An experimental study on tournament design

Christine Harbring\textsuperscript{a,*}, Bernd Irlenbusch\textsuperscript{b,1}

\textsuperscript{a}Department of Economics, University of Bonn, BWL II, Adenauerallee 24-42, Bonn D-53113, Germany
\textsuperscript{b}Department of Economics, Microeconomics, University of Erfurt, Nordhäuser Str. 63, Erfurt D-99089, Germany

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Abstract

Since recently, rank order tournaments have become quite popular for providing incentives in employment relationships. However, the consequences of different tournament designs are widely unexplored. This paper experimentally investigates different tournament design alternatives along two dimensions: tournament size and prize structure. We find that average effort tends to increase with a higher proportion of winner prizes. Additionally, variability of effort is lower if the number of winner prizes is high. Especially two-person tournaments are prone to collusion. Furthermore, we observe a restart effect.

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1. Introduction

Incentive schemes in organizations are more and more often implemented as competitive compensation mechanisms like rank order tournaments, e.g. in which high payments or promotions serve as prizes given to the winners (for an overview, see Gibbons, 1998; Lazear, 1999; Prendergast, 1999). Thus, understanding the implications of different tournament designs becomes increasingly important for personnel economics. The most crucial design issues of tournaments are their size, i.e. the number of agents that
participate and the prize structure, i.e. the number of winner prizes. Imagine, for example, the situation where an employer wants to set up a tournament among her employees. She is confronted with several essential questions, which, however, are difficult to answer: Is one single tournament with many participants more profitable than assigning employees to multiple tournaments with only a few participants? Does it pay off to induce agents to strive for many winner prizes? Or does a tournament with only few winner prizes induce a stronger competition among employees? This paper reports on an experiment, which is designed to gain insights into how tournament size and prize structure may influence the behaviour of participating agents.

In tournaments, competition is used as an incentive device where agents are rewarded according to their relative performance compared to other agents with prizes that are fixed in advance. In their seminal paper, Lazear and Rosen (1981) show by an elegant model with two risk neutral agents that tournaments—like piece rates—can induce efficient effort levels like piece rates. They analyse tournament situations in which the employers cannot directly observe the effort of the employees. Therefore, in their model, the output of an employee depends on his effort and an individual random component. They derive an equilibrium with symmetric agents where effort positively depends on the prize spread, i.e. the difference between winner and loser prize. Moreover, ceteris paribus effort decreases with an increasing influence of the random component. Their work has been generalized and extended in many other studies with respect to different aspects: e.g. Green and Stokey (1983) and Nalebuff and Stiglitz (1983) show the dominance of tournaments in the presence of common shocks, O’Keeffe et al. (1984) analyse the appropriate design of the prize structure to prevent the wrong type of people to participate in a tournament, Rosen (1986) examines sequential tournaments resembling a hierarchy where the losers of each stage game are eliminated from subsequent play. A large prize spread at the end of the career game offers an incentive to exert high efforts throughout the stages. The results offer an explanation for the extraordinarily high pay of top managers. Clark and Riis (1998) and Moldovanu and Sela (2001) allow for more than two types of prizes. Tournaments are especially popular if only ordinal, relative performance information is available at reasonable cost. A further advantage of tournaments arises when it is difficult for a third party (e.g. a court) to verify performance. In this case, a principal may be tempted to claim that performance was lower than it actually was. If it is nevertheless possible to observe whether promised tournament prizes have been awarded, a tournament enables the principal to credibly commit herself/himself ex ante. Tournaments, however, also suffer from severe drawbacks: basing pay on relative performance creates incentives for agents to apply for jobs with less able reference groups or agents may try to collude, i.e. to cooperate by collectively exerting effort on a very low level. Moreover, agents can improve their relative position by reducing their opponents’ output (Lazear, 1989).

The trade-off between competitive and cooperative behaviour in tournaments at its very heart is an empirical question because the degree of how real human actors are willing to rely on strategic alliances in different institutional designs cannot be investigated by theoretical reasoning. Therefore, several empirical studies exist which contrast the predictions from tournament theory with field data. This work can be classified into
two categories. First, there are several very interesting studies that use data from sports tournaments in which the competitive structure is naturally given by the contest, and the performance of participants is measured by final scores. In general, those studies confirm findings from tournament theory especially the result that an increase in the prize spread induces higher performance. Ehrenberg and Bognanno (1990), for example, analyse data from the 1984 men’s United States Professional Golf Association tour. The structure of prize money, i.e. the percentage of total money attributed to a certain rank, is identified to be the same in all tournaments. Only the absolute amount of prize money awarded differs between tournaments. As a measure for performance, the player’s score is used. They find that scores are lower, i.e. performance is improved, if the prize level is high. In a follow-up study, Orszag (1994) analyses data from the 1992 men’s United States Professional Golf Association tour and finds results, which contradict those presented by Ehrenberg and Bognanno. The deviation is attributed to a different handling of variables that reflect weather conditions. Becker and Huselid (1992) study the performance of drivers in auto-racing tournaments with a special emphasis on their willingness to take risk, Lynch and Zax (1998) check for sorting effects in Arabian Horse-Racing, and Fernie and Metcalf (1999) analyse the incentive effects of different jockeys’ contracts. The prize structures and the institutional designs considered in these studies are given by the specific tournament and/or by the framework of the particular sport rules. In general, the sporting performance serves as an approximation of the effort level exerted, i.e. the final score a player obtains in the specific discipline.

The other category of empirical studies constitutes investigations on executive compensation in firms. In these studies, the pay structures in firms are analysed with regard to tournament theory, e.g. O’Reilly et al. (1988) and Main et al. (1993). Eriksson (1999), for example, focuses on the pay of managers on different levels in the corporate hierarchy of Danish firms. Among others, he finds that pay differences increase in the hierarchy level in a firm which is consistent with tournament theory. To approximate the effort of the managers, the firm performance is taken, i.e. average profits divided by sales. Bingley and Eriksson (2001) study the data of Danish companies and check for links of pay distribution and individual efforts, that are approximated by taking the inverse of the average rate of absenteeism, and firm performance. Although all these empirical studies provide valuable findings regarding the behaviour induced by existing tournaments, it becomes clear that field data cannot easily be used to compare different designs because it is hard to find real world tournaments that vary only in the characteristics under consideration. This restriction prevents a more rigorous comparison of systematically varied tournament designs. Therefore, we deliberately opt for an investigation based on a controlled experiment.

Up to now, there are surprisingly few experimental studies on tournaments which nevertheless gained valuable insights. Bull et al. (1987) compare tournaments and piece rates in an experiment and find a large variance in behaviour if tournaments are

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2 Beside the studies that fall in these two categories, there are very few additional analyses from business settings where adequate data are available. For example, Knoeer and Thurman (1994) analyse the performance of broiler producers as tournaments are explicitly used to compensate growers. They find that changes in the prize level that leave prize spreads constant do not change performance.
implemented. Weigelt et al. (1989) as well as Schotter and Weigelt (1992) analyse reactions to discrimination and unfair rules in tournaments. Interestingly, discriminated individuals show higher efforts than theoretically predicted. In Nalbantian and Schotter (1997), different collective compensation schemes are compared, among others a tournament with groups of players. It is confirmed that tournaments can provide much stronger incentives but also induce a higher variance of behaviour than other collective schemes. The study of Orrison et al. (1997) approaches research questions similar to ours. They conduct an experiment on tournaments of different sizes and prize structures. One main finding is that effort decreases if there is a high proportion of winner prizes. In all experimental studies mentioned so far, participants have to choose a number that represents the effort, which induces costs according to a convex function. The study of van Dijk et al. (2001) uses a real effort setting exerted by participants in the laboratory. They compare piece rates, team and tournament compensation schemes. Again, the effort exerted in tournaments is higher than in the other reward schemes but also more variable.

The strategic situation of the tournaments implemented in this study is similar to the situation in all-pay auctions or contests (Hillman and Riley, 1989; Baye et al., 1996). Gneezy and Smorodinsky (2000) conduct an experiment on repeated all-pay auctions where they vary the number of participants. They find among others that participants tend to overbid, i.e. the auctioneer’s revenue frequently exceeds the prize. Furthermore, the number of people who decide not to participate in the auction—indicated by submitting the minimum bid—increases with the size of the auction.

From these example studies, it becomes clear that experiments have the advantage that one can compare different, clear cut institutional designs without abstaining from behaviour shown by real actors. Thus, one is able to sharply separate different institutional design factors and analyse their influence on actual behaviour. Furthermore, by choosing an appropriate experimental design, one can focus on exactly those research questions, which one is interested in without perturbing interferences from real world situations. In addition to the advantage of controlling certain design features of tournaments in experiments, there exists another pro of the experimental method. The effort level analysed in empirical field studies has always to be approximated by some more or less adequate measures like the score in sport matches or firm performance measures. This is necessary because, for obvious reasons, one cannot exactly quantify the intensity of effort and the implicated cost of effort in a natural environment. In experiments, however, one is able to design strategic contexts where players’ decisions are quantifiable and thus, one can exactly measure how much effort participants are willing to exert in order to increase their chance of winning in different tournament settings. This may be done by modelling effort as a strategic decision like in theoretical studies or as a real effort task (see, for example, van Dijk et al., 2001). Thus, experiments appear to be an appropriate tool for the strategic analysis of tournament design. Naturally, we are aware that by choosing the experimental method, we are forced to boil down the real world setting to its very essentials—as it is always the case with economic modelling.

This paper aims to fill a gap in the literature which becomes visible from the overview given above: we present results from an experiment which is designed to analyse tournament size and prize structure in a systematic way. With only one exception, all experimental investigations on tournaments that we are aware of consider two agents and
thus implement only one winner and one loser prize. They focus on varying the prize spread, induce heterogeneity of agents or compare relative performance remuneration with other compensation schemes. The interplay between potentially competitive and/or cooperative behaviour of agents in tournaments, however, may crucially depend on the number of competitors and the promised rewards, i.e. the number of winner prizes. Thus, we investigate tournament size by implementing tournaments with two, three and six participants. Along a different dimension, we vary the prize structure, i.e. we implement tournaments of a given size with different numbers of winner prizes. On the one hand, this procedure enables us to analyse the effect of a variation of the fraction of winner prizes while keeping the size of the tournament constant. On the other hand, we can compare different tournament sizes with the same prize structure. Our aim is to model a very simple competitive situation in which participants are able to focus on the essential strategic considerations in a tournament, i.e. to exert more effort than others. Thus, in our experiment, we decided to abstract from individual random shocks known from other tournament models. Such shocks may disturb the perception of the pure competitive tournament situation at least if shocks are high in relation to the effort choices because high individual shocks are likely to make participants believe that effort has only a marginal effect on the outcome of a tournament. In a sense, our tournament model constitutes one extreme of the type of tournament models proposed by Lazear and Rosen (1981) in which the influence of the random component is reduced to zero. In addition, we inform the participants about the effort chosen by the other agents of the same tournament. This resembles the fact that agents very often observe effort quite accurately among themselves.

Besides our main focus on the amount of effort exerted under different tournament designs, there are still other questions that arise in the analysis. A puzzling observation made in some experimental studies (e.g. Bull et al., 1987) is that effort in tournaments is much more variable than in other compensation schemes, e.g. piece rate or team compensation. Thus, it is important to gain insight into how far the variability of effort...
may be influenced by different tournament designs. We derive a theoretical benchmark for the settings implemented in our experiment, which suggests that agents should either exert maximal effort or no effort at all. Therefore, in our study, we will have a closer look at the occurrences of non-participants\(^5\) and max-performers, which may to some extent explain variability of effort.

When we planned the experiment, we conjectured that agents would show a decreasing tendency to cooperate if the tournaments are repeated. This would mean that effort increases over time. Such an effect can be explained by at least two different reasons. The first one is that agents learn that they should exert higher effort to obtain the winner prize. Another explanation is that subjects are inclined to behave more cooperatively in the beginning to establish a profitable “social norm”. Of course, such strategic considerations are not applicable towards the end of the experiment, which may lead to higher effort in later rounds. In order to be able to differentiate between strategic behaviour and behavioural changes due to learning in our tournaments, we implemented a restart procedure.\(^6\) We are not aware of any other tournament experiment, which is conducted with a restart.

2. Model and experimental design

This section describes the strategic model which is reflected in our experiment. We focus on tournaments used in environments with a deterministic production technology. Hence, the output of an agent is in essence exclusively determined by his effort, and the employer is able to monitor the agents’ performance quite well. Such tournaments are often found in industrial production contexts where distortions like technical breakdowns are well observable rather than in contexts of salespersons whose success is strongly influenced by non-verifiable market conditions. In our experiment, we inform the participants of the other players’ decision after each round, which represents the fact that colleagues often know the performance of their competitors quite well. Thus, by abstracting from a random component and letting participants observe the others’ decisions, we induce a competitive environment which focuses on the very essentials of the strategic interaction in tournaments, i.e. the necessity to exert more effort than the competitors. We allow only for two types of prizes: a winner and a loser prize, i.e. we restrict our study to indivisible prizes like promotions.

2.1. Model

Our experiment is based on a non-cooperative game with complete information. A number of \(n \geq 2\) homogenous agents participate in a tournament. They simultaneously

\(^5\) In the context of an employment relationship, the behaviour of non-participants should not be interpreted as exerting no effort at all. Our tournament setting abstracts from a “baseline effort” which agents have to deliver in any case. Tournaments should provide incentives for effort on top.

\(^6\) Restart effects are known from public goods experiments (e.g. Andreoni, 1988; Croson, 1996). Average individual contributions start at a relatively high level and decrease over rounds. If the experiment is stopped and restarted after a short break, participants typically show again high contributions in the beginning with a decreasing tendency afterwards.
have to choose an effort level $e$ out of the integer set $\{0, e^{\text{max}}\}$. Making an effort $e$ produces cost which is given by the function $c(e)$ with $c(0) = 0$ and $c'(e) > 0$. The agents compete for $w < n$ winner prizes $M$, i.e. those $w$ agents receive a prize of $M$ who choose the $w$ highest effort levels. Correspondingly, there are $(n - w)$ loser prizes $m$ with $M > m \geq 0$. With $\Delta$, we denote the prize difference $\Delta = M - m$. If the winners cannot unambiguously be determined—because there are several agents who choose the same effort level—a fair random move selects the winners among them.

The following proposition gives a normative solution for those tournaments which are modelled in our experiment. It is shown that in pure strategy equilibria, only zero effort or full effort is made provided that the cost of one additional effort unit and the cost for $e^{\text{max}}$ are sufficiently small compared to the prize difference $\Delta$. To simplify the theoretical analysis, we assume that all agents are risk neutral and aim to maximise their expected payoff.

**Proposition 1:** For all $e'$ out of the integer interval $[1, e^{\text{max}}]$, let

$$c(e') - c(e' - 1) < \Delta/(w + 1).$$

Assume that there is a $k^*$ defined as the maximal $k$ out of the integers $1, \ldots, (n - w)$ such that

$$c(e^{\text{max}}) \leq w\Delta/(w + k).$$

Then in all Nash equilibria in pure strategies $(w + k^*)$, agents choose the maximal effort level $e^{\text{max}}$ and all others choose an effort of zero.

**Proof:** See Appendix A.

Condition (1) ensures that it is worth to exert an additional unit of effort, if this results in obtaining the winner prize compared to the situation in which the winner prizes are shared among oneself and $w$ other agents at a one-step lower effort level. Condition (2) guarantees that agents choose the maximal effort level when there are not too many other agents to share the winner prizes with. In all equilibria in pure strategies, agents either choose the maximal effort level, or they choose the minimal effort level of zero and receive the loser prize. It is obvious that all efforts between the minimal and the maximal effort level cannot be equilibrium strategies. To illustrate the intuition behind this proposition, consider the simplest case of a two-person tournament. If one agent chooses an effort below the maximal effort level, it is always the best reply to exert one more unit of effort than the other agent because of condition (1). If one iteratively applies this argument, one ends up in a situation where both agents choose the maximal effort level, and because condition (2) holds no agent has an incentive to choose an effort level of zero.

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7 Usually, continuous efforts are analysed in theoretical models. In an experiment, however, continuous effort choices cannot be implemented. Therefore, we have to derive a proposition for a discrete set of strategies. We limit the highest eligible effort level. A similar cap is analysed in Che and Gale (1998). This cap on effort can be interpreted as a restriction of working time enforced by a union or by legal institutions.

8 For simplicity, we concentrate on pure strategies. Note, however, that if effort is unrestricted, only equilibria in mixed strategies may exist in tournaments with no random shocks (Jost and Kräkel, 2002).
2.2. Experimental design

The experiment was conducted in the Laboratorium für experimentelle Wirtschaftsforschung at the University of Bonn. A total of 216 students of different disciplines were involved in the experiment, 36 in each treatment. Every candidate was only allowed to participate in one session. One session lasted for about 2 to 2.5 h. During the experiments, the payoffs were given in the fictitious currency “Taler” and were changed into DM after the experiment by a known exchange rate of 40 “Taler” per 1 DM. Payment was anonymous. In our experiment, we use a maximal effort level \( e^{\text{max}} = 100, c(e) = e^2/200 \) as the cost function, a winner prize \( M = 150 \), and a loser prize \( m = 50 \). With these parameters, the proposition is applicable to our six treatments HL \((n = 2, w = 1)\), 2HL \((n = 3, w = 2)\), H2L \((n = 3, w = 1)\), 2H4L \((n = 6, w = 2)\), 3H3L \((n = 6, w = 3)\), and 4H2L \((n = 6, w = 4)\).

**Corollary 1:** The pure strategy equilibrium solution of the games modelled in the treatments HL, 2HL, 3H3L and 4H2L prescribe that all agents show the maximum effort level. In a pure strategy equilibrium in treatment H2L, two agents and in treatment 2H4L four agents choose the maximum effort level and the remaining agents choose zero, i.e. they are non-participants.

The corollary follows immediately from the proposition derived in the last section and it reveals that the game theoretic analysis of our setting predicts an extreme behaviour of participants, i.e. they should either give everything or nothing. This all-or-nothing situation is especially interesting because it is quite essential for a designer of a tournament to know when and under which conditions participants exert maximal effort or are willing to give up and drop out of a contest. The essential characteristics of the six different treatments are summarised in Table 1. The treatments’ names indicate the number of winner (H for high) and loser (L for low) prizes. The column “\( p \)” denotes the fraction of winner prizes in a tournament, i.e. the number of winner prizes divided by the number of participants in a tournament.

<table>
<thead>
<tr>
<th>Treatment</th>
<th>No. of participants</th>
<th>No. of observations</th>
<th>Size</th>
<th>No. of winner prizes</th>
<th>Fraction of winner prizes ( p )</th>
<th>Equilibria in pure strategies</th>
</tr>
</thead>
<tbody>
<tr>
<td>HL</td>
<td>36</td>
<td>18</td>
<td>2</td>
<td>1</td>
<td>1/2</td>
<td>( (100, 100) )</td>
</tr>
<tr>
<td>H2L</td>
<td>36</td>
<td>12</td>
<td>3</td>
<td>1</td>
<td>1/3</td>
<td>( (0, 100, 100) )</td>
</tr>
<tr>
<td>2HL</td>
<td>36</td>
<td>12</td>
<td>3</td>
<td>2</td>
<td>2/3</td>
<td>( (100, 100, 100) )</td>
</tr>
<tr>
<td>2H4L</td>
<td>30</td>
<td>5</td>
<td>6</td>
<td>2</td>
<td>1/3</td>
<td>( (0, 0, 100, 100, 100, 100) )</td>
</tr>
<tr>
<td>3H3L</td>
<td>36</td>
<td>6</td>
<td>6</td>
<td>3</td>
<td>1/2</td>
<td>( (100, 100, 100, 100, 100) )</td>
</tr>
<tr>
<td>4H2L</td>
<td>36</td>
<td>6</td>
<td>6</td>
<td>4</td>
<td>2/3</td>
<td>( (100, 100, 100, 100, 100) )</td>
</tr>
</tbody>
</table>

* In treatment H2L, there are 3 asymmetric equilibria in pure strategies; in treatment 2H4L, there are 15 because each of the agent can be the one or one of the two who chooses the minimal effort 0.
Before starting the experiment, the instructions were read to all participants. The language was kept neutral, i.e. we did not use the term “effort” that had to be chosen but a “number”. Moreover, the terms “tournament” and “prize” were not mentioned. During the whole experiment, participants were not allowed to communicate with each other. They received a cost table and a payoff table. The cost table showed the cost for each number (effort), and the payoff table showed the payoff participants earned dependent on their own choice and the chosen number of the other participant(s). During the introduction, three examples were explained to illustrate the rules of the game.

Participants were randomly and anonymously matched to groups of two, three or six candidates, respectively. The group assignment was kept fix during the experiment. One session consisted of 20 rounds with 20 identical tournaments. Thus, we had 18, 12 or 6 (5) independent observations (see Table 1). In the beginning, the participants were only informed that there were 10 rounds of play, and afterwards, the experiment would continue in a way not revealed in advance. After the first 10 rounds, there was a short break in which we offered small snacks and something to drink. During the break, participants had to stay in their cubicles and were not allowed to talk. Afterwards, it was announced that another 10 rounds with the same group matching would be played. We implemented this schedule in order to analyse restart effects.

In their cubicles, participants found a decision sheet on which they had to write the number they had chosen. The sheets were given to the experimenters after each round. If at least two participants chose identical numbers and a winner/loser could not uniquely be determined, a fair chance move decided who received a winner prize. Participants were always returned the complete sheets on which they could find their own payoff and the chosen numbers and payoffs of all group members. On the decision sheet, there was room to fill in 10 rounds. After the break, a new decision sheet was passed out. The decision sheet for the previous 10 rounds was left in the cubicle. Thus, participants could easily trace the complete history of play.

### 3. Results

This section reports our findings with a special emphasis on two central features of tournament design: tournament size and prize structure. The former reflects the number of agents, which take part in a single tournament, the latter focuses on the fraction of winner prizes. Table 2 summarises the treatments with regard to these two features. Our design enables us to compare at least two treatments along each dimension: the six-agent tournament 3H3L is a triplicate of the two-person tournament HL and the

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9 Original instructions were written in German. A translation of the instructions is available under http://www.db-thueringen.de/servlets/DerivateServlet/Derivate-1352/instructions.pdf.

10 The numbers for the examples were chosen randomly by a participant before the introduction started. By doing so, we tried to keep possible suggestive influences as small as possible.

11 In one treatment, we have only five observations because after two rounds one participant decided not to continue.
tournaments 4H2L and 2H4L are duplicates of the two three-person tournaments 2HL and H2L. In the six-person tournaments, we can even compare all three treatments each implementing a different prize structure. We start our analysis by focusing on the aggregated behaviour over all 20 rounds. We then have a look at the development of play over rounds including the analysis of restart effects. Furthermore, we describe the behaviour in the last round to check the robustness of our findings.

3.1. Behaviour aggregated over all rounds

3.1.1. Effort

Let us start with the following question: Does the amount of effort systematically vary with tournament size or/and prize structure? According to the theoretical prediction, in four of the six treatments, all agents choose the same maximal effort of 100. In the two treatments with asymmetric equilibria, only 2/3 of the agents exert the maximal effort while the remaining agents show no effort (see Table 1). Thus, from a theoretical point of view, there should be a difference in the amount of effort due to prize structure. Additionally, one can argue that the behaviour of agents is likely to differ between treatments with \( p = \frac{1}{2} \) and \( p = \frac{2}{3} \). In treatments with \( p = \frac{1}{2} \) in equilibrium, one of the agents is indifferent between choosing the minimal effort 0 and the maximal effort

<table>
<thead>
<tr>
<th>Sizes</th>
<th>Fraction of winner prizes</th>
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<tbody>
<tr>
<td>2</td>
<td>1/3</td>
</tr>
<tr>
<td>3</td>
<td>H2L</td>
</tr>
<tr>
<td>6</td>
<td>2H4L, 3H3L, 4H2L</td>
</tr>
</tbody>
</table>

![Fig. 1. Average effort per round in six-person tournaments.](image-url)
Thus, we hypothesise that the amount of effort chosen in the treatments with $p = 1/2$ lies in between the amount of effort exerted in treatments with $p = 1/3$ and $p = 2/3$. Visual inspection of Figs. 1 and 2 already suggests one of our main results.

**Result 1:** The average effort increases with the number of winner prizes.

Table 3 shows the amount of average effort per treatment aggregated over all 20 rounds. We find that the average effort varies systematically with the proportion of winner prizes $p$: in the six-person tournaments, average effort over all rounds increases with the proportion of winner prizes (Jonckheere–Terpstra test, $0.001\%$ level, one-tailed). Moreover, the data show that average effort in six-person tournaments varies weakly significantly between the two treatments with $p = 1/2$ and $p = 2/3$ (Mann–Whitney $U$-Test, significance at 10\% level, one-tailed). Also, the three-person tournaments can clearly be ordered according to the proportion of winner prizes. The null hypothesis that the average efforts in 2HL and H2L are equal can be rejected. The average effort is significantly higher in 2HL than in H2L (Mann–Whitney $U$-Test, significance at 1\% level, one-tailed). Thus, our experimental results confirm our theoretical finding that the average effort in a tournament is higher if there is a high proportion of winner prizes.

When comparing treatments with the same proportion of winner prizes, we find that effort does not differ significantly. The only exception is that effort in HL is significantly lower than in 3H3L, which indicates an effect resulting from different tournament sizes (Mann–Whitney $U$-Test, 1\% level, one-tailed). Table 3 reveals that average effort in HL

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12 Condition (2) from the above proposition is fulfilled with equality in treatments with $p = 1/2$ and $k^* = n/2$.

13 The Jonckheere–Terpstra test is a non-parametric test for ordered differences among classes. The alternative hypothesis assumes a certain ordering of the medians of $k$ statistically independent samples. In the six-person tournaments, all average efforts—each of a statistically independent observation from a treatment with the same proportion of winner prizes—are assigned to one class.
(51.6) is the lowest of all treatments. This is mainly due to extreme collusion in some groups which appears to be surprisingly stable: agents choose zero effort in almost each round and let chance decide who is going to receive the winner and the loser prize. Similar collusion effects cannot be observed in the other treatments with more than two agents. Obviously, full cooperation is more likely in very small tournaments where agents are able to coordinate on low efforts.

### 3.1.2. Effect of price structure on non-participants and max-performers

When looking at the individual efforts in each treatment, it becomes obvious that agents tend to choose either very high or very low efforts (see Fig. 3), which illustrates the distribution of all efforts over 20 rounds. On the one hand, some agents clearly strive for winning the tournament by exerting very high effort, i.e. in most cases, the maximal effort of 100. On the other hand, some agents seem to quit the contest by choosing very low effort while minimising costs. This polarisation is in line with the theoretical prediction according to our proposition. Therefore, we categorise the choice of an effort between 0 and 4 as nonparticipant behaviour. Due to necessary rounding of the cost function, the effort levels in this interval are the ones which do not cause any costs for the agents. On the other hand, we analyse the fraction of participants who choose the maximal effort. Those we categorise

![Fig. 3. Number of chosen efforts over 20 rounds.](image-url)
as max-performers. Thus, we have to look whether differences of average efforts between the treatments may at least to some extent be explained by the occurrences of non-participants and max-performers. This would reflect the results of our proposition. According to our theoretical prediction, we hypothesise that the number of non-participants varies with the proportion of winner prizes. We expect most non-participants in treatments with $p = 1/3$, whereas we predict least non-participants in treatments with $p = 2/3$. The number of non-participants in treatments with $p = 1/2$ should lie somewhere in between because in equilibrium, one agent is always indifferent between participating by choosing 100 and not participating by choosing 0. The analysis of the data gives us our second result:

**Result 2:** The number of non-participants decreases with the number of winner prizes.

The lower the proportion of winner prizes in six-person tournaments, the more agents choose not to participate by exerting very low efforts (Jonckheere–Terpstra test, 0.001% level, one-tailed). We find evidence for analogous behaviour in the other treatments: there are significantly less non-participants in the treatments with $p = 1/3$ than in treatments with $p = 1/3$ and $p = 1/2$ (see Table 4).

Table 3 also states the fraction of max-performers. Given our theoretic prediction, we should expect less max-performers in the treatments with $p = 1/3$. The results indicate, however, no systematic difference with respect to the variation of winner prizes. A tournament designer should be interested to know how the number of non-participants relates to the number of max-performers. According to our proposition, the fraction of non-participants divided by the fraction of max-performers should be 0.5 for treatments with $p = 1/3$ and zero for all other treatments because no non-participants should be observed. Table 3 shows that the average ratio almost exactly meets the prediction of 0.5 in H2L and in 2H4L. Moreover, the ratios of the treatments with $p = 1/3$ are significantly higher than in almost all other treatments. The only exception is the two-person tournament HL where we observe even more non-participants than max-performers due to the extreme collusive behaviour. Thus, the analysis of the fraction of non-participants and the ratio of non-participants and max-performers appear qualitatively to be in line with our proposition.

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<tr>
<th></th>
<th>H2L ($p = 1/3$), NP = 0.24 (%)</th>
<th>2H4L ($p = 1/3$), NP = 0.30 (%)</th>
<th>HL ($p = 1/2$), NP = 0.37 (%)</th>
<th>3H3L ($p = 1/2$), NP = 0.18 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2HL ($p = 2/3$), NP = 0.09</td>
<td>0.1</td>
<td>1</td>
<td>0.1</td>
<td>1</td>
</tr>
<tr>
<td>4H2L ($p = 2/3$), NP = 0.10</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

14 There are more non-participants per max-performer in H2L than in the other treatments, tested with Mann–Whitney U-Test on the following significance levels: 2HL (1%), 3H3L (weakly significant at 10%), 4H2L (5%). This ratio is also higher for 2H4L than for 2HL (5%), 3H3L (5%) and 4H2L (5%).

15 The fraction of non-participants per max-performer in HL is at least weakly significantly higher than in all other treatments. Applying the Mann–Whitney U-Test yields the following significance levels: H2L (weakly significant at 10%), 2H4L (weakly significant at 10%), 3H3L (5%), 2HL (0.1%) and 4H2L (1%).
3.1.3. Effect of tournament size on non-participants and max-performers

Table 3 shows that for a given proportion of winner prizes, there tend to be slightly more non-participants in the six-person tournaments than in the three-person tournaments. The fraction of non-participants in the two-person tournament HL is extraordinarily high because of the colluding agents in some groups. Comparing the number of max-performers, we find that there are weakly significantly more max-performers in the large tournament 2H4L than in H2L (Mann–Whitney U-Test, 10% level, one-tailed) and also more max-performers in 3H3L than in HL (Mann–Whitney U-Test, 10% level, one-tailed). This seems to indicate that efforts are more polarised in large tournaments than in smaller ones. If one assumes that competition in a tournament becomes stronger with a growing number of participants, one can hypothesise that pressure to exert either very high or very low effort is stronger in large tournaments than in smaller ones. In small tournaments, the competitive pressure might be weaker because agents might more easily coordinate on efforts in between, or—in the extreme case of a two-person tournament—they might even coordinate on very low efforts.

3.1.4. Cost of effort for the employer

From the employer’s point of view, cost of effort is a predominant key figure when designing a tournament. Table 5 shows the cost of effort per treatment, which is the sum of prizes divided by the total effort exerted in each group. The costs in HL are significantly higher than in all other treatments. Beside treatment HL, the cost of effort in 2HL and 4H2L is clearly highest. Treatment 2HL is the second most expensive treatment regarding the cost of effort.

3.2. Development of behaviour over rounds

3.2.1. Variability of effort

An important behavioural characteristic of a tournament design is the induced variability of effort. Let us have a look at the average standard deviation of efforts over

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16 Applying the Mann–Whitney U-Test yields the following significance levels: HL and H2L (0.1%), 2H4L (1%), 3H3L (1%), 2HL (5%), and 4H2L (weakly significant at 10%).

17 Comparison with treatment 2HL reveals the following weak significance levels: 10% for H2L, 10% for 2H4L, 10% for 3H3L, comparison of treatments 2HL and 4H2L yields no significant differences, all Mann–Whitney U-Test.
20 rounds per agent. Following our argumentation from above, we conjecture that the variability of effort varies with the proportion of winner prizes. In the treatments with \( p = 1/3 \), one can argue that it is difficult for agents to coordinate on one of the asymmetric equilibria which may lead to a high variability of effort. In the treatments with \( p = 1/2 \), agents are indifferent between choosing the minimal and the maximal effort. Thus, variability of effort is expected to be higher than in the treatments with \( p = 2/3 \).

**Result 3:** The variability of effort over all rounds decreases with the number of winner prizes.

In the six-person tournaments, the variability of effort decreases with the number of winner prizes (Jonckheere–Terpstra test, 0.1% level, one-tailed). A statistical comparison of treatments is given in Table 6. In accordance with our conjecture, the treatments with \( p = 2/3 \) show a significantly lower variability of effort over all rounds than in the treatments with \( p = 1/3 \) and \( p = 1/2 \).

Moreover, variability of effort in H2L is unambiguously higher than in all other treatments. The comparison of the remaining treatments shows that there is essentially no difference at a conventional significance level between the treatments with the same proportion of winner prizes.

3.2.2. Restart effect

As described in Section 2, we told participants in the beginning of the experiment that they would play 10 rounds (part I) and that the experiment would continue in an unknown way. After the break, we informed them that another 10 rounds (part II) would follow without changing the group composition of the tournaments. We implemented this procedure to be able to differentiate between strategic behaviour and behavioural changes due to learning in tournaments.

The two treatments HL and 4H2L are the only ones in which we observe a statistically confirmed increasing trend of average effort in rounds 1–10, i.e. part I: according to the Binomial test, the Spearman rank-correlation coefficients are at least weakly significantly more often positive than negative (Binomial test, 10% level and 5% level, both one-sided). In both treatments, the same is true for part II (Binomial test, 5% level and 5% level, both one-sided). Thus, in both treatments, we find that effort increases over the

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Table 6
Comparison of average standard deviations (S.D.) in effort per agent (significance levels, one-tailed, Mann–Whitney U-Test)

<table>
<thead>
<tr>
<th></th>
<th>H2L ( (p = 1/3) ), S.D. = 41.9 (%)</th>
<th>2H4L ( (p = 1/3) ), S.D. = 38.6 (%)</th>
<th>HL ( (p = 1/2) ), S.D. = 34.3 (%)</th>
<th>3H3L ( (p = 1/2) ), S.D. = 35.8 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2HL ( (p = 2/3) ), S.D. = 30.7</td>
<td>0.1</td>
<td>5</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>4H2L ( (p = 2/3) ), S.D. = 30.1</td>
<td>0.1</td>
<td>1</td>
<td>10</td>
<td>1</td>
</tr>
</tbody>
</table>

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18 The Spearman-rank correlation coefficient is determined for each observation. The plus or minus sign of the coefficient is then taken as an indicator of an increasing or decreasing trend. Then the Binomial test with an event probability of 0.5 is applied to check whether the proportion of positive coefficients is higher than random.
rounds in each part. Interestingly, in both treatments, average effort in part I is significantly higher than effort in round 11 (Wilcoxon-Signed Rank test, 5% level and 0.5% level, both one-tailed). Thus, effort in the two treatments HL and 4H2L increases over the first 10 rounds, drops down substantially after the restart and in part II increases again towards the equilibrium (compare the corresponding graphs in Figs. 1 and 2). We conclude that learning alone cannot explain the observed increase of effort. A similarly strong increase in effort, however, cannot be found in the other treatments.

3.3. Last round behaviour

Behaviour in the last round is of special interest because participants have already gained considerable experience. Moreover, behaviour in the last round is not prone to strategic considerations regarding subsequent interaction. Analogous to Section 3.1, we check for systematic variations with respect to the two central design features: tournament size and prize structure. Basically, we can confirm most results, but sometimes effects are weaker in the last round.

3.3.1. Effort

Analogous to Result 1, we find that the average effort in the last round in six-person tournaments increases with the number of winner prizes (Jonckheere–Terpstra test, 0.5% level, one-tailed). Table 7 shows the amount of average effort per treatment in the last round. Basically, our findings in smaller tournaments are also in line with the effect due to the prize structure described in Result 1: at a conventional significance level, no difference can be found between treatments with the same proportion of winner prizes. In addition, the average effort in 2HL is higher than in 2H4L and HL (Mann–Whitney $U$-Test, significant at 5% and weakly significant at 10% level, one-tailed), and the average effort in 4H2L is significantly higher than in all other treatments except 2HL.19 Interestingly, however, the efforts exerted in H2L and 2HL do not significantly differ. Prize structure seems to play an important role regarding the amount of effort exerted, but the effect is more pronounced in larger tournaments than in smaller ones.

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19 Applying the Mann–Whitney $U$-Test yields the following significance levels: 4H2L and H2L (5%), 2H4L (1%), HL (5%) and 3H3L (weakly significant at 10%).
3.3.2. Non-participants and max-performers

Analogous to Result 2 stated for behaviour aggregated over all rounds, the number of non-participants in the last round also decreases with the number of winner prizes in six-person tournaments (Jonckheere–Terpstra test, 0.5% level, one-tailed). The lower the proportion of winner prizes in six-person tournaments, the more agents choose not to participate by exerting very low efforts. Table 7 already indicates that there tend to be relatively few non-participants in treatments with \( p = 2/3 \).\(^{20}\) Again, there is no significant difference between the treatments with the same proportion of winner prizes. Additionally, there is no difference at any conventional significance level regarding the number of non-participants between the two three-person tournaments H2L and 2HL.

The fraction of max-performers in the last round increases with the proportion of winner prizes in six-person tournaments (Jonckheere–Terpstra test, 5% level, one-tailed). Interestingly, the average number of max-performers in 4H2L and 2HL is significantly higher in the last round than aggregated over all rounds (Wilcoxon-Signed Rank test, both 5% level, one-tailed). Thus, in treatments with a high proportion of winner prizes, participants tend to choose the maximal effort in the last round.\(^{21}\) On the contrary, agents in 2H4L seem to prefer to drop out of the contest by choosing minimal effort in the last round more often compared to the number of non-participants aggregated over all rounds. Both behavioural changes in the last round are reflected in the ratios of non-participants and max-performers (Table 7). The ratio is smallest in 4H2L and highest in 2H4L.\(^{22}\)

3.3.3. Cost of effort for the employer

We find no difference at a conventional significance level between the treatments regarding cost of effort in the last round (the only exception is the weakly significant difference between HL and H2L; Mann–Whitney \( U \)-Test, 10% level, one-tailed).

4. Conclusion

This paper experimentally investigates different design alternatives for rank order tournaments. The analysis focuses on two central setup features: tournament size and prize structure. Along the first dimension, we implemented tournaments with two, three, and six agents. The prize structure is varied by the number of winner prizes, i.e. we consider fractions of 1/3, 1/2, and 2/3. Tournaments are modelled such that agents simultaneously choose their effort out of a given interval. Agents have to bear the cost for the chosen effort according to a convex cost function. In our setting, output equals effort. Participants take repeatedly part in tournaments of the same design, which reflects the situation in

\(^{20}\) The fraction of non-participants is higher in 2H4L than in 2HL and 4H2L (Mann–Whitney \( U \)-Test, 5% level and 1% level, both one-tailed) and higher in 3H3L than in 2HL and 4H2L (Mann–Whitney \( U \)-Test, 10% level (weakly significant) and 5% level, both one-tailed).

\(^{21}\) This observation is in line with the restart effect in 4H2L. Comparing the number of max-performers in the last round of part I and the first round of part II, we find that there are more max-performers before restarting the experiment than after the restart (Wilcoxon-Signed Rank test, 5% level, one-tailed).

\(^{22}\) In six-person tournaments, the ratio decreases with the number of winner prizes (Jonckheere–Terpstra test, 0.5% level, one-tailed).
organizations in which tournaments are frequently used as incentive schemes. After each tournament round, participants are informed about the output of other agents.

As a benchmark, we derive a normative solution for the type of tournaments we consider. Theory predicts that all agents should exert maximal effort if the fraction of winner prizes is 1/2 or 2/3. Interestingly, however, if the fraction is lower, e.g. 1/3, predicted effort is extremely bipartite, i.e. 2/3 of the agents should choose maximal effort, whereas 1/3 should quit the tournament by showing no effort at all. Qualitatively, we find these results supported by our data: the prize structure has an essential influence on the amount of effort exerted in our tournaments. The higher the proportion of winner prizes, the higher is the average effort. One might conjecture that this observation is equivalent to the well-known result from various tournament studies that effort increases with the prize spread. In fact, from a theoretical point of view, a higher fraction of winner prizes provides equivalent incentives as an increase in the winner prize in our tournament model. Note, however, that one should have doubts whether this equivalence carries over to empirical or experimental proof because the equivalence is grounded on the comparison of expected values of winning the tournament without taking the variance into account. Psychological motivations, like relative deprivation,\(^{23}\) may well lead to different effort levels in different tournaments although from a participant’s point of view, the expected value of winning is the same, e.g. it should not be taken for granted that effort exerting behaviour in a tournament with a single, very high winner prize is the same as in a tournament with many smaller winner prizes (for a given number of participants). An experimental analysis of this question promises fruitful insights for future research.

It is worth noting that our findings are opposite to the observations of Orrison et al. (1997). They find that effort decreases in a tournament with a higher proportion of winner prizes and conclude that agents are not willing to exert effort if there is a high probability of receiving a winner prize. Since their tournament model is different from ours—note that they do not implement an identity of effort and output, rather in their study output is determined by effort and a considerable random component—it is not devious to assume that our, at first glance contradicting results, are due to different tournament settings. The comparison of the two experiments opens further roads for future research: How do incentives in tournaments vary with the observability of effort? Is there a “turning point” regarding the fraction of winner prizes if it approaches fixed pay, in the sense that effort increases if the fraction is lower and decreases if it is higher than this point (note that Orrison, Schotter and Weigelt implement a higher fraction of winner prizes, i.e. a fraction of 3/4)?

We categorise subjects who choose very low effort as non-participants and those who exert maximal effort as max-performers. We find a systematic difference between treatments regarding the varying fraction of winner prizes, i.e. the number of non-participants decreases with the number of winner prizes. Furthermore, the data show that the relation of non-participants per max-performer is significantly worse in the treatments with the lowest fraction of winner prizes. It is also indicated that agents are rather polarised in large tournaments than in smaller ones by the observation that there slightly tend to be more

\(^{23}\) As an example for the influence of relative deprivation in a tournament context, see Kräkel (2000).
max-performers in large tournaments. Besides the collusion in two-person tournaments, this is the only size effect we find.

An analysis of the development of behaviour reveals that variability of effort also varies with the prize structure, i.e. variability decreases with the number of winner prizes. Additionally, in two treatments, there is an increasing tendency of effort over rounds. This might be due to learning or due to the strategic consideration that agents try to establish a more collusive behaviour in the beginning, which is left behind towards the end. In order to find out which explanation better fits the data, we implemented a restart procedure, i.e. we restarted the experiment after 10 rounds. In both treatments in which we observe increasing effort in part I, we find that effort decreases substantially after the restart and increases again during part II. This leads us to the conclusion that the explanation of strategic considerations applies more than learning.

One can argue that an employer might prefer a tournament which induces agents to exert high efforts with low variability and moderate costs. Costs of effort aggregated over all rounds are significantly different between treatments. In the two-person tournament, costs are outrageously high because some groups manage to establish a stable collusion. Apart from that, the data show that the treatments with a higher fraction of winner prizes are most expensive. However, those are the treatments with lower variability of efforts. Thus, an employer has to consider carefully the trade-off between higher costs and higher variability.

To conclude, we find that both fundamental features of tournament design—size and prize structure—but especially the latter seem to determine behaviour of agents. When setting up a tournament, enough winner prizes should be provided to achieve high effort with moderate variability. Especially, two-person tournaments might be prone to collusion. However, the present study raises several questions for further research. It is unclear why an increasing tendency of effort over rounds is only observed in two of our treatments. Another topic arises from our observation of non-participants. Although we modelled the agents identically, equilibrium behaviour appeared to be asymmetric in some treatments. The question arises whether the number of non-participants increases if different types of agents compete in our tournament setting, e.g. if some agents have a higher cost function. Furthermore, in this study, we do not consider the principal—who is the one who designs the tournament—to take an active role in the game. It would be quite interesting to introduce a player who is able to deliberately choose between different tournament design features.

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Appendix A

Proof of Proposition 1: Let \( e = (e_1, e_2, \ldots, e_n) \) be a strategy combination, i.e. one effort level for each agent, and for agent \( i \) out of \( 1, \ldots, n \), let \( e_i = (e_1, e_2, \ldots, e_{i-1}, e_{i+1}, \ldots, e_n) \) denote the corresponding \( i \)-incomplete strategy combination. Let \( \phi(e_{-i}, j) \) with \( j \) out of \( 1, \ldots, n \) be defined as the \( j \)th highest effort level of \( e_{-i} \) and \( \phi(e_{-i}, n) = 0 \).

With this notation, the best response function for an agent \( i \) can be characterised as follows:

\[
e_i(e_{-i}) = \begin{cases} 
\phi(e_{-i}, w) + 1, & \text{if } 0 \leq \phi(e_{-i}, w) < e_{\text{max}} \\
e_{\text{max}}, & \text{if } \phi(e_{-i}, w) = e_{\text{max}} \text{ and } \phi(e_{-i}, w + k*) < e_{\text{max}} \\
0, & \text{otherwise}
\end{cases}
\]

This can be seen by the following arguments.

(i) Let us assume that \( 0 \leq \phi(e_{-i}, w) < e_{\text{max}} \).

If agent \( i \) chooses an effort level which is less than \( \phi(e_{-i}, w) \), she/he receives a loser prize \( m \). If she/he chooses \( \phi(e_{-i}, w) \) as well then she/he can expect is \( w\Delta / (w+1) + m \), namely if all \( w \) agents with the highest effort also choose \( \phi(e_{-i}, w) \). On the other hand, if agent \( i \) chooses \( \phi(e_{-i}, w) + 1 \), she/he receives the winner prize \( M \) for sure with minimal cost.

With \( M - c(e'' + 1) > w\Delta/(w+1) + m - c(e'') \iff \Delta/(w+1) > c(e'' + 1) - c(e'') \), it becomes clear that agent \( i \) should choose \( \phi(e_{-i}, w) + 1 \), because the latter inequality is true for all \( e'' \) out of \( 0, \ldots, e_{\text{max}} - 1 \) as assumed in Eq. (1).

(ii) Now let us assume that if \( \phi(e_{-i}, w) = e_{\text{max}} \) and \( \phi(e_{-i}, w + k*) < e_{\text{max}} \) which means that at least \( w \) agents but not more than \( (w+k*-1) \) other agents choose \( e_{\text{max}} \).

If agent \( i \) chooses an effort level less than \( e_{\text{max}} \), she/he receives the loser price \( m \). If she/he chooses \( e_{\text{max}} \), she/he receives at least \( w\Delta/(w+k*) + m \). Thus, it is enough to show that the condition \( m - c(0) \leq w\Delta/(w+k*) + m - c(e_{\text{max}}) \) is true. This is the case because of Eq. (2).

(iii) If more than \( (w+k*-1) \) other agents choose \( e_{\text{max}} \), then by choosing also \( e_{\text{max}} \), agent \( i \) receives at most \( w\Delta/(w+k*+1) + m \). If she/he chooses 0, she/he receives the loser prize \( m \) which is more than what she/he receives by choosing \( e_{\text{max}} \) because \( k* \) is chosen maximal in condition (2).

Given the best response function (3) which is identical for all agents, it is clear that in a Nash equilibrium \((w+k*)\), agents choose the maximal effort level \( e_{\text{max}} \) and all others choose an effort of zero.

\[\square\]

References


