Design and Performance Evaluation of Filtered Multitone (FMT) in Doubly Dispersive Channels

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Abstract—Orthogonal frequency division multiplexing (OFDM) is the most popular scheme for broadband communications. However, it has been shown that OFDM has disadvantages in time-varying channels. Filter bank multicarrier (FBMC) is an alternative multicarrier modulation method that can be designed for robust performance in doubly (time and frequency) dispersive channels. The emphasis of this paper is on a class of FBMC systems that are called filtered multitone (FMT). We note that although unlike the more popular FBMC systems that use offset quadrature amplitude modulation (OQAM) or cosine modulated techniques to achieve full bandwidth efficiency, FMT does not have full bandwidth efficiency, its possible use in multiple-input multiple-output (MIMO) channels still makes it an attractive method. In a recent work, we have noted that the FBMC systems that are based on OQAM/cosine modulated techniques are very restrictive in MIMO channels and introduced a class of FMT systems with robust performance in doubly dispersive channels. This paper extends our previous results and shows that further improvement can be obtained by organizing the data symbols in a hexagonal lattice structure, as compared to the more popular rectangular structure. In addition, results that compare our proposed FMT systems with the conventional FMT and OFDM are presented and we show that a substantial gain in performance is achieved.

I. INTRODUCTION

Despite the fact that orthogonal frequency division multiplexing (OFDM) has been the most widely used scheme for broadband communications [1], [2], [3], it has a number of limitations that make it a rather poor choice for some applications. For instance, [4] noted that the relatively large side-lobes of the spectra of the individual subcarriers in an OFDM signal limit its applicability to cognitive radios. Also, a number of researchers have noted the limitation of OFDM in dealing with time-varying channels and have suggested the use of alternative methods that employ filter banks for multicarrier modulation.

Filter bank multicarrier (FBMC) techniques were introduced in the 1960s [5], [6]. Le Floch et al. [7] noted the significance of FBMC techniques in dealing with the channels that exhibit spreading both in time (due to multipath effects) and in frequency (due to time variation of the channel); namely, the doubly dispersive channels. It is argued, an FBMC system that addresses both time and frequency dispersions should be constructed using a prototype filter whose time and frequency dispersions match those of the channel.

A few authors have discussed and developed design methods for prototype filters that allow ISI/ICI free transmission. The most popular method is, the so called, isotropic orthogonal transform algorithm (IOTA) filter design [7]. This method was first developed by Alard [8]. Haas and Belfiore [9] noted that the members of a subset of the Hermite functions satisfy the isotropic properties, and thus concluded that a class of isotropic filters can be constructed by linearly combining the members of this subset. Haas and Belfiore called filters designed in this way Hermite pulses. In this paper, we use the term isotropic for both the IOTA design of Alard and the class of filters proposed by Haas and Belfiore.

We note that most designs of the prototype filter \( p(t) \) in the past are for a density of \( \frac{1}{TF} = \frac{1}{2} \), where \( T \) is the symbol interval in time and \( F \) is the subcarrier spacing. This choice of the density has been found to be appropriate as the more popular FBMC implementations based on offset quadrature amplitude modulation (OQAM) and/or cosine modulated filter banks (CMFB), effectively, multiply the density by a factor of 2 and, thus, a maximum density of 1 is achieved; see [10] for a recent review of this type of FBMC systems. However, unfortunately, OQAM/CMFB-based FBMC systems are limited to single-input single-output channels and studies so far have indicated that they are not extendable to multiple-input multiple-output (MIMO) channels, [11]. On the other hand, FBMC systems that are implemented to transmit conventional QAM symbols, as in OFDM, cannot achieve the maximum density of one, but are extendable to MIMO channels. In a recent work, [12], the significance of this type of FBMC systems has been discussed and the method of Haas and Belfiore [9] has been modified to design prototype filters that are more robust to doubly spread effects of channel and also can achieve densities of higher than \( \frac{1}{TF} = \frac{1}{2} \). Such designs, although significantly different with respect to sensitivity to doubly dispersive channels, are somewhat similar to what has been referred to as filtered multitone (FMT) in the literature, [13]. To differentiate the FMT designs that have been proposed in the past, [13], with the FMT designs of [12], we refer to the former as FMT-c (conventional FMT) and to the latter as FMT-dd (FMT for doubly dispersive channels).

In this paper, we extend the results of [12] and study the performance of FMT-dd designs in systems where data symbols are spread in a hexagonal time-frequency lattice structure. Moreover, we present results that compare FMT-dd with the conventional FMT as well as OFDM.

This paper is organized as follows. To keep the paper self-
II. REVIEW

A. Generalized Nyquist criterion

In an FBMC system, the transmit signal is generated as

\[ x(t) = \sum_{n} \sum_{k \in \mathcal{K}} s_k[n] p_k(t - nT) \]  

(1)

where \( p_k(t) = p(t)e^{j2\pi tf_k} \), \( \mathcal{K} \) is the set of integers that specify the subcarriers in \( x(t) \), and \( s_k[n] \) are data symbols that are spread across time (represented by the index \( n \)) and frequency (represented by the carrier index \( k \)), as depicted in Fig. 1.

It is well known that if \( p(t) \) is a root-Nyquist filter and \( P(f) = p(f) \),

\[ < p_k(t-mT), p_l(t-nT) > = \delta_{kl} \delta_{mn} \]  

(2)

where \( \delta_{kl} \) is the Kronecker delta function, we have used the inner product definition

\[ < p_k(t-mT), p_l(t-nT) > = \int_{-\infty}^{\infty} p_k(t-mT)p_l^*(t-nT)dt. \]

and \( * \) denotes complex conjugate. Also, for obvious reasons, we refer to (2) as the orthogonality condition. Moreover, using (2), one finds that the data symbols \( s_k[n] \) can be extracted from \( x(t) \) using the orthogonalization step

\[ s_k[n] = < x(t), p_k(t-mT) >. \]  

(3)

The orthogonality condition (2) and further results to be developed in the rest of this paper can be best explained through the ambiguity function of \( p(t) \), defined as

\[ A_p(\tau, \nu) = \int_{-\infty}^{\infty} p(t + \tau/2)p(t - \tau/2)e^{-j2\pi\nu t}dt \]  

(4)

where \( \tau \) is a time delay and \( \nu \) is a frequency shift. One can show that (2) leads to

\[ A_p(nT, lF) = \begin{cases} 1, & n = l = 0 \\ 0, & \text{otherwise.} \end{cases} \]  

(5)

This we refer to as the generalized Nyquist criterion.

B. Isotropic prototype filter design for FMT-dd

The term isotropic, in the context of filter design, refers to the class of filters whose time and frequency responses are the same within a scaling factor. The design method proposed in [12] constructs an isotropic filter according to the equation

\[ p(t) = \sum_{k=0}^{L} a_k h_{4k}(t) \]  

(6)

where \( h_n(t) \) are the set of Hermite functions defined as

\[ h_n(t) = \frac{1}{(2\pi)^{n/2}} \frac{d^n}{dt^n} e^{-t^2}. \]  

(7)

It can be shown that the set of functions \( h_n(t) \) for \( n = 4k, k = 0, 1, 2, \cdots \) are isotropic. This implies that by construction \( p(t) \) in (6) is always an isotropic function.

In [12] it is noted that the presence of channel will result in a disturbed ambiguity function, \( A_p^e(\tau, \nu) \), in which the null points of \( A_p(\tau, \nu) \) are smeared out and, thus, is argued that to design a robust prototype filter, the constraints on the nulls of the ambiguity function \( A_p(\tau, \nu) \) may be relaxed. Each null point is replaced by a region in the \((\tau, \nu)\)-plane, that we call null region. It is thus proposed to design \( p(t) \) one should choose to minimize the cost function

\[ \zeta = \gamma_0 \int_{A_0} |A_p(\tau, \nu) - 1|^2 d\tau d\nu \]

\[ + \sum_{k=1}^{N} \gamma_k \int_{A_k} |A_p(\tau, \nu)|^2 d\tau d\nu. \]  

(8)

where \( \gamma_k \)'s are a set of positive weighting factors, \( A_0 \) is the region around \((\tau, \nu) = (0, 0)\) over which the peak of \( A_p(\tau, \nu) \) remains approximately equal to a non-zero constant, and \( A_k, \) for \( k = 1, 2, \cdots , N, \) are a set of null regions.

III. HEXAGONAL VERSUS RECTANGULAR LATTICE

The spread of data symbols in Fig. 1 follows an orientation that is referred to as rectangular lattice. In [14], it is argued that better designs may be obtained by adopting the hexagonal orientation depicted in Fig. 2. The argument in [14] follows the fact the hexagonal orientation allows maximum separation of points for a given symbol density. We refer an interested reader to the detailed discussions in [14].

The definition (4) is for the case where \( p(t) \) is a real-valued function of time. In the more general case where \( p(t) \) is a complex-valued function, \( p(t - \tau/2) \) is replaced by \( p^*(t - \tau/2) \).
In [12], it is proposed and shown that good designs are obtained by limiting the constrained points according to those depicted in Fig. 3. In this paper, to design prototype filters for an orientation that follows the hexagonal lattice, we choose the constrained points according to those depicted in Fig. 4. We also set $T = 2F$ and note that for this choice enforcing design constraints at the points indicated by solid circles with an additional circle around each of them is sufficient. As in the case of rectangular lattice, [12], once the constraints are applied to these points, similar constraints will be automatically imposed to the rest of solid circles, following the isotropic property of the designs (when $F = T/2$). Moreover, the remaining grid points, indicated by empty circles, will be close to zero, within a good approximation, thanks to the exponential decay of the designed pulse $p(t)$ as $t$ increases. Finally, a time scaling can be applied to $p(t)$ to set $T$ to any desired value.

### IV. Numerical Results

To demonstrate the effectiveness of the proposed design procedure, we present a few numerical examples. We consider a channel model whose impulse response spans over an interval $(-\Delta \tau, \Delta \tau)$ and consists of a number of independent paths that are uniformly distributed in this interval and all have the same power, i.e., a constant delay power profile is assumed. Each path is assumed to be subject to an independent Doppler shift according to the Jake’s model [15] with a maximum Doppler shift $f_d = \frac{\Delta \nu}{2}$. This is equivalent to saying there is a frequency spreading in the interval $(-\frac{\Delta \nu}{2}, \frac{\Delta \nu}{2})$ with a distribution that follows that of the Jake’s model. We use the time-delay product $\Delta \tau \Delta \nu$ to quantify the channel spreading in both time and frequency. We also note that for a given $\Delta \tau \Delta \nu$ our channel model may be thought as a worst case scenario, since a realistic channel with a typical exponential delay power profile and with a time span $\Delta \tau$ is characterized by a much smaller effective spread when compared to the assumed uniform power profile. Similarly, in a practical channel not all the paths may suffer from the same maximum Doppler shift.

Fig. 5 and Fig. 6 present the results of the robust isotropic designs for the cases of rectangular and hexagonal lattices, respectively. These designs are obtained for the channel model that was introduced above and for the choice of the parameters $\Delta \tau = 0.2T$ and $\Delta \nu = 0.2F$. Also, the designs are for the density $D = \frac{1}{TF} = 0.5$. More specifically, the designs are obtained by setting $T = F = \sqrt{2}$ and, thus, $\Delta \tau = \Delta \nu = 0.2\sqrt{2}$ and running the procedure discussed in [12]. The results clearly show the desired performance of the designs. As seen,
Fig. 5. Results of a design based on the method of this paper and a rectangular lattice. (a) Mesh plot of the ambiguity function $A_p(\tau, \nu)$ and its cross section along the indicated line. (b) The disturbed plots after adding the channel effect.

Fig. 6. Results of a design based on the method of this paper and a rectangular lattice. (a) Mesh plot of the ambiguity function $A_p(\tau, \nu)$ and its cross section along the indicated line, denoted by the variable $\eta$. (b) The disturbed plots after adding the channel effect.
wide nulls appears in the ambiguity function $A_p(\tau, \nu)$ and this clearly leads to robust performance in the presence of channel, as discussed in [12]. Moreover, as one would expect, the superior performance of the design when the grid points are set in a hexagonal lattice instead of a rectangular lattice is clearly observed; the nulls in both $A_p(\tau, \nu)$ and $A_p^H(\tau, \nu)$ are deeper in Fig. 6 than their counterparts in Fig. 5.

Fig. 7 presents a set of results that compare the signal-to-interference ratio (SIR) of the robust isotropic designs, i.e., FMT-dd, with those of OFDM and FMT-c. Three sets of results, corresponding to the density values $D = 1/TF = 1/2$, $1/1.5$ and $1/1.25$, are presented. The robust isotropic designs are obtained for the channel model introduced above and the parameters $\Delta \tau = 0.2T$ and $\Delta \nu = 0.2F$. These designs are then examined for values of $\Delta \tau \Delta \nu$ in the range of 0 to 0.1. For FMT-c, the prototype filter is designed in each of the three cases with the aim of achieving a stopband attenuation of 60 dB or better. Also, following the basic principle of FMT, the roll-off factors of the prototype filters for the density values $D = 1/2$, $1/1.5$ and $1/1.25$, are set equal to 1, 0.5 and 0.25, respectively. For OFDM, the density $D = 1/TF$ is set by adjusting the ratio of cyclic prefix length, $T_{CP}$, over the length of FFT, $T_{FFT}$. More particularly, we note that since in OFDM $F = 1/T_{FFT}$ and $T = T_{CP} + T_{FFT}$, $D = T_{FFT}/(T_{CP} + T_{FFT})$ and, thus, $T_{CP}/T_{FFT} = 1/D - 1$.

![Fig. 7. The ambiguity function of the prototype filter $p(t)$ designed using the method of this paper. The design is for the density $\frac{1}{T_F} = \frac{1}{2}$.](image)

The results presented in Fig. 7 lead to the following observations:

- In high mobility environments, FMT-dd designs significantly outperform OFDM and FMT-c.
- FMT-c outperforms OFDM. This is in line with a few recent reports that have compared FMT-c and OFDM in mobile environments, [16], [17].
- The isotropic designs that are based on the hexagonal lattice consistently perform around 2 dB better than their rectangular lattice counterparts. This shows 1 dB improvement over the earlier results in [14].
- As density $D$ decreases, the performance of FMT-c approaches that of the FMT-dd with rectangular lattice.

**V. Conclusion**

This paper extended our previous results in [12] and showed that further improvement could be obtained by organizing the data symbols in a hexagonal lattice structure, as compared to the more popular rectangular structure. The results that compare our proposed FMT systems with the conventional FMT and OFDM were also presented. The presented results showed that a substantial gain in performance is achieved. For some fast time-varying channel, the proposed FMT outperforms the conventional FMT and OFDM by a gain that could be as much as 5 and 8 dB, respectively.

**References**