Collective Action and Network Change§
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Abstract
Network models of collective action commonly assume fixed social networks in which ties influence participation through social rewards. This implies that only certain ties are beneficial from the view of individual actors. Accordingly, in this study we allow that actors strategically revise their relations. Moreover, in our model actors also take into account possible network consequences in their participation choices. To handle the interrelatedness of networks and participation, we introduce new equilibrium concepts. Our equilibrium analysis suggests that structures that tend to segregate contributors from free riders are stable, but costless network change only promotes all-or-nothing participation and complete networks.

Keywords: Collective action; Social dilemmas; Social networks; Network dynamics; Social control; Structural balance; Local interaction games

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1. Introduction

Why and under which social conditions are groups successful in mobilizing collective action? Voluntary participation in collective actions, such as fund raising, strike movements or political uprising, seems often to contradict the narrowly defined self-interest of the participants. Yet, empirical examples of successful mobilization abound. Students of collective action point to social networks as an important answer (see e.g., Oberschall, 1973; Tilly, 1978; Oliver, 1984; McAdam, 1986; Marwell, Oliver, and Prahl, 1988; Gould, 1993a; Sandell and Stern, 1998; Chwe, 1999; 2000; and for an overview Diani, 2003a). Dense networks of communication and interaction between prospective participants may greatly facilitate group mobilisation (Opp and Gern, 1993; Gould, 1993b; Marwell and Oliver, 1993). The view that social networks facilitate collective action relies on the assumption that individual network members have a “regulatory interest” (Heckathorn, 1988; Kitts, 2006) to enforce others’ contribution to the collective action. Particularly in dense or closed networks, actors can effectively employ their social ties for this purpose (Hechter, 1987; Coleman, 1990) both because group members have more information about one another and because they have more social means to provide rewards for compliance or punish deviance.

But it has also been argued that network ties have a “double edge” (Flache, 1996). Heckathorn (1996) has pointed out how peer pressure may take the form of “oppositional control”, when network members have strong incentives to insulate themselves from pressures to contribute. Based on different behavioral assumptions, other theoretical studies supported the view that dense networks may sometimes undermine rather than facilitate the enforcement of contribution (Flache and Macy, 1996; Flache, 1996, 2002; Kitts, Macy, and Flache, 1999). The theoretical argument focuses at the desire of actors to obtain social rewards from other group members including those who “free ride”. The desire to retain relationships with or attain behavioral confirmation from free riders may often compromise actors’ willingness to exert social control towards contribution, especially in a closely knit network.

To clarify under what conditions networks are positively or negatively related to collective action success, models of collective action need to incorporate explicitly how individual actors make purposive decisions to use their social relations to foster their goals, be it to enforce compliance or to resist peer pressure. But studies who combine positive and negative effects of social ties on cooperation (e.g., Oberschall, 1994; Heckathorn, 1996; Takács, 2001) have neglected a crucial implication of this perspective. Purposive action implies that network members in a collective action situation not only use existing ties to attain their goals, but they may also make or break ties if this serves their purposes.

In general, most models of collective action that address social network effects implicitly assume that there is a fixed set of interpersonal relations that do not change over time. Relational ties are in these studies exogenously given and at most, static comparisons are made. We argue that the relationship between collective action and social networks can not be properly studied without addressing endogenous network change driven by individual
interests. Regulatory interests do not necessarily lead to enforcement, but possibly also to avoidance of unpleasant control and optimization of contacts. For illustration, consider the situation of a defector who is particularly sensitive to conformity pressure and who has her ties mostly with compliant group members. In a static network, it is likely that pressure to contribute brings this defector back into line. But if networks can change, the defector faces an incentive to maximize the number of ties she has with other defectors and thus break ties with cooperators and build new ties with other defectors. If the latter mechanism prevails, the outcome on the collective level may be a disconnected network with a deviant clique on which cooperators can not effectively impose peer pressure. But if the first mechanism is more important, the result may be a dense network in which free riders are effectively sanctioned by their compliant peers. In this example, models of collective action that neglect endogenous network changes might arrive at questionable conclusions.

In this study we emphasize that networks change over time and that is also reflected in how people behave in collective action (Kim and Bearman, 1997; Diani, 2003b; Osa, 2003; Gould, 2003). Besides being driven by independent network dynamics, networks may also change because of the internal determinants of collective action. People might choose their structural relations strategically in order to maximize rewards and minimize punishments that originate in social control (cf. Harsanyi, 1969). Furthermore, in order to facilitate participation, strategic establishment of communication channels and other linkages might take place (McAdam and Paulsen, 1993; Diani, 2003c). Strategic tie formation goes very far in the proliferation of social movement politics. Campaigning and lobbying often involves tie-formation strategies, popularly called as “networking” (e.g., Tilly and Wood, 2003). On the other hand, another branch of literature emphasizes individual opportunities that arise from deleting relations and from structural holes (e.g., Burt, 1992; 2005; Buskens and van de Rijt, 2005; Burger and Buskens, 2006). Severing links is also considered as a purposeful action in different lines of social psychological literature (see Williams, Forgas, and von Hippel, 2005 for an overview). Studies show that in collective action, ostracism and the threat of exclusion increases participation (Olson, 1982; Hirshleifer and Rasmusen, 1989; Ule, 2005; Ouwerkerk et al., 2005).

There is also empirical evidence on how collective action might restructure individual relations. Activism has certainly a role in changing the meaning and impact of interpersonal ties (Gould, 2003). As a result of the dynamics of dyadic relations, macro properties of the network and thus perspectives of (further) collective action also change. Our contribution along this line is to integrate the perspective of network change and the resulting network dynamics in a theoretical model of collective action.

Independently from the literature we cited so far, considerable progress has been made recently in modeling the dynamics of social networks as a result of strategic individual action (Jackson and Watts, 2002a; Dutta and Jackson, 2003; Ule, 2005). Previous game theoretic models, however, concentrated on strategic network formation without collective interdependencies (Bala and Goyal, 2000; Bonacich, 2001). Most studies use games of strategic network formation that assume values for established ties in cooperative situations (Dutta, Nouweland, and Tijs, 1998; Slikker and Nouweland, 2001) or indirect benefits from
the chain or network of contacts (Jackson and Wolinsky, 1996; Jackson and Watts, 2002b). In this literature on network formation, equilibrium concepts have been developed that describe equilibrium networks in which no individuals have incentives to delete existing ties or build new ones (Jackson and Wolinsky, 1996; Watts, 2001; Gilles and Sarangi, 2004; Ule, 2005; Buskens and van de Rijt, 2005; van de Rijt and Buskens, 2005; Goyal and Vega-Redondo, 2007). If network stability and behavioral stability are embraced and adjusted to collective interdependencies, similar concepts could help us to model and understand collective action and stability of social networks. The main added value of this study for the literature on network formation is the development and application of such equilibrium concepts in \( n \)-person games.

Another line of work explicitly combines collective action with network change (Kitts, Macy and Flache, 1999), but uses “backward-looking” models of structural learning that describe the process by which individuals strengthen ties that are beneficial for them and abandon ties with negative experience (Macy et al., 2003). These models do not assume that decision making is strategic and purposive in the sense that individual actors weigh the costs and benefits of changing their contribution behavior against the costs and benefits of making or breaking relationships, or a combination of both. This makes it difficult to address effects of exogenous constraints, such as given access structures of networks or communication costs, within the structural learning framework.

In this paper, we propose a game theoretical model of collective action in dynamic networks and arrive at equilibrium predictions. Our main innovation is that we incorporate into a collective action framework the possibility of tie formation and deletion, and model these decisions as interdependent with participation decisions. We draw on game theoretic studies of network evolution and model both collective action and relational change as the result of strategic and purposive decision making that explicitly reflects the costs and benefits of the various decision options involved. The game theoretic approach allows us to arrive at equilibrium predictions without the need of using simulation techniques. We will use these equilibrium predictions to identify exogenous conditions that explain why sometimes network dynamics may undercut the effectiveness of peer pressure, while network influences successfully foster collective action at other times. Examples of phenomena our model can tackle are the formation of deviant cliques that remain stable despite strong social control, or work teams where team-segregation develops along a division line between workers and shirkers, even when high team performance would be in everybody’s interest.

Our work is a first step towards addressing such phenomena in a framework that integrates collective action dynamics and network dynamics with the assumption of individual strategic behavior and the notion that network ties are conduits of social rewards in collective action. Accordingly, the major contribution of the paper will be theoretical. To provide the tools to tackle the substantive questions we are interested in, we develop an equilibrium concept that embraces Nash equilibrium in the \( n \)-person collective action game and network stability at the same time. The concept, called strongly robust network equilibrium, delineates structural conditions under which group outcomes are stable both in terms of the network and in terms of the collective action decisions of the group’s members.
In the remainder of the paper, we proceed as follows. In section 2, the basic model of social control and collective action is lined out. In section 3, conditions are derived under which strategic actors will not delete any tie or form new ones in a given strategy profile of collective action decisions. Section 4 contains a derivation of the conditions for strongly robust network equilibria. Finally, we discuss our results in section 5, where we also formulate conclusions and suggestions for further research.

2. The model of social control and collective action

This section provides a description of our model of decisions on participation in collective action. Collective action is modeled as a one-shot n-person public goods game with a linear production function and binary decisions and it is integrated with a model of dyadic interdependence between directly related players. In section 3, we extend this model by a network game, in which players can change interpersonal connections.

We define N to be the set of actors, where N contains n \( n>2 \) players. Individual actions are binary and summarized in vector \( \mathbf{\sigma} \), in which the action of \( i \) is denoted by \( \sigma_i \), where \( \sigma=1 \) is contribution and \( \sigma=0 \) is non-contribution or defection (for all \( i \in N \)). We use \( (\sigma, \sigma_i) \) to refer to the action profile chosen by all group members, where we distinguish between \( i \)'s action and the actions chosen by other members of the group, \( \sigma_i \). Each participation (contribution) provides a unit of public good \( \alpha \) for all individuals. This means that the more people participate, the higher the public good reward each players receive. Participation has an individual cost \( c \), which is higher than one unit of public good \( (c>\alpha) \), but smaller than \( n \) units \( (c<n\alpha) \). These payoff restrictions impose a Prisoner’s Dilemma structure in the public goods game. When we neglect social network payoffs, universal contribution is Pareto superior to universal defection, but unilateral defection is individually rational.

Actors play this game not in isolation. They might have connections (links) to each other, which define a social network. For the sake of simplicity, we consider undirected and unvalued connections, which imply that every link is symmetric and equally important. The social network \( (N, R) \) is characterized by the set of actors \( N \), and by the set of connections \( R \subset R_N \), where \( R_N = \{ij \mid \{i, j\} \in N \times N, i \neq j \} \) and \( ij=ji \). Sometimes we will refer to \( j \) as \( i \)'s contact or friend if \( ij \in R \). As we consider a fixed network size, we will simply refer to the social network \( (N, R) \) as \( R \) and to the network from which the relation \( ij \) is deleted as \( R \setminus \{ij\} \).

The set \( R_i = \{ij \mid ij \in R, i \in N, i \neq j \} \subset R \) contains all ties of \( i \) for which \( r_{ij} = 1 \), and the set \( \tilde{R}_i = \{ij \mid ij \in R, i \in N, i \neq j \} \) denotes the null-dyads of \( i \). The size of \( R_i \) is the number of ties (degree) \( i \) has and is denoted by \( r_i \). Actor \( i \) is an isolate if \( r_i = 0 \).

If \( ij \in R \), actors \( i \) and \( j \) influence each other. First, we assume that actors prefer to act the same way as their contacts (cf. Chong, 1991; Oberschall, 1994), which we will refer to as

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1 Sets will be denoted by bold capitals and their sizes by corresponding small letters in italics. For example, \(|N| = n\).
behavioral confirmation. This assumption is supported by empirical studies that show that individuals even gather information on-the-spot to assess whether their friends will participate or not (Oberschall, 1993; Dixon and Roscigno, 2003). When the actions of contacts match, we assume that they obtain a higher subjective payoff than when their action is different. This kind of strategic interdependence is also modeled in coordination games played on a network (see e.g., Berninghaus and Schwalbe, 1996; Morris, 2000; Chwe, 2000; Jackson and Watts, 2002a; Buskens and Snijders, 2005). Our study, however, moves beyond this framework and introduces strategic interdependence not only in terms of the need of coordination, but also in terms of the more challenging problem to achieve cooperation between self-interested actors in an n-person Prisoner’s Dilemma.

We assume that behavioral confirmation might take two different forms. First, we assume that the subjective payoffs \( \pi_i \) for all \( i \in N \) are increased by the value of mass conformity \( b_1 \) for each \( j \) if \( ij \in R \) and \( \sigma_i = \sigma_j \). This means that an additional contact has the same influence on the actor as other contacts, and hence the number of contacts with identical choice determines the strength of this type of influence. Empirical research on social influence supports that individuals are more likely to choose a certain behavior when the number of relevant others who behave this way increases (see for instance, Ploeger, 1997 and Snijders and Baerveldt, 2003 for delinquency; Blum, Beuhring, and Rinehart, 2000 for alcohol use; Simons-Morton and Chen, 2005 for substance use). For example, when mass conformity operates, an individual, who intends to participate in a demonstration, is pleased when there are many friends in the crowd. On the other hand, in case this individual prefers to stay at home, she is pleased to be assured that many friends choose to stay away from the demonstration. As effective control from more contacts pushes stronger towards participation, individuals face “local thresholds” of participation similar to the impact of critical mass of participants on individual participation in other models of collective action (Macy, 1991; Marwell and Oliver, 1993; Chwe, 1999).

Second, behavioral confirmation is also dependent on the proportion of contacts with identical action (from all friends of \( i \)). We assume that the subjective payoffs \( \pi_i \) for all \( i \in N \) increase by \( b_2/s_i \) for each \( j \) if \( i \) and \( j \) have a network tie (\( ij \in R \)) and their actions are identical \(( \sigma_i = \sigma_j \)). The parameter \( b_2 \) scales the strength of this proportional conformity effect. Different lines of arguments justify this assumption. In-degree and the proportion of relevant others behaving the same way (e.g., mean friendship delinquency) are important predictors of peer influence (Haynie, 2001). Furthermore, a study using working unit-level data found that the proportion of participants in the working unit is a significant predictor of individual strike participation (Dixon and Roscigno, 2003). Including a proportional term in the model is also in line with theories on cultural change that emphasize that individuals are influenced by relatively more frequent cultural traits around them (e.g., Boyd and Richerson, 1985) and with network models of coordination (Buskens and Snijders, 2005).

In addition, structural relations are the sources of positive social selective incentives \( (s) \) that reward contribution. We assume that the subjective payoffs \( \pi_i \) for all \( i \in N \) increase by \( \sigma_i s \) from each \( ij \in R \) connection, hence by \( \sigma_i s r_i \) in total. This implies that only contributors receive
selective incentive rewards. If the provision of selective incentives is costly, actors would not only face a public goods problem, but also a second order free rider problem (Oliver, 1980; Heckathorn, 1989). As we concentrate, however, on selective incentives of a social character like respect or status, we deem it plausible that these are produced without costs (Chong, 1991; Lovaglia, Willer, and Troyer, 2003) and that therefore no second order free-rider problem arises (see also Coleman 1990). This assumption receives additional support by evidence that individuals punish defectors voluntarily, in spite of sanctioning costs (Fehr and Gächter, 2000; 2002; Boyd et al., 2003).

We use these assumptions to express the subjective payoff players obtain given their choice for participation or defection. The subjective individual payoff \( \pi_i \) of \( i \in N \) will depend on the number of her contacts who participate, \( r_{ic} \) (where \( r_{ic} \) is the size of \( R_{ic} = \{ ij \mid ij \in R, \ \sigma_j \neq I \} \subset R \)), and on the number of her contacts who do not participate \( (r_{id} = r_i - r_{ic}) \). Formally, \( \pi_i \) for all \( i \in N \) with \( r_i > 0 \) in the given network \( R \) and given strategy profile \( \sigma(\sigma_i, \sigma_{-i}) \) is determined as:

\[
\pi_i(\sigma_i = 0, \sigma_{-i}, R) = r_{id} b_1 + \frac{r_{id}}{r_i} b_2 + \alpha \sum_{j=1}^{n} \sigma_j \quad \text{and}
\]

\[
\pi_i(\sigma_i = 1, \sigma_{-i}, R) = r_i s + r_i b_1 + \frac{r_{ic}}{r_i} b_2 + \alpha \left( \sum_{j=1}^{n} \sigma_j + 1 \right) - c ,
\]

which can be compactly written as

\[
\pi_i(\sigma, R) = \left( \alpha + r_i s + \left( r_{ic} - r_{id} \right) \left( b_1 + \frac{b_2}{r_i} \right) - c \right) \sigma_i + r_{id} \left( b_1 + \frac{b_2}{r_i} \right) + \alpha \sum_{j=1}^{n} \sigma_j , \quad (1)
\]

where \( j \in N \setminus \{i\} \). If \( r_i = 0 \), \( \pi_i(\sigma, R) = (\alpha - c) \sigma_i + \alpha \sum_{j=1}^{n} \sigma_j \). In the main text, we assume that contributions have positive externalities, \( \alpha > 0 \), and social control generates rewards \( (b_1 \geq 0, b_2 \geq 0, \text{and} \ s \geq 0) \). Definition of payoff functions if social control appears in terms of punishments and the analysis of such situations is left to the Appendix.

Now we can derive the conditions for individually rational participation in collective action given a certain network position and given a vector of participation decisions of group members. Broadly, an actor will participate if the subjective payoffs from contribution exceed the subjective payoffs from free riding in the given network and strategy profile. This is achieved when the selective incentives and behavioral confirmation rewards she receives in her present network position are sufficient to compensate for the costs of contribution. Formally, \( i \) participates if \( \pi_i(\sigma_i = 1, \sigma_{-i}, R) \geq \pi_i(\sigma_i = 0, \sigma_{-i}, R) \) that is when
\[
\alpha + r_s + (r_e - r_m)(b_1 + \frac{b_2}{r_j}) \geq c.
\]

Expression (2) asserts that the effect of selective incentives on participation increase in the number of ties of the given individual. Behavioral confirmation promotes contribution only when more contacts contribute than defect.\(^2\)

3. Network changes

As a next step, we include the possibility that actors might delete their existing connections and might form new ones. For the time being, we treat participation decisions as fixed. This allows to identify stable network structures at a given strategy profile of the public goods game.\(^3\) We continue to assume that social control parameters are rewards \((b_1 \geq 0, b_2 \geq 0,\) and \(s \geq 0)\). The costs of abandoning one tie is denoted by \(a\) and the cost of forming one new tie is denoted by \(f\) \((a \geq 0, f \geq 0)\).

A connection \(ij\) is not stable if \(i\) or \(j\) prefers to delete it.\(^4\) No compensation payments are possible to save the connection (cf. Bala and Goyal, 2000). Note that it might be beneficial for a player to delete a set of her links at once. Even if a deletion of \(ij\) does not result in a higher payoff, it can be part of the set of links that is rewarding to be deleted.

On the other hand, we exclude the possibility of one-sided tie formation, because we use non-directed relations (cf. Bala and Goyal, 2000; Buskens and van de Rijt, 2005). To keep our analysis simple, we abstract from the possibility that actors may bargain about which actor or which coalition of group members may bear the costs of the formation of some subset of ties. Tie formation is assumed to have the same costs for both parties and no coalition possibilities beyond the dyad are taken into account.\(^5\) When analyzing network equilibria, we will assume that a new tie is initiated only if it is part of an (extended) individual network that provides higher benefits than the present network, taken into account the costs of change and under the assumption of no other change in the network. Besides this basic and simplified

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\(^2\) A more refined analysis of this static model can be found in Janky and Takács (2005). Main results for the entire network show that the minimum degree of the network is a strong determinant of overall collective action in case selective incentives operate. Network clustering has a strong influence when behavioral confirmation mechanisms are strong and might undermine mass collective action. Clustered networks are more likely to have partial contribution equilibria, in which participants and free riders are segregated. The smaller the number of free riders in the partial contribution equilibrium, the less likely that full contribution is a payoff dominant equilibrium. Moreover, the payoff dominance of full contribution equilibrium is not likely in centralized structures when mass conformity is strong, but it is possible in case proportional conformity is prevalent. A further interesting result is that not only behavioral confirmation, but also selective incentives might have a non-monotonic effect on the existence of full contribution equilibria.

\(^3\) A version of the model allowing only for network decline is also discussed in Takács and Janky (2007).

\(^4\) When behavioral confirmation and social selective incentives are negative, more links are not stable than when behavioral confirmation and selective incentives are positive (see Appendix).

\(^5\) Bargaining could be incorporated with a coalition analysis of network formation including the use of equilibrium concepts from cooperative game theory, such as the core. However, studies that use this approach (e.g., Jackson and Wolinski, 1996; Dutta, van den Nouweland, and Tij, 1998; Slikker and van den Nouweland, 2001; Jackson and van den Nouweland, 2005) do not discuss collective action.
cost-benefit analysis, following previous work, we will assume that consent of the other party is required to form a new connection.

We are also concerned with the possibility that players might benefit from the combination of deleting and building some ties. Consider, for instance, a defector $i$ who has only a single tie $ij$ and this leads to a contributor $j$. Since proportional conformity rewards in this case are zero, player $i$ has no incentive to break $ij$ if $a>0$. She also has no incentive to be tied with other defectors when $f>b_1+0.5b_2$. Replacing $ij$ with a connection to another free rider, however, is beneficial for $i$ in case $b_1+b_2>f+a$. Such a situation is not unlikely when the cost of deleting a relation is small.

Most equilibrium formulations in the research on games of network formation,\(^6\) such as the stable network concept of Watts (2001) or pairwise stability of Jackson and Wolinsky (1996), posited two requirements (for a review, see Dutta and Jackson, 2003). The first requirement prescribes that no player intends to delete a connection and the second requirement is that no player intends to add a new relation (Watts, 2001) or no new tie could be formed for the mutual benefit of the players involved (Jackson and Wolinsky, 1996). A stronger version of the latter concept is strong pairwise stability that combines pairwise stability and strong link deletion proofness by allowing multiple ties to be deleted, but only a single tie to be built at a time (Jackson and Wolinsky, 1996; Gilles and Sarangi, 2004). More complex stability concepts, such as strong stability, allow for a coalition of players that is larger than two to deviate (see Dutta and Mutuswami, 1997; Jackson and van den Nouwenland, 2005; Jackson, 2004).

For our purposes, we need a simple concept that is still non-cooperative in nature and unlike in strong pairwise stability, considers multiple tie formation. The point of departure is based on the Nash logic and states that in equilibrium it should not be in anyone’s interest to make “any change” in her network. By “any change” we mean any combination of deleting existing and of forming new relations in which the individual is involved. This stability concept is useful for the discussion of conditions under which a network is free of tie deletions and initiations. Applied to collective action situations, this stability concept would characterize networks in a given strategy profile in the (collective action) game, if there is no actor $i$, for whom any change in the set of her relations would result in a better outcome given that relational contacts outside of $i$ are fixed. Formally, we would call $R$ resistant to changes in a given $\sigma$, if $\pi(\sigma, R) \geq \pi(\sigma, (R \cup G_i) \setminus H_i)-h_i a-g_i f$ for any $G_i \subset \hat{R}_i$, any $H_i \subset R_i$ and for all $i \in N$, where the notations $H_i$ are introduced for the set of ties to be deleted and $G_i$ for the set of ties to be built by $i \in N$. The sizes of these sets are denoted by $h_i$ and $g_i$, respectively.

It is not our main interest to deal with situations in which actors can impose ties on others (as in directed networks). We also do not want to focus on whether initiations of new ties take place, but we want to address whether new ties are built or not, which requires consent also from the other party. Accordingly, we need to move beyond the use of a purely Nash based concept. As ties are symmetric and tie formation requires mutual consent, we

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\(^6\) This line of research requires a characteristic function defined for the network and an allocation rule, but its basic definitions can also be applied to our case.
build our equilibrium concept on the same logic as monadic stability (Gilles and Sarangi, 2004) and unilateral stability (Buskens and van de Rijt, 2005). These concepts take it into account that a new tie is only formed if it is to the benefit of the partner, as well. Monadic stability is based on the idea that players take it as granted that other players respond affirmatively to an initiation if the new link is profitable to them, but no further consequences are taken into account (Gilles and Sarangi, 2004). Buskens and van de Rijt (2005) defines a network unilaterally stable if no actor would be better off by changing her ties or if anyone was, then at least one actor whom she proposes a new tie is worse off in the new network than in the original network. We will adopt this definition and adjust it for our purposes when applying to collective action problems and network formation.

We define a social network unilaterally stable in a given strategy profile in the (collective action) game, if there is no actor $i$, for whom any change in the set of her relations would result in a better outcome or if a change would result in a better outcome then it does not satisfy some of the new partners given that relational contacts outside of the scope of $i$ are fixed. This stability concept does not allow coalition formation beside the involved dyads, but requires consent from new partners involved.

**Definition:** $R$ is unilaterally stable in a given $\sigma$, if

$$\pi_i(\sigma, R) \geq \pi_i(\sigma, (R \cup G_i) \setminus h_i) - h_i a - g i$$

or exists such $j \in N$ that $ij \in G_i$: $\pi_j(\sigma, R) > \pi_j(\sigma, (R \cup \{ij\}) - f$

for any $G_i \subset \hat{R}_j$, any $H_i \subset R$ and for all $i \in N$.

From the first sight, next to considering a given strategy profile in the collective action game, there is a further difference compared to the original formalization of unilateral stability. Here, new partners compare subjective rewards from the original network with subjective rewards from the network to which their new tie with $i$ is added. Note that in our case this is just a simplified formalization: as strategies are fixed and there are no indirect rewards from network changes, $\pi_j(\sigma, R \cup \{ij\}) = \pi_j(\sigma, (R \cup G_i) \setminus H_i)$ for all such $j \in N$ that $ij \in G_i$.

For the simple case where $a=0$ and $f=0$, there is a clear-cut answer to the question which networks can be stable. Only a disconnected network with a complete component of contributors and a complete component of defectors can sustain unilateral stability. The reason is that defectors always gain from abandoning all ties to contributors and contributors always gain from building as many as possible new ties to fellow contributors. On the other hand, if selective incentives are large enough relative to proportional conformity, then even a disconnected network might not be resistant to changes, because cooperators would prefer to connect to defectors as well. We do not obtain such straightforward results, however, if there are costs of deleting and forming ties. In these cases, networks that contain ties between defectors and contributors can also be unilaterally stable.

A contributor might profit from a new tie to a defector, but such a tie will never be beneficial for the defector. As consent from both parties is required for a new relationship, such a tie will not be realized. In general, a defector $i \in N$ with $r_{it}>0$ is better off by a structural change in which she deletes $h_i$ ties to contributors and newly forms $g_i$ ties to defectors, if
\[ g_ib_1 + b_2 \frac{g_ir_i + h_ir_id}{r_i(r_i - h_i + g_i)} > h_ia + g_if \]  

holds. This condition implies that for a defector it is always more profitable to abandon all relations with contributors rather than just breaking up some of them, even when tie formation is simultaneously possible. The reason is that for a free rider who has at least one contact to another defector breaking contacts to actors with dissimilar strategies improves proportional conformity rewards and does not yield any loss of mass conformity or selective incentive rewards. The more ties to cooperators the defector breaks, the larger is the improvement of the ratio of defectors to cooperators in the defector’s personal network. This implies that any additional tie abandoned yields a greater benefit than the previous one, whereas the costs are the same for each deletion. Technically, after substituting \( r_{ic} \) for \( h_i \), the necessary conditions of a beneficial structural change for defector \( i \) with \( r_{id}>0 \) are given as

\[ b_2 \frac{r_{ic}}{r_i} > r_i a + g_i(f - b_1) \]  

Similarly, for a contributor \( i \in N \) with \( r_i>0 \) a structural change in which she abandons \( h_i \) ties to defectors and newly forms \( g_i \) ties to contributors is beneficial, when

\[ \left(g_i - h_i\right)s + g_i b_1 + b_2 \frac{g_i r_{id} + h_i r_{ic}}{r_i(r_i - h_i + g_i)} > h_i a + g_i f \]  

holds. From (5) it can be seen that for a contributor forming the first new relation with another contributor is always at least as profitable as further ones, even when considering the simultaneous possibility of deleting ties. Again, this is caused by proportional conformity. The less ties a cooperator has (but at least one to a defector), the larger is the improvement in the ratio of defectors to cooperators from ego’s point of view if an additional tie is established with a similarly acting group member. Hence, the first new tie is the most valuable for a cooperator, in case she has at least one connection to a defector (or if she is an isolate). Technically, after substituting 1 for \( g_i \), the necessary conditions of a beneficial structural change for contributor \( i \) with \( r_{id}>0 \) are given as

\[ b_1 + b_2 \frac{r_{id} + h_ir_{ic}}{r_i(r_i - h_i + 1)} > h_i a + f + \left(h_i - 1\right)s. \]  

A network is unilaterally stable in a given strategy profile, if there is no defector for whom equation (4) is satisfied with any values of \( g_i \), and there also is no contributor for whom equation (6) holds with any values of \( h_i \). Networks that contain ties between defectors and
contributors are most likely to be unilaterally stable when costs of link deletion (a) and link formation (f) are high and every individual is tied to several others.\footnote{In case only negative social control operates, calculating which networks are unilaterally stable is easier. In the Appendix we show that in this condition no combination of deleting and building ties can be individually profitable. An individual either has an incentive to delete all her ties or has an incentive to form a new one. Hence, a strong link deletion proof network (see formal definition later) in which no dyad is interested to form a new connection will be \textit{resistant to changes} (and unilaterally stable).} Substantively, such conditions can be interpreted as high constraints on network change imposed by the given network.

Besides these general results, we can formulate some illustrative statements that characterize equilibria. Theorem I. summarizes three results concerning deleting a tie \(ij\) in a given strategy profile \(\sigma\) (see Appendix for the proof). A tie \(ij \in R_i\) will be called \textit{not stable} if there exist \(G_i \subset \tilde{R}_i\) and \(H_i \subset R_i\) such that \(ij \in H_i\) and \(\pi_i(\sigma, (R \cup G_i) \setminus H_i) - h_i \alpha - g_i f > \pi_i(\sigma, R)\).

\textbf{Theorem I:} In any \(R\) and given \(\sigma\), for all \(i,j \in N\) and \(ij \in R\):

a) If \(\sigma_i = \sigma_j\), then \(\pi_i(\sigma, R) \geq \pi_i(\sigma, R \setminus \{ij\})\).

b) If \(\sigma_i = 0\), \(\sigma_j = 1\), \(r_{id} > 0\), and \(b_2 / r_1 > a\), then \(ij\) is not stable.

c) If \(\sigma_i = 0\), \(\sigma_j = 1\), \(b_2 > 0\) and for sufficiently small \(a\) and \(f\), \(ij\) is not stable.

Part a) of Theorem I claims that no player can increase her subjective payoff by deleting a link to another player who acts the same way as she does in any network and given strategy profile. Part b) states that if there is a tie that connects a defector and a contributor, if the defector has at least one tie to another defector and costs of tie deletion are under a threshold that is determined by proportional conformity and the individual degree of the defector, then this tie is not stable (at least the defector wants to delete this tie). Furthermore, part c) asserts that relations between defectors and contributors are not stable, if proportional confirmation is positive and tie deletion and formation costs are under a specific threshold. To analyze macro-level consequences of Theorem I, we introduce stability concepts for the entire network that concern link deletions:

\textbf{Definition:} \(R\) is \textit{link deletion proof} in a given \(\sigma\), if \(\pi_i(\sigma, R) \geq \pi_i(\sigma, R \setminus \{ij\}) - a\) for all \(i \in N\) and \(ij \in R\).

That is, a social network is link deletion proof in a given strategy profile (in the collective action game) if there is no actor for whom deleting a single relation would result in higher subjective payoffs, assuming exactly the same actions and no other change in the network. This concept of link deletion proofness, similar to stable networks (Watts, 2001) and pairwise stability (Jackson and Wolinsky, 1996), concerns only a single change in the network at once. A stability concept that allows players to delete any set of links at once is called \textit{strong link deletion proofness} (Gilles and Sarangi, 2004; Belleflamme and Bloch, 2004).

\textbf{Definition:} \(R\) is \textit{strong link deletion proof} in a given \(\sigma\), if \(\pi_i(\sigma, R) \geq \pi_i(\sigma, R \setminus H_i) - h_1 \alpha\) for any \(H_i \subset R_i\) and for all \(i \in N\).
That is, a social network is strong link deletion proof in a given strategy profile (in the collective action game) if there is no actor for whom deleting any subset of her relations would result in higher subjective payoffs, assuming exactly the same actions and no other change in the network. Adopting the concept of strong link deletion proofness, we can form two corollaries of Theorem I:

**Corollary I.a.** Any \( R \) is strong link deletion proof if \( \sigma = 0 \) or \( \sigma = 1 \).

**Corollary I.b.** If \( b_2 r_{d_{\max}} > a \geq 0 \) (where \( r_{d_{\max}} \) is the highest individual degree among defectors), then in a strong link deletion proof \( R \) for all \( i \in N \) with \( \sigma_i = 0 \) holds that if \( ij, ik \in R_i \), then \( \sigma_j = \sigma_k \).

The corollaries summarize typical cases under which a network is strong link deletion proof. Corollary I.a states that every network is strong link deletion proof in a full contribution and in a full defection strategy profile. Corollary I.b claims that if proportional conformity rewards are positive and costs of tie deletion are under a threshold value, then in a strong link deletion proof network in the collective action game every defector has ties only to defectors or only to contributors, but not to both.

The key parameter that underlies Theorem I and the corollaries is proportional conformity (\( b_2 \)). Proportional conformity is responsible for the result that deleting all relations to contributors is more beneficial for a defector than just deleting one or some of them. The more links to contributors a defector deletes, the higher is the improvement of the ratio of defectors to contributors among her ties. For example, if this ratio is 5:10, then deletion of one link improves the ratio by about 0.0556, while deletion of two links to contributors yields an improvement of about 0.125, deletion of three yields an improvement of 0.214, etc.

The threshold conditions in Corollary I.b for the costs of tie deletion \( a \) illustrate that a high level of proportional conformity makes networks in which defectors are tied both to contributors and defectors subject to link deletion. The conditions imply that the highest degree among defectors who have connections to contributors is decisive for strong link deletion proofness. The higher the maximum degree within this subset, the more likely it is that a non-segregated structure can also be strong link deletion proof. This implies that highly centralized and very dense networks that contain individuals with a high degree are more likely to be strong link deletion proof.

Corollary I.b. shows that if costs of tie deletion are low, a disconnected network in which defectors are only tied to defectors and contributors are only tied to contributors will be strong link deletion proof. One should note, however, that a bipartite network, in which all defectors are only tied to contributors and contributors are only tied to defectors will also be link deletion proof in this case.

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8 In case of negative social control, mass conformity and selective incentives also play a role in strong link deletion proofness. Furthermore, contributors might also have an incentive to delete all their ties to free riders. Hence, there are stricter requirements for strong link deletion proofness - see Appendix.

9 The sign of the social control parameters does not alter our main conclusions, although the maximum degree among contributors with connections to defectors is also relevant for strong link deletion proofness (see Appendix).
The level of segregation of contributors and defectors, however, does not have an unambiguous impact on link deletion proofness. If the network is perfectly segregated, then it has a component of defectors and a component of contributors and hence it is strong link deletion proof. But this does not imply that for those networks that contain ties between contributors and defectors, a highly segregated network is necessarily prone to become even more segregated, or a moderately segregated network is more likely to be stable than a highly segregated one. The examples in Figure 1 illustrate how a less segregated network (Figure 1a) can be more subject to link deletion than a more segregated one (Figure 1b). The two networks are identical concerning the sets of contributors and defectors and have the same density. The numbers of connections of $D_1$ and $D_3$ influence network changes in the less segregated network and only $D_1$'s connections matter in Figure 1b. The improvement of network composition in terms of proportional conformity is smaller for $D_1$ in network 1b than it is in network 1a. Hence, at a wide range of parameter values, only the latter network is (strong) link deletion proof. Consequently, the initially less segregated network (in Figure 1a) becomes more segregated after individual structural decisions. The driving mechanism is again proportional conformity. Bridging actors with many ties do not benefit as much in terms of proportional conformity from deleting links to individuals with dissimilar choices, as compared to the benefits that less integrated bridging actors can obtain from deleting such ties.

Figure 1 around here

We now turn to some equilibrium properties that concern tie formation. The main results concerning forming ties are summarized in Theorem II (see Appendix for the proof). A tie $ij \not \in R$ is initiated by $i$ if exist $G_i \subseteq \hat{R}_i$, and $H_i \subseteq R_i$ such that $ij \in G_i$ and $\pi_i(\sigma, (R \cup G_i) \setminus H_i) - ga-gf > \pi_i(\sigma, R)$.

Theorem II: In any $R$ and given $\sigma$, for all $i, j \in N$ and $ij \not \in R$:

a) If $\sigma_i = 0$ and $\sigma_j = 1$, then $\pi_i(\sigma, R) \geq \pi_i(\sigma, R \cup \{ij\})$.

b) If $\sigma_i = \sigma_j$ and $\pi_i(\sigma, R) \geq \pi_i(\sigma, R \cup \{ij\})$, then $\pi_i(\sigma, R) \geq \pi_i(\sigma, R \cup G_i) - ga-gf$, where $ij \in G_i$ for any $G_i \subseteq \hat{R}_i$.

c) If $\sigma_i = \sigma_j = 1$ and $s + b_1 + b_2 \frac{r_{i,j}}{r_i(r_i + 1)} > f$, then $ij$ is initiated by $i$.

d) If $\sigma_i = \sigma_j = 0$ and $b_1 + b_2 \frac{r_{i,j}}{r_i(r_i + 1)} > f$, then $ij$ is initiated by $i$.

Part a) of Theorem II states that new relationships are not formed between contributors and defectors. The reason is that the defector does not gain anything from a new tie to a contributor. Part b) follows from that the marginal benefits of forming more ties are decreasing in the number of ties. Again, this is caused by proportional conformity. Hence, the first new tie is the most valuable, in case there is at least one connection to a defector. If a single new tie is not beneficial, then no larger new set that contains this tie can be feasible.
Similarly, assuming a coalition of defectors in which multiple ties are formed, the marginal benefits of forming new ties are decreasing.\textsuperscript{10}

Parts c) and d) yield implications for the characteristics of those network positions in which actors are most likely to form new ties. To begin with, the number of defector contacts increases the chance that a new tie is formed between two contributors. Furthermore, contributors with many connections (high $r$) are less likely to form new connections to other contributors, as it does not give them sufficient marginal benefits (if $r_{id}>0$). Considering two defectors that are not tied with each other, the likelihood of a new connection increases with the number of relations to contributors. Again, individuals with many connections are less likely to form new ties (if they are tied at least to one contributor). Individual network parameters are effective because of proportional conformity. Proportional conformity benefits of a new tie are the highest for an individual with just one existing tie to a dissimilar actor. For this individual, the proportion of similar actors is improved by a half if a new tie is formed. Starting from more ties or from more ties to similar actors mean less improvement in the composition and hence less proportional conformity gains.

In a utopian setting in which contacts are formed freely, all contributors would be interested to be matched with all other contributors and all defectors would be happy to build relations with other defectors to enjoy higher behavioral confirmation rewards. In case selective incentives are more important than behavioral confirmation, contributors would even be interested to get any kind of connections including also ties to defectors. A symmetric relationship requires a mutual agreement of the parties, however, and defectors would veto this, because additional cross group ties may reduce the benefits they enjoy from proportional conformity. Moreover, in a full contribution strategy profile highest benefits would come from a network in which everyone is tied to everyone else.\textsuperscript{11}

Like for link deletion, network characteristics influence tie formation only within the subset of those members who have abridging ties. Within this group, strongly embedded, central actors will be less likely to change their networks. Unlike in the case of link deletion, however, the level of segregation between contributors and defectors has a more clear-cut effect on network changes: more segregated networks are less likely to grow. Nevertheless, the impact of individual degree on changes is stronger than the impact of the number of abridging individual connections.

To sum up, our analysis of tie deletion and link creation has suggested that relations in collective action tend to build up slowly and break up easily. We found that the most profitable strategy for a defector is to abandon all of her ties to contributors (assuming that she prefers to delete any tie and has at least one relation to another defector). On the other hand, we also showed that forming the first new tie to another defector has the highest marginal benefits, in case there is at least one connection to a contributor.

\textsuperscript{10} Similar results are obtained also for negative social control (see Appendix).

\textsuperscript{11} Much less (if any) tie formation can be expected if social control is expressed only as punishments (see Appendix).
For illustration, we highlight the main predictions of our model with network changes with a stylized example. Consider a wild cat strike in a factory with a dense but not complete network of informal social ties among workers. The strike can be modeled as a one-shot public good game in which only informal social control fosters participation. When only a few workers participate, tensions between strikers and goons may emerge afterwards and might even result in breaking old relations. The model assumes that strikers who are only related to strikers (goons only related to goons) get positive feedback from their peers, and their relationships do not come under pressure by the event. Those who have a contact from the opposite camp, however, feel shame, guilt or are simply embarrassed by the conflict with some of their other contacts, which generates an incentive to break the relationship. Our model implies that the ‘clearing’ of such a ‘mixed’ ego-network is more difficult to the extent that ego has many connections. Hence, our analysis suggests that a more dense community is more likely to remain cohesive even after the heated times of the wild cat strike. Where workers are less embedded, however, contacts between strikers and goons might dissolve and segregated fractions may be formed.

The model also predicts that in a loosely tied group of workers, the collective experience of the demonstration might bring strikers closer to each other. Nonetheless, similar mechanisms operate among those who did not participate in the wild cat strike. They also seek reinforcement, and may form the group of “moderates” or “rational egoists”. In a denser community, however, it is less likely that such an event can contribute to the building of a larger or an even denser social network, because the relative improvements workers can attain in terms of proportional conformity are small if they are related to many cooperators and many defectors at the same time. Nevertheless, those who have many abridging relations may seek new acquaintances even in a dense network.

Nonetheless, not only structural characteristics matter. Costs of changing ties also have an impact on decisions. Large values of $a$ and $f$ can be interpreted as high constraints on network change imposed by the given network. For example, consider the case of the strike in a project team whose members are tied by a network of such dyadic task interdependencies that are important for their future work performance and thus also their career prospects. In such a situation, team members would not easily segregate along the lines of strikers versus goons, because other rewards besides selective incentives and behavioral confirmation are at stake when relations change.

### 4. Simultaneous social control and strongly robust network equilibrium

In the previous section we relaxed the traditional assumption of models of collective action that the social network is given and individuals cannot change their relations. The analysis we provided is in particular suitable for situations in which social control mechanisms are delayed compared to participation decisions in collective action. There are situations, however, when structural changes and behavior in the collective action game are simultaneous. Furthermore, even when this is not the case, actors can anticipate structural
changes at the time of their participation decision in collective action. Under such circumstances, these actions are part of the same strategy; structural decisions and network stability should be considered together with individual decisions and equilibria in collective action. For example, workers who participate in a wild cat strike may take into account the risk of losing friends and the opportunity of finding new ones. Let us consider a worker, who works in a peripheral unit in which the majority does not prefer to join the strike of the major workshop. If this worker has some friends in the major workshop, then she has to decide about participation but also about the community she wants to be embedded in at the same time.

To address situations like this one, we need an equilibrium refinement that embraces the concepts of unilateral stability and Nash equilibrium in the context of games played in social networks. For this purpose, we propose the notion of strongly robust network equilibrium. A network of social relations and a strategy profile in the (collective action) game are in strongly robust network equilibrium, if there is no actor, for whom any combination of changes in her contribution decision and in her ego-network would result in a better outcome; requiring consent for every new relation from partners.

Definition: We define the combination of $\mathbf{R}$ and strategy profile $\mathbf{\sigma}^*(\sigma^*_i, \sigma^*_j)$ a strongly robust network equilibrium,

\[
\pi_i((\sigma^*_i, \sigma^*_j), \mathbf{R}) \geq \pi_i((\sigma_i, \sigma^*_j), (\mathbf{R} \cup G_i) \setminus H_i) - h_{ij} - g_{ij}
\]

or exists such $j \in \mathbf{N}$ that $ij \in G_i$: $\pi_j((\sigma_i, \sigma^*_j), \mathbf{R}) > \pi_j((\sigma_i, \sigma^*_j), (\mathbf{R} \cup \{ij\}) - f$

for any $G_i \subset \mathbf{R}_i$, any $H_i \subset \mathbf{R}_i$ and for all $i \in \mathbf{N}$.

Note that just as in unilateral stability, the concept of strongly robust network equilibrium requires that no deviations are individually beneficial given that strategies of others and relations in which $i$ is not involved are fixed. From a new partner $j$, consent is required only for the formation of $ij$. A new partner $j$ would not give consent if her subjective rewards in the network without $ij$ were higher than her subjective rewards in the network with $ij$ considering the new strategy profile, in which only the action of $i$ might be different than the original choice. This formalization is the straightforward way to capture that strategy choices and network choices are simultaneous and not independent.

What are the conditions under which strongly robust network equilibria can occur in collective action? It follows immediately that only a Nash equilibrium strategy profile and only unilaterally stable networks can be in strongly robust network equilibrium. However, unilateral stability and Nash equilibrium in the collective action game are necessary but not sufficient conditions for strongly robust network equilibrium. Consider for instance, a situation in which a disconnected network with a complete component of contributors and a complete component of defectors is unilaterally stable. Given sufficiently high benefits from receiving selective incentives and behavioral confirmation, the strategy profile can also be in Nash equilibrium. In this situation, nobody has an incentive to abandon relations, to form ties and receive agreement from the new partners, or to change the decision in the collective action game. Restructuring relations and changing the action in collective action, however, can be beneficial for some players. If forming new ties and abandoning existing relations are
free, there would always be players for whom such changes were beneficial. Part a) of Theorem III expresses that when there are no costs of network change and social control is expressed as rewards, the number of strongly robust network equilibria is restricted to extreme configurations: *only full contribution and full defection with complete networks can be strongly robust network equilibria*. Part b) states that when the group is small and building and deleting costs $a$ and $f$ are relatively small compared to selective incentives and mass conformity a partial contribution profile cannot be strongly robust network equilibrium (see Appendix for the proof).

**Theorem III:**

a) If $a=0$, $f=0$ and $(b_1>0$ or $s>0)$, then in strongly robust network equilibrium $(R, \sigma^*)$: $R=R_N$ and $\sigma^*_i = \sigma^*_j$ for all $i,j\in N$.

b) If $s+2b_1>n(f+(n-2)a$ and $b_1\geq f$, then in strongly robust network equilibrium $(R, \sigma^*)$: $R=R_N$ and $\sigma^*_i = \sigma^*_j$ for all $i,j\in N$.\(^\text{12}\)

To see the intuition underlying this result, consider a network in which there are both defectors and contributors. Without costs for link changes, either contributors or defectors would profit from changing their contribution decision, abandoning all ties to their ex-group and build connections to every member of their new group. Contributors are better off by becoming “integrated” defectors if in the original network there are a sufficiently large number of defectors. In this case, the related gain in behavioral confirmation exceeds the loss in terms of foregone provision of the collective good and lost selective incentives. Conversely, if the number of defectors falls below this critical level, then all defectors would gain from turning into contributors and complete their network with fellow contributors. Hence, the network in which there are both defectors and contributors is not in equilibrium, because depending on the proportion of contributors in the entire network either all defectors or all contributors are pulled towards an equilibrium with uniform choices. One should note that if forming and deleting ties are free or have relatively low costs, then the assumption of simultaneous decisions about relationships and participation makes the initial network structure irrelevant. When looking at network changes only, the initial network structure matters as it constrains the possible sets of contributors and free riders. But the initially given network can also be important for collective action if behavioral changes are considered simultaneously with network updates. The key assumption needed for this is that there are costs for deleting or building ties that represent structural constraints embodied in the existing network.

Costly network changes imply that not only complete networks can be strongly robust network equilibria. When a cost of a new tie exceeds mass conformity benefits ($f>b_1$), any initial network without isolates is in strongly robust network equilibrium in a full defection profile. Furthermore, any initial network with a minimum individual degree of $r_{\text{min}}$ is in

\(^{12}\) In case of negative social control, when deleting ties is free, only full defection with complete isolation can be strongly robust network equilibrium (see Appendix).
strongly robust network equilibrium in a full cooperation profile if the cost of a new tie exceeds selective incentives and mass conformity benefits \((f>s+b_1)\) and social control rewards exceed contribution costs \((a+b_2+r_{\text{min}}(b_1+s)>c)\) to assure that switches to defection are not beneficial).

The exact condition for part b) of Theorem III also determines the cost constraint above which partial contribution with complete components of defectors and contributors can be a strongly robust network equilibrium. In combination with the arguments above it can be also stated that not only complete components can characterize partial contribution in strongly robust network equilibrium. For not complete components of defectors \(f>s+b_1\) should hold, for not complete components of contributors \(f>s+b_1\) should be satisfied. Besides, in equilibrium, every contributor including the one with the fewest connections \((r_{\text{min}})\) must be better off by remaining contributor than by switching to defection \((\alpha+b_2+r_{\text{min}}(b_1+s)>c)\) or by switching to defection and integrating with any number of defectors \((a+r_{\text{min}}(b_1+s+a)>c+b_1f)\); this condition is relevant if \(b_2\) is large.

Finally, if multiple conditions are met, tie formation and deletion costs also make it possible that ties between contributors and defectors are preserved in strongly robust network equilibria.\(^{13}\)

5. Discussion

Previous research on social network effects in collective action has shown that networks may have a double edge. Sometimes, social influence processes in networks may facilitate collective action, but at other times social ties may be instruments for deviants to resist conformity pressure or to affirm each others’ deviant behavior. This suggests that actors face not only incentives to contribute or defect, but also incentives to make or break social relations that are interdependent with their contribution decisions. Hence, an analysis of the conditions under which networks promote collective action can not be disentangled from an analysis of the dynamics of network relations between prospective participants. Previous models of collective action have considered social relations as given. Individuals change their network ties, however, and changes are partly consequences of collective interdependences.

\(^{13}\) First, deletion costs are sufficiently high such that the defector has no interest to delete this tie \((a>b_2(r_{\text{ad}}+I))\). Second, the defector has no interest to change to contribution with any combination of network update. This means that the subjective payoffs \(r_{\text{ad}}b_1+r_{\text{ad}}b_2f(r_{\text{ad}}+I)\) should be larger than

a) the subjective payoffs from a pure switch \(a+s+b_1+b_2f(r_{\text{ad}}+I)\);

b) the subjective payoffs from a switch and deletions \(a+s+b_1+b_2-r_{\text{ad}}f\);

c) the subjective payoffs from a switch and integration \(a+r_{\text{ic}}(s+r_{\text{ic}}b_1+r_{\text{ic}}b_2f(r_{\text{ad}}+r_{\text{ic}}))-r_{\text{ic}}f\);

d) the subjective payoffs from a switch, deletions and integration \(a+r_{\text{ic}}(s+r_{\text{ic}}b_1+b_2-r_{\text{ad}}a)-r_{\text{ic}}f\).

Third, the contributor is better off by staying at contribution than by switching to defection in combination with any update of her network. This means that the the subjective payoffs \(r_{\text{ic}}b_1+(r_{\text{ic}}+I)s+r_{\text{ic}}b_2f(r_{\text{ad}}+I)+\alpha\) should be larger than

a) the subjective payoffs from a pure switch \(c+b_1+b_2f(r_{\text{ad}}+I)\);

b) the subjective payoffs from a switch and deletions \(c+b_1+b_2-r_{\text{ad}}a\);

c) the subjective payoffs from a switch and integration \(c+r_{\text{ic}}b_1+r_{\text{ic}}b_2f(r_{\text{ad}}+r_{\text{ic}})-r_{\text{ic}}f\);

d) the subjective payoffs from a switch, deletions and integration \(c+r_{\text{ic}}b_1+b_2-r_{\text{ad}}a-r_{\text{ic}}f\).
This paper investigated the stability of collective action and network structures that are subject to endogenous changes.

We proposed an integrated game theoretic model and derived equilibrium predictions without the need of using simulation methods. In the proposed model, network effects were incorporated in the standard $n$-person public goods game through different social control mechanisms that assume social rewards are transmitted through interpersonal relations. Forms of social control, namely selective incentives and forms of behavioral confirmation were modeled as rewards (and punishments) that influence individual decisions through actors’ relationships to relevant others and make collective action possible. Social control that goes together with collective action may also lead actors to break relations in the network and build new ones. In order to avoid unpleasant influence and to enjoy more rewards of social control, individuals might strategically revise their network relations. Interestingly, this does not imply that networks that show at the outset a clear tendency towards segregation of contributors and defectors may have the highest potential for network segregation in the course of the collective action. We demonstrated in particular that if it is possible to delete ties, then initial segregation will not be a predictor of final segregation. On the other hand, while initially sparser sub-networks of contributors and defectors may be subject to link deletion, more densely knit subgroups may be stable and preserve bridging ties between dissimilar actors. This can be the case even if the relative measures of segregation and degree variance are the same in the original networks compared. This is caused by the behavioral assumption of proportional conformity. Proportional conformity implies that a weakly integrated bridging actor who deletes bridging ties will achieve a larger improvement in the composition of her relations than a well integrated bridging actor who deletes the same number of bridging ties.

Another model implication is that denser networks are less likely to be subject to link deletion as well as to tie formation than sparser networks. Furthermore, individual level analysis of the possibility of forming new ties demonstrated that in partial contribution those actors can particularly improve their situation by connecting to others with similar choices who have only few connections and relatively many of them lead to dissimilar actors. This means that there is an effect of initial segregation on tie formation: less segregated networks that contain actors who have several ties to dissimilar others are more likely to be exposed to tie formation than more segregated ones. The impact of segregation on tie formation, however, is not as strong as the effect of individual degree: new ties are most likely formed between two similar actors who have only few ties.

For a synthesis of analyses, we introduced a new equilibrium concept of strongly robust network equilibrium that combines equilibrium in collective action and network stability. The use of this concept is not restricted to the structurally embedded public good game. It can be applied for the analysis of any non-cooperative game in which payoffs are partly dependent on decisions about interpersonal relations that are defined within the set of players. Real-word interactions often involve simultaneous decisions about actions and connections that influence the payoffs of those actions (see Bramoullé et al., 2004; Goyal and Vega-Redondo, 2005). Collective action is just an example at hand. Game theoretical analysis
of any such situation could be helped by the adoption of a concept like strongly robust network equilibrium.

Our equilibrium concept applied to collective action has yielded some interesting implications. We have shown that in the ideal typical case where there are no costs for structural change, only full contribution and full defection with complete networks can be strongly robust network equilibria assuming positive social control. In case of negative social control and no costs of deleting relations, full defection and isolation is the only strongly robust network equilibrium. For the more realistic case where network change is costly, we could show that network structures that integrate contributors and defectors can be stable. Hence, the assumption that actors in collective action make purposive decisions about both their contributions and their social relations does neither imply an inexorable tendency towards “deviant cliques”, nor does it suggest that universal contribution or universal defection are likely outcomes in collective actions that are embedded in existing networks.

Our aim was to provide a foundation for subsequent research that recognizes the interrelation of collective action and network structure. The presented framework can be extended to similar situations with different collective structure of interdependence, including public good provision with a different production function, sustaining a public bad (cf. Kuran, 1995) and other n-person games. Another assumption that can be relaxed in future research is the binary character of social relations (two individuals are either tied or not). An alternative approach would be to assume weighted ties that model differences between, e.g., good friends and mere acquaintances in the network. In such a framework, ties are not abandoned or built, but weights are reconsidered. This model extension would also allow considering asymmetric ties (for a similar dynamic analysis see Kitts, Macy and Flache, 1999).

We adopted a model based on strategic individual actions about cooperation and network relations. Strategic decision making in a situation where both network relations and contributions decisions might change may impose an implausibly high cognitive load on decision makers. Possible ways of relaxing the strict cognitive assumptions of the model is to incorporate boundedly rational learning in repeated decision making, such as reinforcement learning (cf. Macy, 1993; Erev and Roth, 1998; Macy and Flache, 2002), or myopically forward-looking belief learning (or “fictitious play”, cf. Fudenberg & Levine, 1998), or modify the model with empirically justified assumptions on behavior and available information. Another further step would be an analysis of strategic behavior in collective action with sequential individual decisions and in the repeated public goods game. Consequently, a dynamic interrelated analysis of repeated collective action problems and structural dynamics that might explain self-reinforcing spirals of participation could also be performed. For the complexity of this problem, however, agent-based simulation techniques would be more appropriate than analytical methods. These proposed developments are purely theoretical. The potential of our approach to also inform empirical research is demonstrated by a recent study that proposes laboratory experiments with artificial networks to test some of the model predictions outlined in this paper (Takács and Janky, 2007).
Appendix

Proof of Theorem I

a) If $\sigma_i = \sigma_j = 0$, then by deleting $ij$, $\pi_i$ decreases by a non-negative proportion of behavioral confirmation and there is no source of compensation. Even when behavioral confirmation incentives are zero, there is no improvement by breaking $ij$. If $\sigma_i = \sigma_j = 1$, then by deleting $ij$, $\pi_i$ decreases by by some proportion of behavioral confirmation and by $s$ and there is no source of compensation.

b) The payoff of defector $i$ in any strategy profile $\sigma$ can consist of only the following elements: $c, n, a, r_i b_1$, and $r_i b_2$. The first three elements do not change if she abandons a relation to a contributor. The last element has an increment of $r_i b_2 / r_i (r_i - 1)$, which is always positive if $b_2 > 0$ and $r_i > 0$. If this increment is larger than the cost $a$ of deleting a tie, then $\pi_i(\sigma, R\{ij\}) > \pi_i(\sigma, R)$. Note that $ij$ could also be not stable even if this latter condition is not met as it might be an element of a set $H_i$ of links to be deleted that increases the subjective reward of the defector. The number of elements in $H_i$, is $h_i (r_i \geq r_i \geq h_i \geq 0)$, where individual $i$ should choose $h_i$ as to maximize the net benefits of her action. This means that we should find the maximum value of

$$b_2 \frac{h_i r_{id}}{r_i(r_i - h_i)} - h_i a,$$

which is the difference between the benefits and the costs of abandoning $h_i$ relations to contributors. As $h_i$ increases, the net benefits are also increasing (if $r_i > 0$). Hence, the optimal strategy for a defector who has at least one connection to another defector is to delete all her ties to contributors, which holds also for the case of negative social control (see later in the Appendix). When the benefits of deleting the first connection outweigh the costs $a$, deleting $h_i > 1$ connections will outweigh $h_i a$ with a larger margin. Hence it might happen that deleting one or few connections does not outweigh the costs, but deleting all links to contributors becomes beneficial. In short, relation $ij$ is not stable, if at least one of the actors $(i)$ benefits from deleting all of her $r_i$ connections. After substituting to (3) it means that $ij$ is not stable, if

$$\frac{b_2}{r_i} > a$$

holds, given that $r_i > 0$.

c) If $r_{id} = 0$, deleting $ij$ might be beneficial for $i$ in combination with building some new ties to defectors. This is the case when the costs of change are compensated by behavioral confirmation benefits. As marginal gains are highest for the first newly formed tie, $ij$ is not stable and a new tie to another defector is formed, if $b_1 + b_2 r_{id} > a + f$. Note that deleting a larger subset of links to contributors here does not provide higher subjective rewards for $i$ than just deleting $ij$ and forming a new relation to a defector. On the other hand, deleting all $r_i$ links to contributors and forming a new tie to a defector is beneficial if $b_1 + b_2 > r_i \ a + f$, which provides the same marginal benefits as just deleting $ij$.

Proof of Theorem II

a) Follows immediately as there are no gains for the defector.
b) Consider a contributor \(i\) who has the option to form multiple ties to other cooperators at the same time. The cost of forming one tie (\(f\)) is the same for every new relation and for both sides. Denoting the number of new ties of individual \(i\) by \(g_i\), the benefits of structural change for \(i\) are given by

\[
s + b_1 + b_2 \frac{r_{id}}{r_i(1 + g_i)} - f. \tag{9}
\]

As \(g_i\) increases, the marginal benefits are decreasing (if \(r_{id} > 0\)). Hence, the highest marginal benefits for a contributor come from a first new tie to another contributor.

c) Substituting 1 to \(g_i\) in (5) and expressing \(f\), provides the condition.

d) The result for defectors is obtained the same way as for contributors.

**Proof of Theorem III**

For the proof of parts a) and b) of the theorem, we demonstrate that in partial contribution equilibria with \(n_c\) contributors and \(n_d\) defectors (\(n_c + n_d = n\)), either contributors or free riders would have the incentive to abandon all their existing ties, form new ties with every member of the other camp and change their action in the collective action game. Contributors are better off by remaining in the complete component of contributors than by switching to defection and connect to all defectors if

\[
\alpha + (n_c - 1)s + (n_c - n_d - 1)b_1 \geq c - (n_c - 1)a - n_d f. \tag{10}
\]

On the other hand, defectors are better off by remaining in the complete component of defectors than switching to contribution and connect to all contributors, if

\[
\alpha + n_c s + (n_c - n_d + 1)b_1 \leq c + (n_d - 1)a + n_c f. \tag{11}
\]

Equations (10) and (11) cannot be simultaneously satisfied when \(a=0, f=0,\) and \(b_1>0\) or \(s>0\), which completes the proof of part a). These equations cannot be simultaneously satisfied also when \(s + 2b_1 > nf + (n-2)a\). If there are costs of tie formation, for the consent of defectors it is required that \(b_1 \geq f\), which provide the cost conditions for part b).

**Main results in case social control appears in form of punishments**

We assume in the following calculations that selective incentives and behavioral confirmation are punishments that decrease individual payoff. These calculations can be contrasted with those in the main text that hold for positive rewards of social control. The subjective payoffs of defection and contribution for \(i\) are now:

\[
\pi_i(\sigma, R) = \left(\alpha + r_i s + \left(r_{ic} - r_{id}\right) \left(b_1 + \frac{b_2}{r_i}\right) - c \right) \sigma_i - r_i s - r_{ic} b_1 - \frac{r_{ic}}{r_i} b_2 + \alpha \sum_{j=1}^{n} \sigma_j, \tag{1}
\]

where \(j \in N \setminus \{i\}\). From these equations it follows that the conditions for participation of individual \(i\) to be beneficial are exactly the same as in equation (2).

**Theorem I.** In any \(R\) and given \(\sigma\), for all \(i, j \in N\) and \(ij \in R\):
a) If $\sigma_i = \sigma_j = 1$, then $\pi_i(\sigma, R) \geq \pi_i(\sigma, R \setminus \{ij\})$.

b) If $\sigma_i = 0$ and $s + b_1 + \frac{b_2}{r_i} > a$, then:
   - if $\sigma_j = 1$, then $ij$ is not stable.
   - if $\sigma_j = 0$ and $s > a$, then $ij$ is not stable.

c) If $\sigma_i = 1$, $\sigma_j = 0$, and $b_1 + \frac{b_2}{r_i} > a$, then $ij$ is not stable.

Proof. Part a) states that a contributor always prefers to keep the links with other contributors. This follows from equations (1).

Part b) states that if deletion costs are under a threshold determined by selective incentives, behavioral confirmation, and individual degree of the defector, then the defector $i$ prefers to delete the $ij$ link to the contributor. She prefers to delete a link with a contributor, if

$$s + b_1 + \frac{b_2}{r_i} > a.$$  \hspace{1cm} (7d)

She prefers to sever all links with contributors, if

$$s + b_1 + \frac{b_2}{r_i} > a.$$  \hspace{1cm} (8d)

The left side of equation (8d) is never smaller than the left side of equation (7d), hence the most beneficial for $i$ is to delete all her ties to contributors.

When behavioral confirmation has an effect, deleting a tie to a contributor always gives higher benefits for the free rider than deleting a tie to another free rider. This has the consequence that a free rider might have an incentive to delete a tie to another free rider only once all ties to contributors have already been deleted. When this is the case, a free rider will delete all relations, if $s > a$ holds; and will delete no relations to other free riders, if $s < a$.

Part c) states that contributors prefer to delete their links to defectors if deletion costs are under a threshold determined by behavioral confirmation and individual degree of the contributor. A contributor $i$ prefers to delete a link with a defector, if

$$b_1 + \frac{b_2}{r_i} > a.$$ \hspace{1cm} (7c)

holds.

A contributor prefers to delete all links with defectors, if

$$b_1 + \frac{b_2}{r_i} > a.$$ \hspace{1cm} (8c)

Hence, the most beneficial for a contributor is to delete all her ties to defectors.

Corollary I. a): Any $R$ is strong link deletion proof if $\sigma_i = 1$ for all $i \in N$.

Corollary I. b): If $s > a$, then $R$ is strong link deletion proof if there are links only between contributors.

In general, in case of negative social control, the network will be strong link deletion proof in a strategy profile, if there is no defector for whom (8d) holds, and there is no contributor for whom (8c) holds.

Theorem II.: In any $R$ and given $\sigma$, for all $i,j \in N$ and $ij \notin R$:

a) If $\sigma_i = 0$ and $\sigma_j = 1$, then $\pi_i(\sigma, R) \geq \pi_i(\sigma, R \cup \{ij\})$. 

b) If $\sigma_i = \sigma_j$ and $\pi_i(\sigma, R) \geq \pi_i(\sigma, R \cup \{ij\}) - f$, then $\pi_i(\sigma, R \cup \{ij\}) - g$ if $ij \in G_i$ for any $G_i \subseteq \tilde{R}$.

c) If $\sigma_i = 1$, $b_2 \frac{r_{id}}{r_i(r_i + 1)} > f$, then $ij$ is initiated by $i$.

d) If $\sigma_i = 0$, $b_2 \frac{r_{ic}}{r_i(r_i + 1)} > f + s$, then $ij$ is initiated by $i$.

Proof. Part a) follows directly from the payoffs. For other parts of Theorem II, consider that contributor $i$ has an incentive to be matched with contributor $j$, assuming a cost of forming a tie $f$, if

$$b_2 \frac{r_{id}}{r_i(r_i + 1)} > f.$$ 

which follows from a comparison of marginal benefits and costs. Defector $i$ has an incentive to be matched with defector $j$, assuming a cost of forming a tie $f$, if

$$b_2 \frac{r_{ic}}{r_i(r_i + 1)} > f + s.$$ 

Consider now that contributors can form multiple ties at the same time. The cost of forming one tie $f$ is the same for every new relation and for both sides. Denoting the number of new ties of contributor $i$ by $g_i$, this structural change is beneficial for $i$, if

$$b_2 \frac{r_{id}}{r_i(r_i + g_i)} > f$$

is satisfied. This shows that the marginal benefits of forming more ties are decreasing. Similarly, the marginal benefits of forming new ties are decreasing also for defectors.

As it was discussed, when $s > a$ holds, a defector prefers to delete all of her relations. When $s < a$ and $(8d')$ hold, a defector deletes all relations to contributors and builds no ties. When $(8d')$ is not satisfied, but $(9')$ holds (high costs of link deletion and cheap tie formation), then defectors are interested to build at least one tie to another defector, but keep all their ties to contributors.

A similar result can be obtained for contributors. When $(8c')$ holds, a contributor prefers to delete all ties to free riders and has no interest to form new ties. When $(8c')$ is not satisfied and $(9')$ holds, a contributor would like to form a new tie to another contributor, but is not interested to delete any of her ties.

Hence, a combination of deleting and forming ties is never profitable in case of negative social control. This has the corollary that a strong link deletion proof network in which no dyad is interested to form a new connection is resistant to changes (and unilaterally stable).

Theorem III.: If $a = 0$ and $s > 0$, then in strongly robust network equilibrium $(R, \sigma^*)$: $R$ is an empty set and $\sigma^* = 0$. Theorem III. states when deleting relations is free, only full defection with no ties can be strongly robust network equilibrium, irrespective of the costs of forming new ties. This follows directly from (1'): as there are no positive rewards for maintaining relations and tie deletion is free, all defectors are better off by deleting all their relations by at least $s$ and contributors are gaining at least $c - \alpha$ by switching to defection and deleting all ties. (Cooperation
can only be maintained in strongly robust network equilibrium if tie deletion costs are high compared to costs of cooperation and punishments of social control are serious.)
References


http://nw08.american.edu/~hertz/Spring%202004/NetworkTrust3.pdf


