby increasing their self-confidence in the e-commerce systems.
system. Finally, the buyer(s) will have an ordered set of products with numerical strength of preferences under the changed scenario.

In the existing e-commerce systems, the sellers’ and the buyers’ agents do negotiate amongst themselves for a product (or product attribute(s)) in order to arrive at a consensus business deal. During the course of negotiation the buyers’ and sellers’ express their views (buyers’ and the sellers’ agents views) for a particular product or product attributes. These views are only for a negotiated settlement and should not be interpreted as the buyers’ feedbacks on the products. The buyers’ feedbacks are basically his/her reaction after he/she comes across and feels about the actual product. There are some literatures on the negotiating agents and they are in [2,5,21,28].

A difficulty in handling Internet business is due to the fact that the buyers’ product assessments are of multi dimensional nature. For example in a CAR purchasing problem the multiple attributes may be price, design, color, popularity, re-sale value, maintenance cost, mileage, etc. In this case, the buyers need to select a CAR with maximum satisfaction over all the dimensions. However, as the attributes in general are conflicting, non-commensurable and fuzzy in nature for a buyer it is difficult to select a car satisfying all the attributes. It requires some sort of tradeoff of one attribute over the other in the selection process. In traditional markets, through interaction with the sales personnel it is possible to make tradeoffs. But in online business this is next to impossible, as there are no sales personnel available for interface. We have used the concepts of Multiple Attribute Decision Making (MADM) [14,24,25,32,34,35] and following the procedure [31] to emphasize this case in the e-commerce system. The MADM is one of the pioneer methods in solving the problems consisting of several objectives. To name a few they are available in [14,15,22,24–26,31,36]. The proposed method in our work evaluates the multiple attributes of the products initially and thereafter aggregates them to obtain the final rating of the products.

Other drawbacks in the existing e-commerce systems are that in many occasions the buyers’ opine their product requirements in rigorous quantitative aspects as well as fuzzy and inexact qualitative features. Majority of the existing systems do not count the buyers’ evaluations of product preferences in qualitative or linguistic terms. The problem is how to deal with the buyers’ quantitative and qualitative facets in the e-commerce systems. One way of dealing is by converting the linguistic information to their numerical counterparts and vice versa. These conversions will make the online business more parallel to the traditional ones and help in increasing the confidence level of the buyers in the e-commerce system. This is necessary in the e-commerce as many features of the business deals are vague, imprecise and they are defined qualitatively in day to day language terms. It is essential to express these terms in numeric forms in the e-commerce system not only to analyze the business conditions realistically but also to fully satisfy the customers. However, the assessments of linguistic terms in their numerical counterparts and vice versa are very complex and it is difficult to establish in the business system. It needs a phenomenon, to deal with the both quantitative and the qualitative concepts. Following the procedure given in [9], our paper uses fuzzy linguistic approach to convert a linguistic term to a numeric one and the vice versa to resolve the above problem. This paper [9] helps in interpreting the buyers’ linguistic assessments in qualitative terms by means of linguistic variables, that is, variables whose values are not numbers but words are sentences in natural or day to day languages. The use of words or sentences more suits to the buyers’ as in general, it is less specific, more flexible, realistic and very adequate to express the buyers’ qualitative estimates.

In the literature there are many agent based e-commerce systems. In [3,23], agent based personalized recommendation systems are given. The paper [23] explains how the personalized recommendations are made in an interactive way, with the help of fuzzy cognitive agents. In [3], the consumer specifies the product preferences in each dimension and the system provides the optimal products according to his/her personal preferences. Though in the works [3,23], the recommendations are based on the customers’ choices, it hardly accounts the customers’ feedbacks after the products are being recommended. Also the works do not consider the linguistic evaluations of the customers. In the paper [6], automated intelligent agents of the trading partners negotiate on several issues with an aim to arrive at a consensus business agreement. Though the work [6] maintains flexibilities in the agents’ strategies during the process of negotiations, it is inflexible in incorporating the imprecise requisites of the business partners in the online system. Moreover, the methodology [6] is inactive towards the changing scenarios of buyers’ reactions after the negotiated agreements. In [15], Fuzzy Logic and Game Theory have been used to develop e-business strategies in a competitive online market. The methodology given in [15] is very customer friendly, as it takes into account the buyers’ linguistic terms as an input in the e-business processing and suggests the products to them thereafter. However, it is deficient in understanding the buyers changed inclinations in their preferences after the initial delivery of the products. In [29], for the trading agents, an auction protocol along with its trade evaluation procedure through utility functions, are devised for determining the optimal trading strategies. The auction test-bed is evaluated through a series of experiments. Though, the work [29] is very innovative in formulating a market driven agent for on line auctions, it is short of acknowledging the fuzzy inputs of the trading partners in its various auctions. Moreover, the procedure does not have any mechanism in appraising the buyers’ changed reactions if any, in their product preferences during the auctioning. Later the authors in [30], made a big enrichment, by introducing the market driven negotiation agents that react to the changing market situations in the external market environment. However, in [30], the changes in mindset of the buyers after the offer has been made are not discussed. Paper [11], assesses the impact of Internet agents on the end users. These are mainly measured in terms of time savings, decision quality, confidence in decisions and the cognitive effort. One can interpret this as the buyers’ feedbacks after the final purchasing decisions. However, in actual terms it is not the buyers’ complete opinion, as the buyers’ changing attitude in his/her product preferences are not considered in the process of business settlement. In [16], an agent based job market place is created in which the agents can negotiate in multilateral aspects. However, the methodology lacks the information about the reactions of the persons to be employed, after the job
offer is made. In [24], the customers’ preferences of product attributes, in qualitative or fuzzy terms are considered. Depending on their preferences the methodology [24], suggests the products in a preference hierarchy to the customers. The drawbacks here [24] are that, (1) the methodology requires the preference hierarchy of the product attributes from the buyers’ first, in order to arrive at a final product ranking, and (2) the second one is the buyers’ responses after the product ranking is made, is not accounted. In [4] a market named eGora was developed. The key issues in the paper [4] are to make it a customer oriented e-market place through multi-issue negotiations. This work is more related to the issue based negotiation and does not consider the comments of the customers after he/she feels or realizes about the product. The paper [12], discusses about the use of fuzzy intelligent agents to develop the modular products. While developing the product the agents use the customers’ opinion. However, the methodology is silent on the customers’ preference reaction after the product being developed. In [13] a software agent is developed which not only facilitate the online trading but also does the agent-to-human persuasion under different customer characteristics. The feedback characteristics of the buyer on the preferred products are not counted in the paper [13]. In [18] an Internet-enabled multi-agent prototype system, called AgentStra is developed for competitive marketing strategies. The business strategy corresponding the buyers’ reaction after the products are presented are not accounted here. In [19], an e-commerce system developed to capture the buyers’ needs through an opportune advertising system. This is a good investigation [19] to attend the buyers’ real needs in the e-market. However, the work [19] does not incorporate the buyers’ reactions when they come across the products. The work in [20], attempts to discover the useful information for the business players (buyers and sellers) using Fuzzy Mobile Agents. Though the work is helpful and efficient in the buyers’ shopping decisions, it does not reflect the buyers’ feedbacks while making final product selections. The paper [17] uses agent technology and web mining for product searching in the Internet. During the search process, the work [17] incorporates customers’ requirements, fuzzification scheme, fuzzy based agent negotiation, fuzzy product selections and product de-fuzzification. Though this work [17], takes into consideration many of the buyers’ aspirations, it does not contemplate the buyers’ reactions in the business system. The work given in [1], presents an overview of the field of recommender systems and the recommendations are classified as content based, collaborative and hybrid. Though the methodology given in [1] incorporates the contextual information, supports the multicriteria rating, and main-
Price of the car should be around US$ 20,000. Maintenance cost should not be high. Mileage should be OK. The popularity should be above high.

Past market transaction data along with the buyers’ choices of fuzzily specified attribute values are taken to articulate the buyers’ purchasing behavior in the e-commerce system. The marketing data tells us to what extent a buyer(s) compromises in his/her attribute values (preferences) of a particular attribute given the choice of the availability of some other attribute(s) of the same product or altogether a different product.

Example. (1) To what extent, a buyer(s) will compromise on the price of a computer, when a printer is available in a lesser price along with it. (2) Similarly, how a consumer would compromise on the attribute price of car when the car gives a better mileage.

In general we can say that, to what extent an attribute’s presence supports the other attributes of the product in the market transactions. We have derived this by using the concepts of fuzzy membership functions and association rule notions of data mining [7]. That is, if there are three attributes $a_1$, $a_2$ and $a_3$ of a product; we can calculate the level of added liking; a consumer will have on the attributes $a_2$ and $a_3$ because of the presence of $a_1$ by using the following association rules.

\[
\begin{align*}
(1) & \quad a_1 \rightarrow a_2 \\
(2) & \quad a_1 \rightarrow a_3 \\
(3) & \quad a_1 \rightarrow a_2a_3
\end{align*}
\]

The rule (1) tells that, a consumer has extra favor for $a_2$ because of the presence of $a_1$. Similarly we can have for $a_3$ from (2) and $a_2a_3$ together from (3). These association rules indicate that, poor or stumpy level of $a_1$ in the product may not buy added liking of buyers to attributes $a_2$ and $a_3$ either individually or in combination. In the process, the importance of the attribute $a_1$ gets enhanced. This makes the system to attach more weights to $a_1$. The magnitude of the weight depends upon the degrees of association the attribute has with the other attributes. Similarly, by having the association rules with antecedents as $a_2$ and $a_3$ we can attach the weights to the attributes $a_2$ and $a_3$ accordingly. But the question is how to form the association rules? In our paper, we have taken the market transaction data as the basis for forming the association rules. Now by using the principle of association rules of data mining, pair wise comparison method of Analytical Hierarchy Process (AHP) [26] and following the work given in [15], we can calculate the importance of the attribute $a_1$, $a_2$ and $a_3$. The weighted average of the attributes in each product gives the product preference value and hence the ranking of the products with numerical strength of preference.

The product ranking and the buyers’ fuzzy membership values of the product preferences are used to compare the products pair wise. The pair wise comparison matrix of the products is nothing but the subjective information of the buyers’ product preferences.

Fig. 1. Agent based e-commerce system architecture.
The objective information of the products is obtained following the procedure given in [31]. Let’s take “n” products $P_1, P_2, \ldots, P_n$ are in the market, and each product is evaluated through “m” attributes $a_1, a_2, \ldots, a_m$. The normalized attribute values are:

$$z_{ij} = \frac{x_{ij} - x_{ij}^{\min}}{x_{ij}^{\max} - x_{ij}^{\min}}, \quad i = 1, 2, \ldots, n, \quad j \in \Omega_1$$

$$z_{ij} = \frac{x_{ij}^{\max} - x_{ij}}{x_{ij}^{\max} - x_{ij}^{\min}}, \quad i = 1, 2, \ldots, n, \quad j \in \Omega_2$$

where $x_{ij}$ and $z_{ij}$ are respectively, the real value and the normalized value of the $j$th attribute of the $i$th product. $x_{ij}^{\min} = \min_{1 \leq i \leq n} \{x_{ij}\}$ and $x_{ij}^{\max} = \max_{1 \leq i \leq n} \{x_{ij}\}$. $\Omega_1$ and $\Omega_2$ are respectively, the sets of benefit and the cost attributes. The matrix $Z = (z_{ij})_{n \times m}$ is taken here as the objective information of the recommended products.

Using Fuzzy Linear Programming, the procedure in our paper attempts to articulate a new set of attribute weights in order to match the buyers’ subjective information to that of the products objective one. If such a set of weights are obtained the buyers’ subjective information and the products objective information are completely matched and the recommended products are most customer focused. If the matching is not complete, depending on the matching scale, the magnitude of the customers’ focus can be determined accordingly.

Further, the new weights are basically the revised attribute weights of the buyers after he/she comes across with the actual products. These weights may give a different product ranking and/or with different preference intensities.

### 3. Fuzzy and linguistic representation of the product attributes

When a buyer selects a product online, the experience tells us that, he/she works with uncertain information about the product or product attribute level choices. Under these situations, on the buyers’ part, it is difficult to estimate the attribute levels with exact numerical values but with natural languages. When the buyers provide the imprecise information about his/her product choices in natural languages, it is most desirable to look for a tool to handle the buyers’ linguistically defined terms in the e-commerce system for the business success. Fuzzy logic is a viable methodology which in general meant to represent and manipulate the buyers’ linguistic and vague concepts in their product choices. Further, fuzzy sets and linguistic variables are best suited in approximating the buyers’ linguistically defined terms for estimating the product attribute values in numerical numbers as per the buyers’ requirements [22,10]. This approach is appropriate, since it allows a representation of buyers’ opinion in a more direct and adequate form, whether or not they are unable to express the product choices with precision. In the online business the linguistic depiction of buyers’ product choices play therefore a crucial role in the representation and handling of commonsense knowledge in the day to day business (traditional or online) systems.

In product purchases normally a buyer expresses his/her requirements of the product features in fuzzy or linguistic terms [5,13,24,25]. For example in a car purchasing problem, a customer communicates with the sales person about the car attributes price, re-sale value, mileage, comfort, maintenance cost and popularity in the following terms.

*Price*: The price of the car should be around US$ 20,000.

*Re-sale value*: The re-sale value after 3–4 years should be OK.

*Mileage*: Mileage should be around 20 km.

*Comfort*: Overall the car should be comfortable.

*Maintenance*: Maintenance cost should not be very high.

*Popularity*: Popularity of the car should be high.

In the above the italic words are fuzzy or linguistic terms. The attributes price and mileage can be represented through fuzzy numbers. Whereas the attributes; re-sale value, comfort, maintenance cost, and popularity can be expressed using the fuzzy or linguistic terms [9].

It is very difficult for a sales person to judge the buyers’ above terms in the traditional markets. This difficult multiplies to evaluate the above terms in the e-commerce system. However, in our paper we have made a realistic representation of the terms in the e-commerce system by using fuzzy number [24,25] and Fuzzy Linguistic quantifier approach [9].

#### 3.1. Fuzzy number representation of the attributes

The car attributes price and mileage; as the customer specifies are expressed in fuzzy numbers. They are shown in Figs. 2a and 2b below.

In Fig. 2a the price US$ 20,000 corresponds to the membership function value 1. The membership value gradually comes down if the price of the car deviates below or above US$ 20,000. It becomes zero, if the price of the car is US$ 10,000 or below or US$ 30,000 or above. Fig. 2a is a fuzzy number corresponding to the buyer’s car price “around US$ 20,000”. Similarly in
we have represented the buyer’s requirement of mileage “around 20 km”. Exactly at 20 km the membership value of the fuzzy number is 1. It gradually decreases and finally becomes zero when it crosses below 10 km or above 30 km.

### 3.2. Linguistic–numeric conversion

Following the procedure given in [9] we can represent the product attributes in fuzzy or linguistic terms. In order to do this, we need to define a set of basic linguistic terms. The number and the range of linguistic terms in the basic set depend on the context of the problem. For example in our case, that is, in order to describe the buyers’ linguistic requirements in the e-commerce system, we can define basic linguistic terms as follows:

$$S = \{N = \text{not, VL = Very low, L = Low, M = Medium, H = High, VH = Very high, P = Perfect}\}$$

Graphically, through fuzzy numbers they are shown below.

In Fig. 3 the following fuzzy numbers are shown corresponding to the basic linguistic terms.

- **Perfect** = (0.83, 1, 1)
- **Very high** = (0.67, 0.83, 1)
- **High** = (0.5, 0.67, 0.83)
- **Medium** = (0.33, 0.5, 0.67)
- **Low** = (0.17, 0.33, 0.5)
- **Very low** = (0, 0.17, 0.33)
- **None** = (0, 0, 0.17)

From the above, the buyer’s Re-sale attribute value in linguistic terms is “OK”. Considering the term “OK” is in between the basic linguistic terms “high” and “very-high” we can express it as \((H, 0.4)\). Its equivalent numeric term is 4.4. This is calculated by adding the “term number” of “H” (=4) to the value of \(\alpha (\alpha = 0.4)\). Similarly we can put the buyer’s comfortable attribute in between VH and Perfect (say \((VH, 0.02)\)) and can calculate its equivalent numeric form as 5.02. The attributes maintenance cost and the popularity can be written as \((L, 0)\) and \((H, 0)\). The equivalent numeric numbers for the attributes “maintenance cost” and the “popularity” are “2.0” and “4.0”, respectively. Similarly, we can convert a numeric number \(n’ \in [0, g]\), (where \(g\)th term is the highest basic linguistic term, in our case \(g = 6\)), into its equivalent linguistic form.
3.3. Fraction–Linguistic conversion

Many times the e-commerce system needs to convert a fraction into its linguistic form and vice versa. In that case, following the procedure [9], we have described the linguistic-numeric and numeric–linguistic conversion as follows.

Let the basic linguistic terms are:

(Very low = VL, Low = L, Medium = M, High = H, Very high = VH).

We can define them in the form of fuzzy numbers as given below.

VL = (0, 0, 0.25), L = (0, 0.25, 0.5), M = (0.25, 0.5, 0.75), H = (0.5, 0.75, 1) and VH = (0.75, 1, 1). Graphically we can represent the above basic terms as shown in Fig. 4.

Let’s take a number 0.78. We will convert this number into its linguistic form. Taking the value 0.78 in the above fuzzy numbers, we can have its membership values in each of the fuzzy numbers as follows:

\[ TS(0.78) = \{(VL, 0), (L, 0), (M, 0.00), (H, 0.88), (VH, 0.12)\} \]

The numerical number that can be assessed from the fuzzy set TS(0.78) is:

\[ (\text{Term-}VL) * 0 + (\text{Term-}L) * 0 + (\text{Term-}M) * 0 + (\text{Term-}H) * 0.88 + (\text{Term-}VH) * 0.12 = 0 + 0 + 0 + 2 + 0.88 * 3 + 0.12 * 4 = 3.12 \]

This is obtained by multiplying the term number of the basic linguistic terms with the membership values. The number 3.12 can be represented in a linguistic term as (H, 0.12).

3.4. Linguistic–fraction conversion

Following the procedure given in [9], a linguistic term (H, 0.12) can be converted to its equivalent numeric term in [0, 1] as shown below. According to the definition a linguistic term \((s_k, x)\) can be transformed to

\[ \delta(s_k, x) = \{(s_k, 1 - x), (s_{k+1}, x)\} \]

\[ \kappa(s_k, 1 - x), (s_{k+1}, x) = CV(s_k)(1 - x) + CV(s_{k+1})(x) \]

where \(CV()\) represents the characteristic value functions. As an example, take the linguistic term (H, 0.12). Thus we have:

\[ \delta(H, 0.12) = \{(H, 0.88), (VH, 0.12)\} \]

Using the maximum characteristic value we have

\[ \{(H, 0.88), (VH, 0.12)\} = 0.75 * 0.88 + 1 * 0.12 = 0.78 \]

The numeric–linguistic conversion and vice versa will help the agents in the e-commerce system to combine the buyers’ linguistic and numeric terms in order to arrive at an aggregated view of the products.

4. Objective information of the products

Before going to the market (online or traditional), generally the buyers have some preference information about the product features. On these mind-sets, they start negotiating with the sales person or the computer (in the e-commerce system) to find a product(s) which best suits to their requirements. Based on their preferences the sellers’ agents provide a set of products to buyers directly or through the buyers’ agents. The buyers normally give their feedback on the presented products (at the first instance) along with their modified preferences if any. The information of the products which are presented to buyers by an agent (sellers’ or buyers’) initially gives us the products’ objective information following the methodology given in [31]. Mathematically we can have the objective information as given below.

![Fig. 4. Linguistic terms.](image-url)
Let’s take there are “m” number of attribute on which a buyer decides his/her preferences. Out of these “m” attributes some of them are given in linguistic terms and the rest in numeric forms. Using the methodology of linguistic–numeric conversion given in Section 2.2, we can convert all the linguistic terms to its equivalent numeric counterparts. Now taking into account the buyer’s preferences (in numeric and linguistic terms), assume that the buyer’s agent has made an initial presentation of “n” number of products to the buyer. The objective information of these products is given in Eq. (1) in Section 2. Reproducing them again we have:

\[
\begin{align*}
Z_i & = \frac{x_{ij} - x_{ij}^{\text{min}}}{x_{ij}^{\text{max}} - x_{ij}^{\text{min}}}, \quad i = 1, 2, \ldots, n, \ j \in \Omega_1 \\
Z_j & = \frac{x_{j}^{\text{max}} - x_{j}}{x_{j}^{\text{max}} - x_{j}^{\text{min}}}, \quad i = 1, 2, \ldots, n, \ j \in \Omega_2
\end{align*}
\]

where \(x_{ij}\) and \(z_{ij}\) are respectively, the real value and the normalized value of the \(j\)th attribute of the \(i\)th product. \(x_{ij}^{\text{min}} = \min_{1 \leq i \leq n} \{x_{ij}\}\) and \(x_{ij}^{\text{max}} = \max_{1 \leq i \leq n} \{x_{ij}\}\). \(\Omega_1\) and \(\Omega_2\) are respectively, the sets of benefit and the cost attributes. The matrix \(Z = (z_{ij})_{mn}\) is taken here as the objective information of the recommended products.

5. Subjective information of the consumers

A buyer’s subjective information is basically his/her own preferences about the products and its features. The buyers’ agent’s first job is to obtain this preference information from the buyers’ and there after to search the suitable products in the online market. The buyers’ preferences are normally implicit in their mind and it is very difficult to explicit their requirements in the usual markets. This difficulty is multiplied when it comes to explain the same situation in the online market. The buyers’ preferences are normally implicit in their mind and it is very difficult to explicit their preferences. Subsequently, the sellers’ agents (or sellers) do interact with the buyers agents (or with the buyers directly) to uncover the buyers’ desires in product features. However, it is not an easy job to uncover the attribute weights from the fuzzy information given in Eq. (6). Our paper uses the past market transaction data in conjunction with the buyers’ fuzzy preference information to reveal attribute weights in the similar line as given in [15]. This procedure is explained in the following steps.

5.1. Attribute weight calculation

Step 1 Let’s take \(K\) numbers of past transactions \(\{t_1, t_2, \ldots, t_k\}\). In each transaction there are certain numbers of products and the number differs from one transaction to the other.

Step 2 For each transaction, we can find the average attribute totals of each attribute. For example take the transaction \(t_s\) \((s = 1, 2, \ldots, K)\) and let’s take the products \(P_1, P_2, P_3\) and \(P_4\) were transacted in \(t_s\). The attribute totals \(T_{sj}\) of the attribute \(j\) \((j = 1, 2, \ldots, m)\) in the transaction \(t_s\) is:

\[
T_{sj} = \frac{1}{4}\{|\mu_{a_1}(a_{j1}) + \mu_{a_2}(a_{j2}) + \mu_{a_3}(a_{j3}) + \mu_{a_4}(a_{j4})|\}
\]

where \(a_{ji}\) \((i = 1, 2, \ldots, 4)\) represent the \(j\)th attribute level of the \(i\)th product.

Step 3 Calculate the average value of the \(j\)th attribute in the entire business transactions.

\[
T_j = \frac{1}{K} \sum_{i=1}^{K} T_{sj}
\]

Step 4 The \(j\)th attribute totals \(T_{sj}\) in transaction \(s\), is checked to ensure that they are at least to the level of \(T_j\) \((T_{sj} \geq T_j)\). If not the attribute totals \(T_{sj}\) are taken as zero in transaction \(s\). The revised \(T_{sj}\)’s are taken as \(T_{sj,\text{new}}\). This is necessary, as we do not want to consider the attribute totals in any transaction if they are less than the average attribute value of the entire business transaction. If it is, we assume that the attribute has got negligible transaction and zero value is assigned to the attribute totals.

Step 5 Transaction frequency of the \(j\)th attribute is:
The term $\frac{a}{b}$ represents the relative preference of the $i$th attribute to the $j$th attribute ($i, j = 1, 2, \ldots, m$).

The attribute weights give us the light to obtain the pair-wise comparison of the products as per the customers’ subjective information. In order to obtain this information, first we need to determine the rating and ranking of the products in a preferential order. The weighted average of the attribute values of the product gives us the product rating $rP_i$. This is derived in the following equation.

$$\sum_{j=1}^{m} W_j \mu_{a_j} (a_j) = rP_i \quad (i = 1, 2, \ldots, n)$$

As the $rP_i$’s are numerical quantities, they can be ordered from a most preferred product to a least preferred one. Note that this preference order of the products is as per the customers’ choice. This preference order gives us the following matrix which is nothing but the buyer’s subjective information.

$$C = \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1n} \\ C_{21} & C_{22} & \cdots & C_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ C_{m1} & C_{m2} & \cdots & C_{mn} \end{bmatrix}$$

where $C_{ij} = (rP_i/rP_j)$.

6. Integration of buyers’ subjective and products’ objective information

In this section, our paper attempts to determine the changed mindset of the buyer in their product preferences, when they come across the actual products. A buyer normally tries to maintain his/her own preference levels and at the same time does fluctuate after seeing the actual products. In other words he/she tries to bridge the gap between his/her subjective preferences and the products objective realizations. This can be bridged to some extent, if not completely, provided the buyer’s agent could identify a new set of weights, which links the buyer’s subjective information and the products’ objective information. By using the procedure given in [31], our paper articulates such weights by modeling the problem as a fuzzy Linear Programming Problem (LPP). The procedure is as follows.

Take the subjective information matrix in (14) and derive the matrix as shown below.
such weights (from Eq. (5)) and the subjective information matrix \( A \) as shown in the Eq. (15). The following equation helps in determining preferences on the product attributes.

\[
A = \begin{bmatrix}
\sum_{j=2}^{n} C_{ij} & C_{i2} & \cdots & C_{in} \\
C_{i1} & \sum_{j=2}^{n} C_{ij} & \cdots & C_{in} \\
\cdots & \cdots & \cdots & \cdots \\
C_{i1} & C_{i2} & \cdots & n-1 \sum_{j=1}^{n} C_{ij}
\end{bmatrix}
\]

Now we need to find a new set of attribute weights which minimizes the space between objective information matrix \( Z = (z_{ij}) \) (from Eq. (5)) and the subjective information matrix \( A \) as shown in the Eq. (15). The following equation helps in determining such weights \( \delta = (\delta_1, \delta_2, \ldots, \delta_m) \).

\[
AZ\delta = (n - 1) Z\delta
\]

If a weight vector \( \delta \) exist satisfying Eq. (16) a linkage between the subjective and the objective information can be established. However, due to the existence of fuzziness and linguistic terms in the product attributes and the subjectivity of the buyers, it is difficult to identify exactly a set of weights which satisfies the above equation. Therefore allowing the deviation vector we have:

\[
E = [AZ - (n - 1)Z]\delta
\]

where \( E = (e_1, e_2, \ldots, e_m) \) is the deviation vector. Now we will find the weights which minimize the total deviations. Following the procedure given in [31] we have used LPP to get the minimum deviation.

\[
G = \text{Min} \sum_{i=1}^{n} |e_i| \\
|AZ - (n - 1)Z|\delta - E = 0 \\
\sum_{j=1}^{m} \delta_j = 1
\]

The above LPP gives us a new set of attribute weights \( \delta = (\delta_1, \delta_2, \ldots, \delta_m) \) corresponding to the minimum value of the total deviation \( G \). Depending on the magnitude of \( G \) we can have the degree of customers’ focus of the e-commerce system. The zero value of \( G \) indicates the maximum customers’ focus. Thus we have the degree of customer focus is:

\[
1 - \frac{1}{n} \left( \sum_{i=1}^{n} e_i \right)
\]

Now the buyer’s agent uses changed set of weights \( \delta = (\delta_1, \delta_2, \ldots, \delta_m) \), to obtain a new preferential ranking of the products as per the buyer’s revised choice. This revised preference of the buyer is basically the feedback of the buyer to the e-commerce system. Finally, the buyer’s agent can recommend a set of similar products with the buyer’s modified preferential ranking.

7. Numerical example

In this section, we have given a numerical example to highlight the procedure. To begin with let’s assume that a buyer intends to purchase a car in the online market. Let the buyer’s desires is based on the attributes (1) price, (2) maintenance cost, (3) mileage and (4) popularity. The e-commerce system follows the following procedure to recommend a collection of cars (in order of preference) to the buyer.

7.1. Information from the buyer

Initially the buyer informs his/her agent about the car specifications. The agent interacts with the buyer and finally retrieves his/her preferences on price, maintenance cost, mileage and popularity in the following fuzzy or linguistic terms. The information retrieved from the buyer is strictly in terms of the desired attributes and no privacy information of the buyer is realized here. Note that the buyer’s requirements on the popularity level lies fuzzily inside the basic linguistic term set as given below. The interaction between the buyer and the seller (seller’s or buyer’s agents) is basically to collect the buyers’ preferences on the product attributes.

\[
\text{Price} = \begin{bmatrix}
0 & 0.8 & 1 & 0.8 & 0.6 & 0.4 & 0.1 & 0 \\
10000 & 15000 & 20000 & 25000 & 30000 & 40000 & 50000 & 60000
\end{bmatrix}
\]

\[
\text{Maintenance} = \begin{bmatrix}
0 & 0.4 & 0.63 & 0.65 & 1 & 0.8 & 0.8 & 0.4 & 0.2 \\
40 & 50 & 100 & 150 & 200 & 300 & 400 & 500 & 600
\end{bmatrix}
\]
\[
\text{Mileage} = \{0, 0.1, 0.4, 0.5, 0.6, 0.73, 0.8, 1.0, 0.9, 0.8, 0.72, 0.7\}
\]

\[
\text{Popularity} \in \{\text{VL}, \text{L}, \text{M}, \text{H}, \text{VH}\}
\]

where \(\text{VL} = (0, 0, 0.25), \text{L} = (0, 0.25, 0.5), \text{M} = (0.25, 0.5, 0.75), \text{H} = (0.5, 0.75, 1)\) and \(\text{VH} = (0.75, 1.1)\). Graphically they are shown in Figs. 5a–5c, and 5d.

7.2. Identifying the products and presenting them to the buyer

After obtaining the buyer’s preference information, the buyer’s agent interacts with sellers’ agents to find out the appropriate products. Let the buyer agent found five cars (might be of different brands from different sellers) which closely matches the buyer’s requirements. The products and their attribute values are given in Table 1. The linguistic values of the popularity attribute along with its numeric counterparts are also shown in Table 1. These are obtained using the procedure given in [9].
7.3. Satisfaction level of the buyer on the presented products

At first, the buyer's agent presents these products (cars) to the buyer and seeks his/her responses. By using fuzzy membership functions from Eq. (20) and the formula for linguistic–numeric conversions from Sections (3.3) and (3.4) we can get the buyer's attribute wise satisfaction levels on the presented products in Table 2.

7.4. Products' objective information

By using the Eq. (5) and taking the product information from Table 1, we can have the products' objective information as given in the matrix $Z = (z_{ij})$.

7.5. Market transaction data

We have taken the market transaction data to calculate the attribute weights as per the buyer's preferences. Let's take 10 past transactions $T_i (i = 1, 2, \ldots, 10)$, shown in Table 3. In each transaction we have calculated the average attribute totals. For example, the average attribute total of attribute price in 1st transaction, from Eq. (7) is $(1/3)(0.6 + 1.0 + 0.1) = 0.57$. Similarly we can obtain the average attribute totals for the other attributes.

From Table 3, and using the Eq. (8) we have the transaction average for the attributes price, maintenance, mileage, and the popularity as 0.36, 0.61, 0.76, and 0.60, respectively. Now using step-4 of Section 5, in each transaction, the minimum level of
average attribute totals should be at least to the level of transaction average. In case it is not, the corresponding attribute totals are taken as zero value. Accordingly for the attributes price, maintenance cost, mileage and popularity the minimum average attribute total is required respectively 0.36, 0.61, 0.76 and 0.60. After incorporating these minimum requirements the new average attribute totals are shown in Table 4.

Now by using Eq. (9), the frequencies of the attributes transacted are:

- **Price** = \[0.57 + 0.37 + 0.50 + 0.60 + 0.40 + 0.60\] = 3.04, similarly we have for the other attributes as given below.
- **Maintenance cost** = 4.57.
- **Mileage** = 3.89.
- **Popularity** = 2.41.

### 7.6. Association rules amongst the attributes

By using Eq. (10), association rules amongst the price \((A_1)\), maintenance \((A_2)\), mileage \((A_3)\) and popularity \((A_4)\) can be obtained. For example, as explained in Section 5, we have seven association rules with the attribute price as antecedent. They are:

- **Price → Maintenance.**
- **Price → Mileage.**
- **Price → Popularity.**
- **Price → Maintenance, mileage.**
- **Price → Maintenance, popularity.**
- **Price → Mileage, popularity.**
- **Price → Maintenance, mileage, popularity.**

### 7.7. Confidence level

Using Eq. (10) we can calculate the confidence level of the \((Price → Maintenance)\) as: Min \((0.57, 0.69)/3.03 = 0.19\). Similarly we can find the confidence levels of the other association rules. They are shown in Table 5. \(A_1A_2\) represents the association rule \(A_1 \rightarrow A_2 \ (Price → Maintenance)\) and similarly the others.

From Eq. (10), in the last row 0.72 represents the buyer’s additional likings to maintenance cost because of the car prices. By taking the sum over all the association rules and all the transactions, we have the buyer’s improved liking to all the attributes, because of the present car price is \((0.72 + 0.56 + \ldots + 0.40) = 3.59\). The number 3.59 is interpreted here as the support

### Table 4
Average attribute total (revised).

<table>
<thead>
<tr>
<th>Transaction</th>
<th>Price</th>
<th>Maint</th>
<th>Mileage</th>
<th>Popularity</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>0.57</td>
<td>0.69</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>T2</td>
<td>0.37</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>T3</td>
<td>0.00</td>
<td>0.24</td>
<td>0.76</td>
<td>0.00</td>
</tr>
<tr>
<td>T4</td>
<td>0.50</td>
<td>0.00</td>
<td>0.76</td>
<td>0.72</td>
</tr>
<tr>
<td>T5</td>
<td>0.60</td>
<td>0.63</td>
<td>0.80</td>
<td>0.85</td>
</tr>
<tr>
<td>T6</td>
<td>0.40</td>
<td>0.69</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>T7</td>
<td>0.60</td>
<td>0.63</td>
<td>0.80</td>
<td>0.85</td>
</tr>
<tr>
<td>T8</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>T9</td>
<td>0.00</td>
<td>0.64</td>
<td>0.77</td>
<td>0.00</td>
</tr>
<tr>
<td>T10</td>
<td>0.00</td>
<td>0.64</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

### Table 5
Association rule confidence levels.

<table>
<thead>
<tr>
<th>Transaction</th>
<th>A1A2</th>
<th>A1A3</th>
<th>A1A4</th>
<th>A1A2A3</th>
<th>A1A3A4</th>
<th>A1A2A3A4</th>
<th>A1A2A4</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>0.19</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>T2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>T3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>T4</td>
<td>0</td>
<td>0.17</td>
<td>0.17</td>
<td>0</td>
<td>0.17</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>T5</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0</td>
</tr>
<tr>
<td>T6</td>
<td>0.13</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>T7</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0</td>
</tr>
<tr>
<td>T8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>T9</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>T10</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>0.72</td>
<td>0.56</td>
<td>0.56</td>
<td>0.40</td>
<td>0.56</td>
<td>0.40</td>
<td>0.40</td>
</tr>
</tbody>
</table>
of the attribute price for the entire business. Similarly we can form the association rule with the attribute Maintenance, Mileage and Popularity as antecedents and derive their business supports. We have calculated the supports as:

- Maintenance cost = 2.37.
- Mileage = 3.06.
- Popularity = 4.01.

7.8. Weight calculation

After finding the support levels of the attributes, using Eq. (12), we can form a pair wise comparison matrix amongst the attributes as:

\[
\begin{array}{cccc}
A_1 & A_2 & A_3 & A_4 \\
A_1 & 1.52 & 1.52 & 1.54 & 1.53 \\
A_2 & 1.71 & 1.27 & 1.71 & 1.72 \\
A_3 & 1.32 & 1.36 & 1.36 & 1.41 \\
A_4 & 1.28 & 1.36 & 1.36 & 1.27 \\
\end{array}
\]

The Eigen vector corresponding to the maximum Eigen value of the above matrix gives us the weights to the attributes [26]. The normalized weights are found to be 0.27, 0.18, 0.23 and 0.33 for the attributes price, maintenance, mileage and popularity, respectively. Note that these weights are as per buyer's preferences.

7.9. Pair wise comparison of the products

The Buyer's agent can use the attribute weights to rate and rank the products as per the customer's own preferences. Using Eq. (13), we can have the rating for \( P_1 \), as \( rP_1 \) where

\[
rP_1 = 0.6 \times 0.27 + 0.63 \times 0.18 + 0.80 \times 0.23 + 0.85 \times 0.33 = 0.74
\]

Similarly we have \( rP_2 = 0.54, rP_3 = 0.77, rP_4 = 0.51, rP_5 = 0.43 \). As these ratings are in numerical quantities the cars can be ordered. Using the matrix given in Eq. (14) we can obtain the buyer's subjective information matrix below:

\[
\begin{array}{cccccc}
P_1 & P_2 & P_3 & P_4 & P_5 \\
P_1 & 0.74 & 0.74 & 0.74 & 0.74 & 0.74 \\
P_2 & 0.54 & 0.54 & 0.54 & 0.54 & 0.54 \\
P_3 & 0.77 & 0.77 & 0.77 & 0.77 & 0.77 \\
P_4 & 0.74 & 0.74 & 0.74 & 0.74 & 0.74 \\
P_5 & 0.74 & 0.74 & 0.74 & 0.74 & 0.74 \\
\end{array}
\]

Following [31] and after normalizing the above matrix by taking \( C_{ii} = 0.5 \) and \( C_{ij} + C_{ji} = 1 \), we have the subjective information matrix as:

\[
\begin{array}{cccccc}
P_1 & P_2 & P_3 & P_4 & P_5 \\
P_1 & 0.5 & 0.65 & 0.48 & 0.68 & 0.75 \\
P_2 & 0.34 & 0.5 & 0.33 & 0.53 & 0.61 \\
P_3 & 0.52 & 0.67 & 0.50 & 0.70 & 0.76 \\
P_4 & 0.32 & 0.47 & 0.30 & 0.50 & 0.59 \\
P_5 & 0.25 & 0.39 & 0.24 & 0.41 & 0.5 \\
\end{array}
\]

7.10. Subjective and the objective information matching

Using Eq. (15), we have the matrix \( A \) as given below.

\[
A = \begin{bmatrix}
2.56 & 0.65 & 0.48 & 0.68 & 0.75 \\
0.34 & 1.81 & 0.33 & 0.53 & 0.61 \\
0.52 & 0.67 & 2.65 & 0.7 & 0.76 \\
0.32 & 0.47 & 0.30 & 1.68 & 0.59 \\
0.25 & 0.39 & 0.24 & 0.41 & 1.29
\end{bmatrix}
\]

Now using the Eq. (18) the solution of the following LPP gives the buyer’s feedback or the revised set of preferential attribute weights.
The solution to the above problem is:

$$\text{Min} \quad G = \sum_{j=1}^{5} e_{j}^{+} + e_{j}^{-}$$

$$-0.26\delta_{1} + 1.36\delta_{2} + 1.29\delta_{3} + 0.19\delta_{4} + e_{j}^{+} - e_{j}^{-} = 0$$

$$-0.17\delta_{1} - 1.33\delta_{2} - 1.31\delta_{3} - 0.87\delta_{4} + e_{j}^{+} - e_{j}^{-} = 0$$

$$-0.78\delta_{1} + 1.79\delta_{2} + 1.81\delta_{3} + 0.08\delta_{4} + e_{j}^{+} - e_{j}^{-} = 0$$

$$0.67\delta_{1} - 0.97\delta_{2} - 0.24\delta_{3} - 0.45\delta_{4} + e_{j}^{+} - e_{j}^{-} = 0$$

$$0.54\delta_{1} - 0.86\delta_{2} - 2.45\delta_{3} + 1.03\delta_{4} + e_{j}^{+} - e_{j}^{-} = 0$$

$$\delta_{1} + \delta_{2} + \delta_{3} + \delta_{4} + \delta_{5} = 1$$

The customer has given his/her feedback in terms of the revised weights of the car attributes. Now the agent can use these new set of weights to finally recommend the products in a hierarchical preferential order. Thus we have the new ratings as:

$$rP_{\text{new}} = 0.54'0.6 + 0'0.63 + 0.22'0.8 + 0.24'0.85 = 0.70$$

Similarly we have

- \(rP_{\text{P1 new}} = 0.51\).
- \(rP_{\text{P2 new}} = 0.83\).
- \(rP_{\text{P4 new}} = 0.36\).
- \(rP_{\text{P5 new}} = 0.3\).

Thus the new preferential order of the cars is \(P_{3}, P_{1}, P_{2}, P_{4}\) and \(P_{5}\).

The degree of customer focus of the recommended cars is:

$$1 - (1/5)^{\gamma} \left( \sum_{i=1}^{5} (e_{i}^{+} + e_{i}^{-}) \right) = 1 - \frac{1}{5} (0.18 + 0.59 + 0.2) = 0.84$$

8. Conclusions

In our paper, we have developed an agent based system which recommends the products to the buyers in the online system. The buyer’s agent initially collects buyer’s product preferences and combines with the past market transaction data to extract the buyer’s real inclinations towards the products and the product features. This is derived in terms of attribute weights. The weighted average of attribute weights of each product gives us the product rating and hence the ranking of the products suggested by the buyers’ agent. The product ranking helps us to determine the buyers’ subjective preference. From the actual products we get the objective information. We have established the linkage between the buyers’ subjectivity to the products attainability through a new set of attribute weights which were interpreted as the buyers’ feedbacks when they come across with the actual products. The new set of attribute weights gives us a different rating and the ranking of the products. It is expected that, the buyers have the fullest satisfaction with the new set of attribute weights and the preference ranking therein. Using these new weights, finally the agent recommends the products.

Mainly the contributions of this paper are categorized as follows:

- An agent based e-commerce system is introduced here which takes the buyers’ feedbacks as input and there after recommends the products in an ordered sequence of preference.
- This work takes into account the fuzzily and linguistically defined values of the product attributes.
- Our work derives the degree of customer focus on the final recommended products.

There are certain limitations in the paper and they are given below:

- The complexity of the LPP depends on the number of products are suggested initially by the agent. The problem becomes large and complex, if the agent will suggest a big number of products initially.
- Weights are calculated using the market transaction data. A huge number of market transaction data is required to accurately estimate the weights. Possibly some data mining techniques will be helpful for this purpose.
Acknowledgements

The authors would like to thank the anonymous referees for their valuable comments which have improved the paper significantly.

References


