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New TOA Estimators within Energy-based Receivers under Realistic UWB Channel Statistics

J. Youssef, B. Denis, C. Godin, S. Lesecq
CEA-Leti / Minatec
17 rue des Martyrs, 38054 Grenoble Cedex 09, France
E-mails: [joe.youssef, benoit.denis, christelle.godin, suzanne.lesecq]@cea.fr

Abstract—The Impulse Radio - Ultra Wideband (IR-UWB) technology, which benefits from unprecedented ranging capabilities through precise Time of Arrival (TOA) estimation, is now widely considered as a credible solution for Location and Tracking (LT) applications in Wireless Sensor Networks (WSN). In this context, low-complexity receiver architectures such as Energy Detectors (ED) have been favored for the past years. Recent methods have also been proposed to determine ED thresholds for optimal leading-edge detection under simplified channel assumptions. Nevertheless, more realistic indoor multipath profiles still make threshold-based TOA estimation challenging with such receivers. In this paper, we present two new Bayesian TOA estimators that rely on the overall energy profile available at the output of the ED, as well as on realistic channel statistics (typically that of the IEEE 802.15.4a standard). Simulation results are provided for performance comparison with two popular threshold-based estimators, showing the robustness of the new methods at practical Signal to Noise Ratio (SNR) values.

Index Terms—Bayesian Estimation, Energy Detection, Impulse Radio, Non-coherent Receivers, Ranging, Time of Arrival, Ultra Wideband, Wireless Sensor Networks

I. INTRODUCTION

Nowadays, more and more wireless communication systems are able to retrieve location-dependent information, which is a key requirement of numerous emerging applications. For instance, location-enabled Wireless Local/Personal Area Networks (WLAN/WPAN) or Wireless Sensor Networks (WSN) have been considered in indoor environments for smart homes, assisted navigation or security purposes.

Generally speaking, inter-devices distance measurements can be used to locate blind or mobile nodes. Hence, due to their high ranging precision and relatively low power consumption, Impulse Radio - Ultra WideBand (IR-UWB) systems have proved their relevance in the Low Data Rate - Location and Tracking (LDR-LT) WSN context (e.g. [1] - [4]), as put forward in the IEEE 802.15.4a standard (e.g. [5]).

Besides, many low complexity ranging techniques have also been described in the recent literature. As an example in [6], various UWB Time Of Arrival (TOA) estimation algorithms are compared in terms of complexity and performance. Complexity is usually related to hardware capability (e.g. sampling rate), processing time and computational complexity, whereas performance can be theoretically assessed by deriving lower bounds for the estimation errors with unbiased estimators (e.g. [7]). By computing the Mean Square Error (MSE) or Mean Absolute Error (MAE) of the TOA estimation over simulated realistic channels (e.g. [7], [8]), one can even more concretely compare the performance of different TOA estimators, highlighting practical estimation biases.

The stringent complexity requirements imposed in WSN naturally favor non-coherent receivers such as Energy Detectors (ED) on the one hand, as well as simple threshold-based detection algorithms for TOA estimation on the other hand (e.g. [6], [7]). Unfortunately, under realistic indoor channels, TOA estimation through threshold-based ED still strongly suffers from overlapping Multipath Components (MPC) in case of dense received profiles. More particularly, due to the high energy of MPC or further noise components, threshold-based methods can introduce significant errors into TOA estimates. One identified weakness of these threshold-based methods is that they do not benefit from the whole MPC profile observed at the receiver side, but they only depend on the first component that is expected to exceed the threshold.

In this paper we propose two TOA estimators applying to IR-UWB ED receivers, considering prior knowledge of a few indoor channel parameters (e.g. power decay and path amplitude statistics). These solutions can exploit all the received MPC to consolidate the final TOA estimate, and hence, they claim to enable more accurate ranging. In particular, taking the energy samples collected over the receiver time window as available observations, we derive analytical expressions for both the Maximum Likelihood (ML) and the Minimum Mean Square Error (MMSE) TOA estimates under exact channel statistics instead of using the Gaussian approximation to model the energy samples like in [9]. Moreover, unlike the procedure proposed in [10], we do not estimate channel parameters and we do not either consider simplified Gaussian fading, but more representative Nakagami-m fading as recommended in the IEEE 802.15.4a model (e.g. [11]). We also compare the proposed estimators with two other popular threshold-based schemes, namely a) an estimator based on a fixed threshold that is set to minimize the MSE according to the channel model, and b) an estimator based on a normalized threshold that depends on the difference between the minimum and the maximum energy samples collected on the receiver side. All the estimators are evaluated for different kinds of channel, including the complete IEEE 802.15.4a reference model.

The paper is structured as follows. In Section II, we recall the main specificities of existing statistical UWB channel models (e.g. [11], [12]), focusing mostly on the IEEE 802.15.4a standard. In Section III, the energy samples collected on the ED receiver side are modeled as well, along with related decision statistics. Then, classical threshold-based strategies are briefly discussed in Section IV. In Section V, we develop analytical expressions for the two new TOA estimators, which are subsequently evaluated and compared with classical schemes in Section VI. Finally, Section VII concludes the paper.

II. STATISTICAL UWB CHANNEL MODELS

The received UWB multipath signal $r(t)$ is usually modeled as a linear combination of $L_p$ delayed replicas of a transmitted
pulse \( p(t) \) of duration \( T_p \) and unitary energy \( E_p \), as follows

\[
r(t) = \sum_{i=1}^{L_p} \alpha_i p(t - \tau_i - \tau_{TOA}) + n(t)
\]

where \( \tau_i \) and \( \alpha_i = \beta_i \exp j \Phi_i \) are the relative excess delay and the complex gain of the \( i^{th} \) multipath component, with \( \beta_i \) and \( \Phi_i \) denoting the path amplitude and phase respectively. \( \tau_{TOA} \) is the ideal Time of Arrival (TOA) associated with \( d/c \), where \( d \) is the distance between the transmitter and the receiver.

The \( i^{th} \) path amplitude \( \beta_i \) is a Nakagami-\( m \) random variable (r.v.) with parameters \( m = m_i \) and \( \mathbb{E}[\beta_i^2] = m_i \). The received signal is filtered with an ideal Bandpass Filter (BPF) of bandwidth \( W \) to eliminate the out-of-band noise and interferers. The zero-mean Additive White Gaussian Noise (AWGN) with double-sided power spectral density \( N_0/2 \), which is also filtered in the band of interest, is denoted as \( n(t) \) with a variance \( \sigma_n^2 \).

We also define the SNR related to the average energy of the first path and noise power as \( \text{SNR} = \frac{E_p^2}{\sigma_n^2} \), where \( E_p \) is the expected energy value of the First Arrival Path (FAP).

In [12] the Average Power Delay Profile (APDP) is modeled by a single exponential decay for a 2ns sampling period, with a randomly distributed decay constant \( \epsilon \). We depict this Channel Model (CM) by CM0-\( m \), where \( m \) denotes the degree of freedom of the Nakagami-\( m \) r.v., whereas CM\( i \) with \( i \geq 1 \) denotes the channel models defined in the IEEE 802.15.4a (e.g. [11]).

### III. Energy of the Received Signal

On the receiver side, the overall observation time \( T \) is divided into \( N \) time slots of length \( \Delta \approx T_p \) and \( L \Delta \) is the total channel length (including null channel taps). In the following analysis, for the sake of simplicity, we assume non-overlapping replicas of the transmitted pulse so that \( L_p \leq L \), as well as statistically independent noise samples, whenever they are taken in different slots. Finally, without loss of generality we consider that \( \tau_{TOA} \sim \mathcal{U}(0,T) \), where \( \mathcal{U}(\cdot) \) denotes the uniform distribution.

Note that the intervals \( [0, \tau_{TOA}] \) and \( [\tau_{TOA} + L \Delta, T] \), corresponding respectively to the first \( N_{fn} = \lceil \tau_{TOA}/\Delta \rceil \) slots and the last \( N_{ln} = N - \lfloor \tau_{TOA}/\Delta \rfloor - L \) slots contain only noise signal, where \( \lfloor \cdot \rfloor \) denotes the floor function. The remaining interval \( [\tau_{TOA}, \tau_{TOA} + L \Delta] \) may contain dense multipath echoes in addition to noise.

The slots are numbered starting from slot 1 at \( t = 0 \), to slot \( N \) at \( t = T \). Let \( k_{TOA} \) be the slot index to be ideally estimated (i.e. chosen among \( N \) possible slots), which actually contains \( \tau_{TOA} \). Then slots \( \{k_{TOA} - 1, \ldots, k_{TOA} + L - 1\} \) correspond to the multipath region, whereas slots \( \{1, 2, \ldots, k_{TOA} - 1\} \) and \( \{k_{TOA} + L, \ldots, N\} \) correspond to the region only affected by noise.

The energy samples on the output of the energy receiver in Figure 1 can be written at each time slot as follows

\[
\xi_k = \int_{(k-1)\Delta}^{k\Delta} |r(t)|^2 \, dt
\]

According to [8] and under the AWGN assumption, the random integrator output follows a centered chisquare distribution for the noise-only components, or a non-centered chisquare distribution for signal components. Thus, for \( k = \{1, \ldots, k_{TOA} - 1, k_{TOA} + L, \ldots, N\} \), \( \xi_k \) are centered Chi-square r.v. with \( V = 2W\Delta \) degrees of freedom and for \( k = \{k_{TOA}, \ldots, k_{TOA} + L - 1\} \), \( \xi_k \) are non-centered Chi square r.v. still with \( V \) degrees of freedom and a central parameter \( E_p\beta_k^2 \) that represents the useful energy of the signal in slot \( k \). Without loss of generality, we assume that \( E_p = 1 \) here. Note that if the multipath is not resolvable, the following analysis gives a lower bound on the achievable performance.

### IV. Classical Threshold-Based TOA Estimation

Based on the energy-based receiver architecture described above, low-complexity TOA estimation can be directly performed without accurate timing, pulse shaping or prior channel estimation. Indeed, once the energy samples are collected over the observation window, TOA estimation can be considered as a problem of FAP detection in noise.

One first trivial way to estimate TOA is to select an appropriate threshold \( v \). The index of the first energy sample exceeding the threshold is associated to the TOA estimate [7]. We define the Threshold-to-Noise Ratio (TNR) as

\[
\text{TNR} = \frac{v}{\text{SNR}}
\]

where \( v \) is the Cumulative Density Function (CDF) of the chi-square distribution.

The probability of crossing the threshold for each of the remaining \( L \) slots can be computed as the average over the path amplitude of the conditional probability:

\[
P(\xi_k < v) = \mathbb{E}_{\beta_k}[P(\xi_k < v/\beta_k)]
\]

with

\[
P(\xi_k < v/\beta_k) = P_{\text{ncch}}(v/\beta_k)
\]

where \( P_{\text{ncch}} \) is the CDF of the non-centered chi-square r.v. centered on \( \beta_k^2 \).

Note that if \( V/2 \) is an integer then one can express the latter conditional probability with the Marcumq function, as follows

\[
P_{\text{ncch}}(v/\beta_k) = 1 - \text{Q}_{V/2}(\sqrt{2\text{SNR}\beta_k^2}, \sqrt{2\text{TNR}})
\]

The threshold can be simply based on the estimated noise level [13] without considering any channel statistics (e.g. relying on an early false alarm criterion). Another method consists in setting the threshold depending on the received signal kurtosis [14]. In the following, we present two other well known and popular threshold-selection TOA estimation methods for ED receivers. These two solutions will be compared in Section VI with the proposed Bayesian methods described in Section V.

#### A. Absolute Threshold

One first threshold setting strategy consists in choosing the optimal Absolute Threshold (AT) that minimizes the mean error or the deviation of the TOA estimate for a given SNR, as proposed in [7]. We depict as \( \text{SNR}_{opt} \) the threshold that minimizes the estimation MSE. In the case when no energy sample exceeds the threshold, then \( \tau_{TOA} = T/2 \). An analytic expression can usually be derived under simple channel statistics from (4), which is a key to analytically set the threshold as a function of the SNR. In the following we retain systematically \( \text{SNR}_{opt} \) in our simulations for a fair comparison with other methods. For more details on the AT techniques, one can refer to [7], [8].
B. Normalized Threshold

Another convenient method relies on a Normalized Threshold (NT), as described in [8]. The normalization is performed with respect to the difference between the minimum and maximum values among all the collected energy samples, as follows

\[ \text{Th}_{\text{norm}} = \frac{v - \min_n \{\xi_n\}}{\max_n \{\xi_n\} - \min_n \{\xi_n\}} \]  

(7)

For \( \text{Th}_{\text{norm}} = 0 \) the absolute threshold is equal to the smallest energy value, and hence \( \hat{k}_{\text{TOA}} = 1 \). For \( \text{Th}_{\text{norm}} = 1 \) the absolute threshold is equal to the largest energy value, thus index selection simply coincides with Maximum Energy Detection (MED). Note that \( \text{Th}_{\text{norm}} \) can also be optimally selected to minimize the MSE of the TOA estimation. Like for the absolute threshold strategy, we systematically choose the optimal normalized threshold value \( \text{Th}_{\text{opt}} \) in the following simulations for comparison purposes.

V. NEW BAYESIAN TOA ESTIMATORS

Two common Bayesian estimators are the mode estimator, which Maximizes the Likelihood (ML) of available observations, and the mean estimator, which Minimizes the Mean Square Error (MMSE) of the estimation. The ML estimate \( \hat{k}_{\text{ML}} \) of \( k_{\text{TOA}} \) can then be defined as follows

\[ \hat{k}_{\text{ML}} = \arg \max_{k_{\text{TOA}} \in \{1 \ldots N\}} p(\xi_1, \ldots, \xi_N / k_{\text{TOA}}) \]  

(8)

whereas the MMSE estimate \( \hat{k}_{\text{MMSE}} \) is defined as:

\[ \hat{k}_{\text{MMSE}} = \sum_{k_{\text{TOA}} = 1}^{N} k_{\text{TOA}} p(k_{\text{TOA}} / \xi_1, \ldots, \xi_N) = \sum_{k_{\text{TOA}} = 1}^{N} k_{\text{TOA}} p(\xi_1, \ldots, \xi_N / k_{\text{TOA}}) p(k_{\text{TOA}}) / p(\xi_1, \ldots, \xi_N) p(k_{\text{TOA}}) \]  

(9)

Since we assume that the actual \( k_{\text{TOA}} \) is uniformly distributed over all the addressable slot indexes in the observation window (e.g. after coarse temporal synchronization), then the Maximum A Posteriori (MAP) and the ML estimators are strictly similar here. Now, so as to derive \( k_{\text{ML}} \) and \( k_{\text{MMSE}} \), one has to compute the joint probability density function of the energy samples conditioned upon \( k_{\text{TOA}} \), which can be expressed as follows under the assumption of independent energy samples

\[ p(\xi_1, \ldots, \xi_N / k_{\text{TOA}}) = p(\xi_1 / k_{\text{TOA}}) \ldots p(\xi_N / k_{\text{TOA}}) \]  

(10)

For \( k < k_{\text{TOA}} \) and \( k \geq k_{\text{TOA}} + L \), the marginal sample density is given by

\[ p(\xi_k / k_{\text{TOA}}) = p_0 = p_{\chi_2^2}(\xi_k) \]  

(11)

where \( p_{\chi_2^2}(\xi) \) is the Probability Density Function (PDF) of a centered Chi-square variable.

Since \( L_p \leq L \) for \( k = \{k_{\text{TOA}} \ldots, k_{\text{TOA}} + L - 1\} \), we define \( P_p(k) \) the probability of path presence in the \( k^{th} \) slot. Then the overall PDF is given by

\[ p(\xi_k / k_{\text{TOA}}) = (1 - P_p(k)) p_0 + P_p(k) p_1 \]  

(12)

with \( p_1 \) the average (over the path amplitude) of the conditional energy sample PDF:

\[ p_1 \triangleq \mathbb{E}_{\lambda_k}[p(\xi_k / 2\text{SNR}\lambda_k, k_{\text{TOA}})] \]  

(13)

where \( \lambda_k = \beta_k^2 \).

As \( \beta_k \) is Nakagami-\( m \) distributed with a mean value \( \mu_k \), then it can be shown that \( \lambda_k \) is Gamma distributed with a shape parameter \( m_k \) and a scale parameter \( \mu_k / m_k \). Conditioned on \( \lambda_k, \xi_k \) are non-centered Chi-square distributed with \( V \) degrees of freedom, centered around \( 2\text{SNR}\lambda_k \). Accordingly, one can write

\[ p_1 = \int_{0}^{\infty} p_{\text{NCH}_2^2}(\xi_k / 2\text{SNR}\lambda_k, k_{\text{TOA}}) p_{\text{trans}}(\lambda_k) d\lambda_k \]  

(14)

where

\[ p_{\text{trans}}(\lambda_k) = \frac{\exp(-(m_k \lambda_k / \Lambda_k)) (m_k \lambda_k / \Lambda_k)^{n-1}}{\Gamma(m_k) \Lambda_k / m_k} \]  

(15)

with \( \Gamma \) the Euler Gamma function.

Then, according to [15, pp. 227] it is possible to show that

\[ p_{\text{NCH}_2^2}(\xi_k / 2\text{SNR}\lambda_k, k_{\text{TOA}}) = \frac{1}{2} \exp \left( -\frac{\xi_k + 2\text{SNR}\lambda_k}{2} \right) \left( \frac{\xi_k}{\text{SNR}\lambda_k} \right)^{V/4 - 1/2} \times I_{V/2 - 1}(\sqrt{2\text{SNR}\lambda_k} \xi_k) \]  

(16)

where \( I_a(x) \) is the \( a \)-th order modified Bessel function of the first kind. Finally substituting (15) and (16) into (14) and integrating over \( \lambda_k \), it comes

\[ p_1 = 0.5 \exp \left( -\frac{\xi_k}{2} \right) \frac{(\xi_k/2)^{V/2 - 1}}{\Gamma(V/2)} (\text{SNR}\lambda_k/m_k)^{V/2} \times \frac{1}{2} F_1 \left( \frac{mk \cdot V/2}{\text{SNR}\lambda_k/m_k} \frac{\xi_k}{m_k} \right) \]  

(17)

where \( 1 F_1 \) is the Hypergeometric function.

The latter expression is finally incorporated in (8) and (9) for TOA estimation purposes, considering the exact PDF of the energy samples instead of the Gaussian approximation made in [9].

VI. PERFORMANCE ANALYSIS AND DISCUSSION

In this section, we provide simulation results for different energy-based TOA estimation strategies, as previously described. In particular, we compare four different methods, namely the two classical threshold-based detection schemes shown in IV-A and IV-B, and the new MMSE and ML TOA estimators shown in V. Note that all of these estimators assume a prior knowledge of both the SNR and channel statistics.

First, we consider a dense multipath channel with exponential power delay profile CM-0, where the average path power gains are given by \( \Lambda_k = \lambda_{\text{TOA}} \exp(-(k - k_{\text{TOA}})\Delta) / \epsilon \) for \( k = k_{\text{TOA}} \ldots, k_{\text{TOA}} + L - 1 \) [12]. We choose \( \Delta = 2\text{ns}, L = 32 \), and a channel spread \( \epsilon = 12\text{ns} \). Like in [7], we consider Nakagami-\( m \) fading channels with parameters \( m_k = 2 \) and \( m_k = 10 \), which will be respectively represented by CM-0 and CM-10 in the following. For those CM-0 channels, we also set the probability of path presence to \( P_p(k) = 1 \) systematically. IEEE802.15.4a CM1 (residential LOS) and CM2 (residential NLOS) channel models have also been considered in further simulations. Channel realizations are sampled at 8GHz with a bandwidth of 4GHz.

For each channel kind, 50000 random realizations are generated, and each realization has a uniformly distributed TOA within \([0, T]\) with \( T = 100\text{ns} \). The second derivative of a gaussian pulse of duration \( T_p = 1\text{ns} \) is used as the transmitted waveform in all the considered scenarios. Finally,
energy samples are collected within non-overlapping windows to obtain decision statistics as discussed in Section III.

Figure 2 shows the Root Mean Square Error (RMSE) of TOA estimates for the AT strategy (IV-A) as a function of the TNR value under different SNR conditions. Note that the RMSE is computed like in [7]. Since $\tau_{TOA}$ is uniformly distributed over the interval $T$, it can be seen that for small values of TNR (i.e., $k_{TOA} = 1$) the RMSE tends to $T/\sqrt{3}$ whereas it tends to $T/(2\sqrt{3})$ at high TNR values (i.e., $k_{TOA} = T/2$). For each SNR, we remind that the TNR that minimizes the RMSE is retained for comparison fairness.

Similarly, Figure 3 shows the RMSE of TOA estimates as a function of $T_{h, norm}$ for the Normalized Threshold strategy (IV-B). For $T_{h, norm} = 0$ the threshold is equal to the smallest bin energy, then $k_{TOA} = 1$ and the RMSE is equal to $T/\sqrt{3}$. For $T_{h, norm} = 1$ the error is equivalent to that of a MED scheme. For smaller SNR values $k_{TOA}$ tends to have equal probability over all possible indexes and thus the RMSE tends to $T/\sqrt{6}$. For each SNR value, the $T_{h, opt}$ that minimizes the RMSE is retained to be compared in the following with other TOA estimators.

Note that the Nakagami shape parameter $m_k$ for all the paths in the IEEE802.15.4a CM1 and CM2 channel models are actually Lognormal variables following respectively $Log N(0.67, 0.28)$ and $Log N(0.69, 0.32)$. However, the analytic expression of the proposed Bayesian TOA estimators assumes a constant shape parameter for all the paths. Therefore, for the sake of simplicity, we consider such a constant parameter value equal to $m = E[m_k] \approx 2$ for both of the channel models. The advantage of using integer values for both $m = \{2, 10\}$ and $V/2 = W \Delta = 8$ is to approximate $_{1}F_{1}(m, V/2, x)$ (e.g. using Matlab®) with a nonlinear function of $x$, what significantly reduces the computational complexity and thus the simulation time accordingly.

Figure 4 to show the RMSE of TOA estimates for different SNR values and the four estimation methods under the same channel and noise realizations. For instance, Figures 4 and 5 show simulation results under CM0-2 and CM0-10 models. As expected, better performance can be achieved with Bayesian approaches over the whole range of investigated SNR values. More particularly, the MMSE estimator provides the lowest MSE. Under reasonably low SNR values (i.e. in the range of 5dB to 15dB), the proposed estimators even more significantly outperform classical threshold-based ED schemes.

Figures 6 and 7 show simulation results when applying the IEEE 802.15.4a CM1 and CM2 channel models respectively. Again, the Bayesian estimators noticeably provide better performances on the average. But occasionally, under particular SNR values, the ML estimator in Figure 7 exhibits higher MSE than that of the normalized threshold estimator. This might be due to the statistical dependency between successive energy bins under realistic channel realizations and/or mostly to the approximated $m$ Nakagami parameter assumed to be constant for all the path amplitudes. However, in spite of those identified sources of suboptimality, the obtained results show satisfactory improvements in comparison with existing estimators.

VII. Conclusion

In this paper, we have proposed new Bayesian TOA estimators for low-complexity energy-based receivers. First, we have derived the analytic expression of the described estimators considering realistic UWB channel multipath statistics. Then previous Bayesian solutions have been compared with two well-known threshold-based estimators benefiting from optimal threshold setting. The obtained simulation results tend to show that the proposed approach significantly reduces the MSE of TOA estimates for a wide variety of SNR values. Further works should assess robustness with respect to the sampling rate or under more limited knowledge of channel statistics. Further experiments might be also carried out to validate some of the underlying statistical channel assumptions.

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References

Fig. 2. RMSE of the AT TOA estimation method (threshold-based) [7], as a function of TNR, for different SNR values and with CM1.

Fig. 3. RMSE of the NT TOA estimation method (threshold-based) [8], as a function of $T_{\text{th, norm}}$, for different SNR values and with CM1.

Fig. 4. RMSE for different TOA estimators with CM0-2 as a function of SNR.

Fig. 5. RMSE for different TOA estimators with CM0-10 as a function of SNR.

Fig. 6. RMSE for different TOA estimators with CM1 as a function of SNR.

Fig. 7. RMSE for different TOA estimators with CM2 as a function of SNR.