On the development of e-business strategies:
an integrated fuzzy logic and game theoretic approach

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Abstract: This paper presents a methodology to formulate business strategies for the web-based markets. We focus on strategies that involve decisions made by companies about the selection of the appropriate products to be launched on the web, as well as decisions about the right timing to do so. Our approach builds on the concept of a product’s market value. The linguistic or imprecise perception of the customers on the product attributes is handled by using Fuzzy Sets theory, while the problem of exploration and resolution of the strategic issues involved is addressed through a Game Theoretic analysis.

Keywords: e-business; strategy development; game theory; fuzzy sets; electronic business.


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1 Introduction

In the last few years, we have witnessed an unprecedented explosion of electronic commerce. Companies are increasingly implementing online applications to augment their market status. At the same time, the impact of web as a medium for enabling new business models has been proven to be significant (Timmers, 1998). Such models basically aim at satisfying the needs of customers for lower prices and better services (throughout this paper, when speaking about customers, we refer to web customers). The web is expected to increase business competition (Bakos, 1997), as information and control flows are put into the hands of the customers (Rayport and Jaworski, 2001). Thus, formulation of strategies to provide companies with an advantage in the competitive online market space has to be based on market research insights and appropriately accommodate the diverse customers’ desires and attributes. Survival will be more likely for companies that do not lose sight of the end customers, by focusing on satisfying their needs and fulfilling their expectations. Decision making about launching a new product plays an important role in a company’s strategy formulation process. As far as the online market is concerned, the related decisions have to take into account additional parameters compared to its physical world counterpart. Such parameters concern data gathered through monitoring customers’ buying habits and communications with trading partners.

In a competitive setting, such as the one described above, the success in doing business highly depends on the quality of the strategies that a company implements. A strategy is successful if it captures a major market share and acts as a motivating factor for retaining the customers. Customer-oriented approaches and personalisation techniques may contribute significantly to the building of the appropriate strategy. In any case, one needs to analyse and understand the individual requirements of the customers. In the approach described in this paper, we use the concept of personalisation to characterise techniques that exploit information about customers (such as individual customers’ characteristics, interests and preferences) in order to accordingly tailor a business strategy.

A major problem in performing online business stems from the fuzziness of the product assessments by the customers. The opinion of the customers is usually expressed in vague or linguistically defined terms, something which is realistic in physical language and day-to-day transactions. Since it is necessary to incorporate these linguistically expressed assessments in an e-business model, we need to accommodate their associated terms, which actually represent imprecise, qualitative or fuzzily expressed linguistic perceptions. This paper aims at developing appropriate e-business strategies that satisfy the customers’ needs, while reckoning their imprecise desires. We have handled this problem by using concepts from the fuzzy sets theory.

Another problem related to personalisation stems from the customers’ product assessments. Very often, a customer expresses his/her product preferences in terms of its attributes. When a product with the desired attribute levels is available, the customer is satisfied. Otherwise, the customer looks for an alternative product (e.g., by browsing different websites). This paper considers the attribute-wise satisfaction of the customers’ requirements, which are often fuzzily defined. The product, as a whole, is aggregated across its evaluated attributes. The major problem lies in the process of aggregation, as the procedure requires attribute weights. The proposed approach articulates the attribute weights from the customers’ past data through a data mining procedure.
Another problem considered in this paper (also related to personalisation) is how to make predictions about which product is needed, at what period and under what business scenario. To address these issues, a company may like to ponder over alternative business strategies. In our approach, such strategies concern the appropriate selection of products and the timing to be placed in the online market. This is certainly a difficult task, especially when one has to deal with the interpretation of customers’ preferences and expectations.

To address the above problems, our approach builds on techniques coming from the disciplines of Fuzzy Logic (Bandemer and Gottwald, 1996; Zadeh, 1965) and Game Theory (Hui and Tam, 2002; Osborne and Rubinstein, 1994). Concepts adopted from Game Theory may help a company to strategically select and launch a product in the online market depending on the competitor. According to our approach, a company selects the product to be launched aiming at both having a good market share over its competitor and satisfying the customers. Obviously, a company can achieve a good market share if the product is more popular amongst the customers. We have handled this problem by determining the popularity of the product through market values and identifying the business strategy through Game Theory. On the other hand, Fuzzy Sets Theory and Fuzzy Logic have been extensively used in the literature to accommodate diverse fuzzy opinions, or even fuzzily defined weights and preferences, in strategy formulation. However, our approach does not only take into account the customers’ fuzzily defined inputs; instead, it integrates data mining techniques to exploit past market transactions, while it also incorporates the weights of a product’s attributes in the calculation of its popularity or market value. To our knowledge, there is no other methodology for formulating competitive strategies for the online market by jointly considering the above inputs.

In the following, we conceive the marketing suitability of a product as emerging from the weighted average of its attributes, to which we refer in the sequel as the product’s market value. The remainder of the paper is structured as follows: Section 2 elaborates the concept of a product’s market value and gives insights into the problem of associating a product’s attributes. Section 3 deals with the issue of managing the fuzzily expressed customers’ views. Section 4 is devoted to data mining issues concerning the extraction of association rules amongst the product attributes and the related procedure of weight estimation. The overall problem is modelled and solved through a Game Theoretic analysis in Section 5. Section 6 validates the proposed approach through an illustrative numerical example. Finally, Section 7 provides concluding remarks and outlines future work directions.

2 Market value and association rules

The market value of a product is to be interpreted as an indication of the rating given to it by the customers in the market. Since customers’ product preferences are based on the product’s attributes, each product is evaluated through its attributes. To better explain the concept of a product’s market value, consider the following example: Let a customer being interested in purchasing a car and having in his/her mind a set of attributes $A \ (A = \{a_1, a_2, \ldots, a_n\})$, through which he/she intends to evaluate each instance of the product in the market. In this case, the company would like to associate a weight to each of these attributes, in order to assess the product’s market value. Such weights actually
reflect the customers’ perception about the importance of a product’s attribute (Yager, 1996). If \( w_i \) is the weight of the product’s attribute \( a_i \), we conceive the market value \( Q_j \) of a product \( j \) as:

\[
Q_j = w_1 a_{i1} + w_2 a_{i2} + \cdots + w_n a_{in},
\]

(1)

where \( a_{ij} \) is the value of the attribute \( i \) for product \( j \).

The attached weights are overall weights, assessed by the customers and not from a single individual. It is noted that if the product attributes are interactive, we cannot derive a formula for market value like the one shown in equation (1); interactions amongst attributes are not applicable in our approach (as discussed below, we have considered the attribute associations and hence the derived association rules that are based on the past data of the customers).

Market values may help a company in defining their business strategies; the concept is further exploited in our Game Theoretic analysis of the problem (Section 5). However, a major problem emerging at this point is how to determine the above weights, especially when the attributes are fuzzily defined. A variety of weight estimation methods has been proposed in the multi-attribute decision making literature (Mohanty, 1994, 1998; Saaty, 1978). In the approach described in this paper, past data of a customers’ buying habits are used for this purpose. However, the volume of customers’ past buying data is huge and the company must carefully inspect them in order to extract some valuable information. The concept of association rules from the field of Data Mining (Han and Kamber, 2001; Pujari, 2001) helps us to further elaborate the estimation of weights. Such rules attempt to reveal associations between different attributes. Such associations basically respond to questions of the type “if a customer buys a computer, how likely it is that this customer will also buy a printer in the same trip?”. Data extracted from the customers’ past buying habits (obtained either through the internet or the traditional market) may also help companies to determine the existing associations amongst a product’s attributes. For example, consider a product car with the associated attributes cost and mileage. The inspection of past market transactions (or the customers’ buying habits) concerning alternative cars (with respect to the attributes of cost and mileage), together with their processing through the appropriate data mining techniques, will help us to determine the existing degree of association between the above two attributes. In turn, the existing (resulting) associations will help us to predict the future association amongst these attributes and hence the expected buying pattern of the customers.

To give an example, consider three attributes \( a_1, a_2 \) and \( a_3 \) characterising a product. The possible association rules having attribute \( a_1 \) as an antecedent are: \( a_1 \rightarrow a_2, a_1 \rightarrow a_3, a_1 \rightarrow a_2 a_3 \). These associations should be interpreted as “given an attribute \( a_1 \), how likely it is that a customer would prefer another attribute (i.e., \( a_2 \) or \( a_3 \)) or the coexistence of other attributes (i.e., \( a_2 \) and \( a_3 \))”. Or, in other words, “if a product with attribute \( a_1 \) is available, how much a customer would be interested in the attributes \( a_2 \) and \( a_3 \)”. If \( a_1, a_2 \) and \( a_3 \) refer to the attributes cost, maintenance cost and resale value, respectively, the third rule above could be used for representing the question “given that a car costs US$ 20000, how likely it is that a customer would purchase it if the expected maintenance cost and the resale value are not very lucrative?”. Adopting the data mining concepts given in (Pujari, 2001), for the first rule (\( a_1 \rightarrow a_2 \)) we have:
where $c_{1,2}$ represents the degree of confidence of the association $a_1 \rightarrow a_2$, while $s(a_1, a_2)$ and $s(a_i)$ represent the support of $a_1$ and $a_2$ together (coexisting attributes) and $a_1$ (single attribute), respectively, to the association. We interpret the value $c_{1,2}$ as the extent to which a customer is motivated by the attribute $a_2$ given the availability of $a_1$. Similarly, one can derive the degrees of confidence for the rest association rules with $a_1$ as an antecedent (i.e., $c_{1,3}$ for the rule $a_1 \rightarrow a_3$, and $c_{1,23}$ for the rule $a_1 \rightarrow a_2 \rightarrow a_3$). The aggregation of these degrees of confidence provides us with the customers’ preference degree for the attribute $a_1$. As noted above, these degrees of confidence amongst attributes, which are obtained through the consideration of the customers’ past buying habits, act as indicators for the companies to predict the expected buying behaviour of the customers. Hence, they assess the range of potential customers for the product.

The average of the above degrees of confidence is the customers’ preference level $pl_1$ for attribute $a_1$. That is, $pl_1 = \frac{c_{1,2} + c_{1,3} + c_{1,23}}{3}$. Working in the same way, we calculate the customer’s preference levels for the other two attributes. Following Saaty (1978), if $r_{pij} = pl_i / pl_j$ represents the customer’s relative preference of the attribute $a_i$ over $a_j$, we can have the weights as the eigenvector corresponding to the maximum eigenvalue of the following reciprocal matrix (for a more detailed description of this procedure, see Saaty, 1978).

\[
\begin{bmatrix}
    r_{p11} & r_{p12} & r_{p13} \\
    r_{p21} & r_{p22} & r_{p23} \\
    r_{p31} & r_{p32} & r_{p33}
\end{bmatrix}
\]

(3)

Using the weights derived through the above procedure, one can use equation (1) to calculate the market value of a product.

3 Fuzzy representation of the product attributes

Most of the real world decision-making procedures take place in an environment where the goals of the overall process, as well as the constraints imposed and the consequent possible actions are not precisely known. To deal quantitatively with the imprecise nature of the issues involved in such settings, Zadeh (1965) introduced the concept of fuzzy sets, which has received much interest in the research community. This is due to its remarkable usefulness in dealing with the inherent vagueness or ambiguity involved in various realistic systems. As an example, in daily life one often uses the expression ‘around three’ to indicate his/her desire in a vague but practical way (this desire may, for instance, concern the number of years a car is under guarantee). Formally speaking, a fuzzy set is a set of elements with a continuous grade of membership. That is, a fuzzy set contains a class of objects, while there is no sharp boundary between the objects that belong to the class and those that do not. This is represented by a membership function that assigns a grade of membership, ranging between zero and one, to each object of the class. A precise definition of fuzzy sets is as follows:
Definition: Let $X$ be a set of objects. A fuzzy set $A$ in $X$ is defined as a set of ordered pairs $A = \{x, \mu_A(x)\}$, where $\mu_A(x)$ represents the membership function of the fuzzy set $A$, which associates each point $x \in X$ with a real number in the interval $[0, 1]$. The value $\mu_A(x)$ is called the grade of membership of $x$ in $A$.

3.1 Fuzzy attributes

In the majority of market settings and products, customers perceive and rate the products’ attributes in a qualitative or linguistic manner. This is often due to the vague knowledge the customers have about a product, which leads to the customers’ inability or hesitancy to express their preferences with an exact numerical value. On the other hand, the use of words or sentences rather than numbers enables a more flexible and realistic form of adequately expressing day-to-day business terms. Fuzzy set theory provides a flexible framework to deal with the qualitatively defined terms (linguistic assessments) in a quantitative manner (for a representative sample of such approaches, see Adamopoulos and Pappis, 1996; Bordogna and Pasi, 1997; Herrera and Herrera-Viedma, 1997; Liu et al., 1994; Yager, 1993). In our approach, we take the customers’ linguistic assessments or opinions about the product attributes and appropriately represent these imprecisely defined attributes of a product as fuzzy sets or fuzzy numbers (Dubois and Prade, 1978). In fact, in a real e-business system, we need a way to analyse the vaguely defined customers’ judgments and represent them in the form of fuzzy sets or fuzzy numbers.

We briefly introduce the above representation through the following example: Consider the product category car and let customers contemplate (in this product category) over the attributes cost, resale value, mileage, comfort and maintenance cost. It is very often that a customer expresses his/her preferences regarding these attributes in the form:

**Cost:** cost should be around US$ 20,000

**Resale value:** after 3 to 4 years, the resale value should be OK

**Mileage:** mileage should be normal

**Comfort:** overall, the car should be comfortable

**Maintenance cost:** maintenance cost should not be very high.

In fact, the day-to-day business language is often of the above form; customers view all these attributes vaguely or fuzzily, but certainly in a realistic way. We represent each attribute as a fuzzy set $\{\text{attribute}, \mu_{\text{attribute}}\}$. For instance, we represent the attributes cost and mileage as $\{\text{cost}, \mu_{\text{cost}}\}$ and $\{\text{mileage, } \mu_{\text{mileage}}\}$, respectively. Without loss of generality, we can then define these attributes through the fuzzy numbers $(10000, 20000, 30000)$ and $(10, 20, 30)$.

Figure 1 illustrates the fuzzy number representation of these two attributes. Speaking about the attribute cost (left part of the figure), the cost US$ 20000 corresponds to the highest satisfaction level (i.e., $\mu_{\text{cost}}(20000) = 1$). The satisfaction level $\mu_{\text{cost}}$ gradually comes down if the cost of a car either decreases or increases from the above amount, and represents zero satisfaction when the cost is either US$ 10000 or US$ 30000 (i.e., $\mu_{\text{cost}}(10000) = 0$ and $\mu_{\text{cost}}(30000) = 0$, respectively). Similarly, for the attribute mileage, if a customer feels that the normally expected mileage of a car should be
20 miles/gallon, the highest satisfaction level for this attribute is at this point (i.e., \( \mu_{\text{mileage}}(20) = 1 \)). The satisfaction level progressively decreases if mileage gets below or above the 20 miles/gallon point, while at the points 15 miles/gallon and 25 miles/gallon we observe a zero satisfaction level (i.e., \( \mu_{\text{mileage}}(15) = 0 \) and \( \mu_{\text{mileage}}(25) = 0 \), respectively).

**Figure 1** Fuzzy representation of cost (left) and mileage (right)

Linguistically defined product attributes, such as these discussed above, are represented as fuzzy sets (fuzzy numbers) mainly through interaction with the customers. This approach is appropriate for diverse problems in the e-business context, since it incorporates the linguistic judgements of the customers in a more direct and adequate form. A variety of linguistic quantifier models has been already proposed in the literature (Herrera and Herrera-Viedma, 1997; Liu et al., 1994; Yager, 1988).

### 4 Data mining for the estimation of attribute weights

When doing business in the online market and prior to making the final purchasing choice, customers often look for as much as possible information about the product they have in mind. In addition, they want to analyse the alternative offers in terms of their attributes. In such a setting, customers may favour a particular product taking into account their attribute evaluation levels. Such a behaviour requires the need of a methodology to assess the product attributes as per the customers’ requirements. Our paper follows the Analytical Hierarchy Process (Saaty, 1978) and the concept of association rules from the Data Mining discipline to derive the attribute weights.

More specifically, the proposed approach is as follows:

Let \( n \) alternative products \( p_1, p_2, \ldots, p_n \) of a certain product category \( P \) being available in an online market. Products of this category are characterised (evaluated) through a set of attributes \( A \ (A = \{a_1, a_2, \ldots, a_m\}) \). Let \( a_j(i = 1, 2, \ldots, n; \ j = 1, 2, \ldots, m) \) denote the attribute level \( j \) of product \( i \).

**Step 1:** Consider the customers’ product requirements in terms of its attributes. Represent these attributes as fuzzy numbers, as per the customers’ view. This results to the creation of the following \( m \) fuzzy sets or fuzzy numbers.

\[ \{a_1, \mu_{a_1}\}, \{a_2, \mu_{a_2}\}, \ldots, \{a_m, \mu_{a_m}\}, \]

where \( \mu_{a_j}(j = 1, 2, \ldots, m) \) represents the membership function of the fuzzy set corresponding to attribute \( a_j \).
Step 2: Consider the customers’ past purchasing transaction data (buying habits). Let a number \( k \) of transactions \( \{t_1, t_2, \ldots, t_k\} \) carried out so far. Each transaction consists of a set of products. Formally speaking, \( t_i \subseteq \{p_1, p_2, \ldots, p_n\} \) and \( t_i \neq \emptyset \).

Step 3: For each transaction, find the attribute totals. For instance, consider transaction \( t_s \) involving four products \( p_1, p_2, p_3 \) and \( p_4 \). The attribute totals for attribute \( j \) is:

\[
T^{\mu_j} = (\mu_{i_1}(a_j) + \mu_{i_2}(a_j) + \mu_{i_3}(a_j) + \mu_{i_4}(a_j))/4
\]

\((s = 1, 2, \ldots, k; \; j = 1, 2, \ldots, m)\).

\( T^{\mu_j} \) is considered as representing the amount of attribute \( j \) traded in transaction \( k \).

Step 4: Form the association rules amongst the attributes by taking each attribute as an antecedent, i.e., \( a_j \rightarrow \psi \), where \( \psi \in P(A - a_j) \), \( \psi \neq \emptyset \), and \( P(A - a_j) \) is a power set of any subset of \( (A - a_j) \). The confidence level of the above association rule is:

\[
\frac{\sum_{s=1}^{k}[T^{\mu_i} \land T^{\mu_j}]}{\sum_{s=1}^{k} T^{\mu_i}} = \alpha_\psi \quad (j = 1, 2, \ldots, m).
\]

It is noted that in the above equation we have the association between the attribute \( a_j \) and a non-empty subset of the attribute set not containing \( a_j \), that is a non-empty element from \( P(A - a_j) \). By scanning over all the non-empty subsets of \( P(A - a_j) \), we can derive the number of association rules with antecedent \( a_j \) as:

\[
\text{card}\{P(A - a_j)\} = 2^{m-1},
\]

where \( \text{card}\{P(A - a_j)\} \) denotes the cardinality of \( P(A - a_j) \). The number of non-empty subsets in \( P(A - a_j) \) is \( 2^{m-1} - 1 \). The above association rules help us in deriving the degree of confidence \( c_j \) of the customers about the attribute(s) \( \psi \), given the availability of \( a_j \), through the following equation:

\[
c_j = \frac{1}{(2^{m-1} - 1)} \sum_{\psi \in P(A - a_j) \neq \emptyset} \alpha_\psi.
\]

Step 5: If \( r_{p_i} \) represents the customers’ relative preference of \( a_j \) over \( a_j \) \( (r_{p_i} = c_i/c_j) \), the eigenvector corresponding to the maximum eigenvalue of the matrix below gives us the weights of the attributes.

\[
\begin{bmatrix}
r_{p_{i1}} & r_{p_{i2}} & \cdots & r_{p_{im}} \\
r_{p_{21}} & r_{p_{22}} & \cdots & r_{p_{2m}} \\
\vdots & \vdots & \ddots & \vdots \\
 r_{p_{m1}} & r_{p_{m2}} & \cdots & r_{p_{mm}}
\end{bmatrix}
\]

According to equation (1), if \( w_j \) is the weight of the attribute \( a_j \) of a product \( p_i \), the market value of the product is:

\[
Q_i = w_1a_{i1} + w_2a_{i2} + \cdots + w_ma_{im}.
\]
5 The game theory model

According to Game Theoretic approaches, a company needs to choose strategies for its own profit and, at the same time, the satisfaction of its customers. As mentioned earlier, our focus is on strategies that involve decisions about the selection of the appropriate products to be launched on the web, as well as decisions about the right timing to do so. Such a selection has to also take into account products offered by the company’s competitors in the online market. Based on the approach given in Hui and Tam (2002), we present below a Game Theory model to address the above issues.

As above, let again \( n \) products \( p_1, p_2 \ldots p_n \) of a certain product category \( P \) existing in an online market. Each of these products is evaluated through a set of attributes \( A \) \( (A = \{a_1, a_2 \ldots a_m\}) \), while \( a_j \) \( (i = 1, 2, \ldots, n; \ j = 1, 2, \ldots, m) \) denotes the attribute \( j \) of product \( i \). The fuzzy representation of the attributes is (see Step 1, Section 4):

\[
\{a_{i1}, \mu_{i1}(a_{i1})\}, \{a_{i2}, \mu_{i2}(a_{i2})\}, \ldots, \{a_{im}, \mu_{im}(a_{im})\}, \ i = 1, 2 \ldots n.
\]

Similarly, using equation (7) and considering the membership levels of each attribute, we can obtain the market value \( Q_i \) of a product \( p_i \) as:

\[
Q_i = w_1\mu_{i1} + w_2\mu_{i2} + \ldots + w_m\mu_{im}, \ i = 1, 2 \ldots n. \tag{8}
\]

Note that \( Q_i \in [L, H] \) (in our approach, we assume \( L = 0 \) and \( H = 1 \)). The strategy to be developed for the company involves decision making about which product, with what value of \( Q_i \), will be launched in the online market in order for the business to be profitable.

Consumers of different type have different ‘sense’ of preferences for the market values \( Q_i \). If some consumers prefer the products with \( Q_i = \theta_i \), then the company looks for products with \( Q_i \geq \theta_i \). If the company charges \( p_{ri} \) for product \( p_i \), then the net surplus to the consumers is:

\[
U = \begin{cases} 
\theta_i - p_{ri} & \text{if } \theta_i \leq Q_i \\
0 & \text{if } \theta_i > Q_i
\end{cases} \tag{9}
\]

In the above equation, the price \( p_{ri} \) is always less than one. The demand for the product \( p_i \) is made up by consumers whose net surplus is more than zero. It is:

\[
D(Q_i, p_i) = P(U > 0 \land \theta < Q_i) = F(Q_i) - F(p_{ri}) = Q_i - p_{ri} \tag{10}
\]

where \( F \) denotes the cumulative distribution function and \( P(U > 0 \land \theta < Q_i) \) represents the probability that the customers’ utility level in the product is more than zero and the product attributes in total (market value) are higher than that of the customers’ requirements. Following Hui and Tam (2002), if \( M \) is the total market size, the expected demand is:

\[
D(Q_i, p_i) = M(Q_i - p_{ri}) \tag{11}
\]
5.1 Monopoly market

Only one product is available in a monopoly market, thus consumers do not have any alternative choice; they will either purchase or reject the product. The profit $\pi$ for the company is:

$$\pi = M(Q_i - p_r)pr_i - VQ_i,$$

where $V$ is the unit development cost to the company. From the above equation, we can find the optimal values for $Q_i$ and $p_r$ by solving the following derivatives:

$$\frac{\partial \pi}{\partial Q_i} = Mpr_i - V$$
$$\frac{\partial \pi}{\partial p_r} = MQ_i - 2Mpr_i.$$

The above equations give the optimal values $Q^*_i = 2V/M$ and $p^*_r = Q^*_r/2$.

5.2 Duopoly market

If a company launches a product in the online market with market value $Q_1$, while its competitor acts alike with market value $Q_2$, the demand and profit for the company are as follows (for details, see Hui and Tam, 2002):

$$D_i = \begin{cases} 
M(Q_1 - p_r) & \text{if } Q_1 < Q_2 \\
1/2M(Q_1 - p_r) & \text{if } Q_1 = Q_2 \\
M(Q_1 - \max(p_r, Q_2)) & \text{if } Q_1 > Q_2 
\end{cases}$$

$$\pi_i = \begin{cases} 
M(Q_1 - p_r)pr_i - VQ_i & \text{if } Q_1 < Q_2 \\
1/2M(Q_1 - p_r)pr_i - VQ_i & \text{if } Q_1 = Q_2 \\
M(Q_1 - \max(p_r, Q_2))pr_i - VQ_i & \text{if } Q_1 > Q_2 
\end{cases}.$$  

To give an example, the payoffs derived when the first company chooses a product with market value $Q_1$ ($Q_1 \in \{C, D\}$) and its competitor chooses the same product with market value $Q_2$ ($Q_2 \in \{C, D\}$) are shown in Table 1. We have assumed at this point that the values $Q_1$ and $Q_2$ are not very far from each other and that both are on the higher side of the market value i.e., $H = 1$. It is easy to calculate that the firm with lower $Q$ always gets more payoff than the firm with the higher $Q$ value.

In this setting, the two companies have to decide whether the market value of the product will be $C$ or $D$. In case we allow the strategy space to any $Q$ ($Q \in [L, H]$ – in our case, $Q \in [0, 1]$), and provided that the opponent is launching the product in the online market with a high $Q$ value, there is always an advantage for the firm with a lower $Q$ value in terms of better payoffs and higher demand. However, if one firm’s product has very low $Q$ value, the other firm may opt for higher $Q$ values and capture a big market share. That is, if there is a big gap between the market values of the products, the firm having a high $Q$ value may attract a good market share and, consequently, a higher profit. The indifference point between the companies having a product with $Q_1$ market value and the opponent with a very high market value $Q_2$ ($Q_2$ close to $H$) is given by $Q^*$:
Table 1
Payoffs in a duopoly market

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Q_1)</td>
<td>(\frac{MC^2}{8} - VC)</td>
<td>(\frac{MC^2}{4} - VC)</td>
</tr>
<tr>
<td>(\frac{MC^2}{8} - VC)</td>
<td>(\frac{M(D - C)D}{2} - VD)</td>
<td></td>
</tr>
<tr>
<td>(\frac{M(D - C)D}{2} - VD)</td>
<td>([M(D - C)C - VD])</td>
<td></td>
</tr>
<tr>
<td>(\frac{MC^2}{4} - VC)</td>
<td>(\frac{MD^2}{8} - VD)</td>
<td></td>
</tr>
</tbody>
</table>

Payoffs shown in brackets denote when the firm actually raises the price of the product from \(D/2\) to \(C\).

\[
\frac{MQ^2}{4} - VQ_1 = M(1 - Q_1)\frac{1}{2} - V
\]

\[
\Rightarrow Q^* = \frac{2}{M}\left( V - M \frac{2}{2} \right) + \sqrt{\left( V - M \frac{2}{2} \right)\left( V - M \frac{3}{2} \right)}
\]

\[
\pi = -2\left( V - M \frac{2}{2} \right) - \sqrt{\left( V - M \frac{2}{2} \right)\left( V - M \frac{3}{2} \right)}
\]

Equation (16) gives the payoff. According to it, the two companies will benefit if one of them launches a product with market value \(Q^*\) and the other with \(H\). If we observe the limits, we have:

\[
\lim Q^* = \sqrt{3} - 1 = 0.732 \text{ when } V / M \to 0.
\]

This indicates that \(Q^* > 0.5\) and \(Q^* < 1\). That is, the company building the average product has a larger market share than a company launching products with high market values.

In the above payoffs, the two companies followed the optimal pricing strategy. If the competitor’s price \(pr_2\) for the product is lower than \(Q_1\), the competitor can raise its product price to \(Q_1\) to get extra profits. This is explained through the following equation:

\[
\frac{MQ^2}{4} - VQ_1 = M(1 - Q_1)Q_1 - V
\]

\[
\Rightarrow Q^{**} = \frac{2}{5M}\left( (V + M)^{\frac{1}{2}} + \sqrt{(V + M)^{\frac{3}{2}} - 5VM} \right)
\]

\[
\pi^{**} = \frac{2M - 8V}{25M}\left( (V + M)^{\frac{1}{2}} + \sqrt{(V + M)^{\frac{3}{2}} - 5VM} \right) - V
\]
Again here, it is \( \lim Q^{**} = 0.8 \) when \( V/M \to 0 \). That is, the indifference point lies around 0.8. \( \pi^{**} \) is the best payoff for the companies when one chooses the products with market value 0.8(> 0.5) and the other chooses the product with \( Q = H \), while the price is \( Q^{**}/2 \). However, this will not be sustainable as the company choosing the products with market value \( Q^{**} \) may like to switch to \( H-e \) (\( e \) is a small number) to get a better payoff.

We can summarise the above observations as follows: Let a company launching a product with market value \( Q_1 \) and its competitor launching the same product with market value \( Q_2 \). Let their product prices be \( pr_1 \) and \( pr_2 \) respectively.

- If \( Q_2 > Q^{**} \), the company should choose the products with \( Q_1 = Q_2 - e \) and \( pr_1 = Q_1/2 \)
- If \( 0.5 < Q_2 < Q^{**} \), the company should choose products with \( Q_1 = H \) and \( pr_1 = Q_2 \)
- If \( Q_2 \leq 0.5 \), the company should choose products with \( Q_1 = H \) and \( pr_1 = 0.5 \).

### 6 Numerical example

In this section, we describe our overall approach through a comprehensive example. Let customers evaluate the product car with respect to three attributes, namely maintenance cost (per month), mileage (miles/gallon) and resale value depreciation (in percentage). After interacting with the customers and taking their views in the linguistic manner, we can express the views in the form of fuzzy sets (fuzzy numbers) (this has been explained in Section 3).

As graphically represented in Figure 2, the fuzzy sets for the above attributes are as follows:

- **Maintenance Cost**
  
  \[
  \text{Maintenance Cost} = \{0/40, 0.4/50, 0.63/100, 0.65/150, 1.0/200, 0.8/300, 0.4/400, 0.2/500\}
  
- **Mileage**
  
  \[
  \text{Mileage} = \{0/9, 0.1/10, 0.4/12, 0.5/15, 0.6/16, 0.73/17, 0.8/19, 1.0/20, 0.9/21, 0.8/22, 0.72/25, 0.7/26\}
  
- **Resale Value Depreciation**
  
  \[
  \text{Resale Value Depreciation} = \{0.5/5, 0.6/10, 0.8/15, 1/20, 0.8/30, 0.6/40, 0.4/50, 0.2/60\}
  
![Figure 2](image-url)

As graphically represented in Figure 2, the fuzzy representation of maintenance cost (left), mileage (middle) and resale value depreciation (right)

Assume that there are eight products \( (p_1, \ldots, p_8) \) of the type car available in an electronic marketplace. Product data related to the three attributes considered in our example are summarised in Table 2, while data concerning customers’ transactions for these products in ten time periods (i.e., weeks) are given in Table 3.
Table 2  Products available in the electronic marketplace

<table>
<thead>
<tr>
<th>Car</th>
<th>Maintenance cost (US$)</th>
<th>( \mu_{\text{maintenance}} )</th>
<th>Mileage (miles/gallon)</th>
<th>( \mu_{\text{mileage}} )</th>
<th>Resale value depreciation (%)</th>
<th>( \mu_{\text{depreciation}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_1 )</td>
<td>100</td>
<td>0.63</td>
<td>19</td>
<td>0.8</td>
<td>5</td>
<td>0.5</td>
</tr>
<tr>
<td>( p_2 )</td>
<td>50</td>
<td>0.4</td>
<td>25</td>
<td>0.72</td>
<td>40</td>
<td>0.6</td>
</tr>
<tr>
<td>( p_3 )</td>
<td>300</td>
<td>0.8</td>
<td>17</td>
<td>0.73</td>
<td>30</td>
<td>0.8</td>
</tr>
<tr>
<td>( p_4 )</td>
<td>100</td>
<td>0.63</td>
<td>22</td>
<td>0.8</td>
<td>60</td>
<td>0.2</td>
</tr>
<tr>
<td>( p_5 )</td>
<td>150</td>
<td>0.65</td>
<td>25</td>
<td>0.72</td>
<td>20</td>
<td>1</td>
</tr>
<tr>
<td>( p_6 )</td>
<td>200</td>
<td>1</td>
<td>22</td>
<td>0.8</td>
<td>50</td>
<td>0.4</td>
</tr>
<tr>
<td>( p_7 )</td>
<td>500</td>
<td>0.2</td>
<td>12</td>
<td>0.4</td>
<td>15</td>
<td>0.8</td>
</tr>
<tr>
<td>( p_8 )</td>
<td>300</td>
<td>0.8</td>
<td>20</td>
<td>1</td>
<td>10</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Using equation (4), we calculate the total attribute values for each transaction by adding the attribute values individually (see Table 3). For instance, transaction \( t_1 \) includes the products \( p_1, p_3, \) and \( p_8 \). The \( \mu_{\text{maintenance}} \) values for \( p_1, p_3, \) and \( p_8 \) are 0.63, 0.8, and 0.8, respectively. Thus, the average maintenance satisfaction in \( t_1 \) is \((0.63 + 0.8 + 0.8)/3 = 0.74\). Similarly, we calculate the total attribute satisfaction level for any cell of Table 3.

Using equation (5), we can then construct the association rules for all attributes, as explained in Section 4. For instance, for the attribute \( \text{maintenance cost} \), a possible association rule is \( \mu_{\text{maintenance}} \rightarrow \mu_{\text{mileage}} \). The degree of confidence for this rule is:

\[
\frac{(0.74 + 0.42 + 0.64 + 0.5 + 0.71 + 0.57 + 0.31 + 0.67 + 0.55 + 0.81)}{(0.74 + 0.42 + 0.64 + 0.5 + 0.71 + 0.57 + 0.66 + 0.67 + 0.55 + 0.81)} = 0.94.
\]

Similarly, the degrees of confidence for the rules \( \mu_{\text{maintenance}} \rightarrow \mu_{\text{depreciation}} \) and \( \mu_{\text{maintenance}} \rightarrow \mu_{\text{mileage}, \text{depreciation}} \) are 0.81 and 0.76, respectively. We can now calculate the customers’ preference level for the attribute under consideration, which is equal to \((0.94 + 0.81 + 0.76)/3 = 0.84\).
By the same procedure, we can easily obtain the customers’ preference levels for the attributes mileage and resale value depreciation. These are 0.76 and 0.84, respectively. As discussed in Section 4, these preference levels help us to construct the reciprocal matrix given below.

\[
\begin{pmatrix}
0.84 & 0.76 & 0.84 \\
0.84 & 0.84 & 0.84 \\
0.76 & 0.76 & 0.84 \\
0.84 & 0.84 & 0.84 \\
0.76 & 0.76 & 0.84 \\
0.84 & 0.84 & 0.84
\end{pmatrix}
\]

The eigenvector corresponding to the maximum eigenvalue of the above matrix, which gives the weights attached to the attributes, is (0.32, 0.36, 0.32). Using equation (8), we can now find the market value of each available product in Table 4.

Table 4  Market values for the marketplace products

<table>
<thead>
<tr>
<th>Car</th>
<th>Market value $Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_1$</td>
<td>0.65</td>
</tr>
<tr>
<td>$p_2$</td>
<td>0.58</td>
</tr>
<tr>
<td>$p_3$</td>
<td>0.77</td>
</tr>
<tr>
<td>$p_4$</td>
<td>0.57</td>
</tr>
<tr>
<td>$p_5$</td>
<td>0.79</td>
</tr>
<tr>
<td>$p_6$</td>
<td>0.74</td>
</tr>
<tr>
<td>$p_7$</td>
<td>0.46</td>
</tr>
<tr>
<td>$p_8$</td>
<td>0.87</td>
</tr>
</tbody>
</table>

The market values help the company in designing the strategies for a better payoff. According to equations (16)–(18) and the solution steps presented in Section 4, the alternative scenarios concerning the competitor’s and the company’s strategies are given in Table 5.

Table 5  Strategies for our numerical example

<table>
<thead>
<tr>
<th>Competitor’s strategy ($Q_2$)</th>
<th>Company’s strategy ($Q_1$)</th>
<th>Product price for the company</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_2 &gt; 0.8$</td>
<td>$Q_1 = Q_2 - \varepsilon$</td>
<td>$Q_1/2$ (approx. 0.39)</td>
</tr>
<tr>
<td></td>
<td>*Possible cars to be launched in the e-market are $p_3$ and $p_5$</td>
<td></td>
</tr>
<tr>
<td>$0.5 &lt; Q_2 &lt; 0.8$</td>
<td>$Q_1 = 1$</td>
<td>$Q_1/2$</td>
</tr>
<tr>
<td></td>
<td>*Since no car with market value equal to 1 is available, the cars to be launched should be the same with those of the competitor</td>
<td></td>
</tr>
<tr>
<td></td>
<td>*In no case $p_8$ should be launched in the e-market, as this will incur loss</td>
<td></td>
</tr>
<tr>
<td>$Q_2 &lt; 0.5$</td>
<td>$Q_1 = 1$ (approx.) or $Q_1 = Q_2$</td>
<td>$Q_1/2$ (0.23 or 0.44)</td>
</tr>
<tr>
<td></td>
<td>*Possible cars to be launched in the e-market are $p_7$ and $p_8$</td>
<td></td>
</tr>
</tbody>
</table>
From the above table, a company can identify which car(s) should be launched in the electronic market in order to do profitable business. It is stressed here that these strategies resulted from the exploitation of the customers’ historical data.

7 Discussion and conclusions

We have presented a methodology to formulate profitable business strategies for the competitive electronic markets. Data reflecting the buying patterns of customers are exploited to help companies in the above process, thus making their business more customer-focused. Our approach may actually aid a company in capturing the market’s behaviour and trends (extracting data from multiple transactions, carried out by multiple customers), the aim being to decide the products to be launched on the web. The products to be launched may certainly have diverse ‘degrees of freedom’, thus allowing then a particular customer to translate his/her product wish to an instance of such predefined products. In other words, our approach may precede the formulation of a mass customisation strategy (Pine, 1993), according to which companies enable customers to specify their individual product wishes and create, in a virtual and interactive way (e.g., by using web-based configurator and recommender tools built on diverse personalisation techniques), their own instance of a product.

Our approach builds on the concept of a product’s market value. This value facilitates the company in the comprehension of the fuzzily defined customers’ judgment about the market’s products. The weights to be associated to each attribute are derived through the integration of Fuzzy Logic techniques with data mining association rules. Exploration and resolution of the strategic issues involved has been addressed through a Game Theory model, which takes into account the market value of each product.

In our approach, a product’s market value derives as the weighted aggregation of the satisfaction (importance) levels of its attributes. When determining this value, the incorporation of the customers’ profile, the type of competitors and their product brand equity would provide more refined results; such an approach delineates a direction for future research. Another direction for further work concerns the company’s business characteristics to be taken into account when formulating a strategy. In this paper, the development of a company’s business strategy was based on its competitors’ types of products (launched in the marketplace) and their market values. In addition to them, the above process could also take into account other business characteristics of the competitors, such as their advertisement policy, image amongst the customers, business volumes, financial health of the company, etc.

Another advantage of the proposed methodology is that it can simultaneously evaluate a number of alternative products (in a particular product category) and make a viable suggestion of products to the customers’, as per their choice. When a large number of products exists in the e-market, the computation requirements for their evaluation concern the calculation of their market values and a logical comparison of these values (as presented in Section 6) during the definition of the product launching strategies.
The proposed methodology has yet to be applied in real life e-business problems. However, it provides a formal theoretical framework, built around well-tried concepts, which is able to aid companies modifying their business strategies (and the associated e-business sites) to overcome their competitors, while also exploiting the buying patterns of customers.

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References


On the development of e-business strategies


**Note**

1Elaborated discussions on the estimation of weights and the required weighted averaging aggregation operators in multiple criteria decision making settings can be found in Yager (1988, 1996).