Erratum


Başak Karpuz*, Özkan Öcalan
Kocatepe University, Department of Mathematics, Faculty of Science and Arts, ANS Campus, 03200 Afyonkarahisar, Turkey

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In [1], the authors study the stability of the delay dynamic equation

\[ x^\Delta (t) + p(t) x(t - \tau(t)) = 0 \quad \text{for all } t \in \mathbb{T}, \tag{1} \]

where \( \mathbb{T} \) (i.e., a nonempty subset of reals) is a time scale and \( p \in C_{rd} (\mathbb{T}, \mathbb{R}^+) \).

The statements of their theorems and representations include some mistakes as follows:

\( \tau : \mathbb{T} \to (0, r] \) and \( t \in \mathbb{T} \) with \( t - \tau(t) \geq \min \mathbb{T} \), which implies \( t - \tau(t) \in \mathbb{T} \). The forward jump operator \( \sigma(t) := \inf \{s > t : s \in \mathbb{T}\} \) and the graininess function \( \mu(t) := \sigma(t) - t \).

Theorem 0.1 ([1, Theorem 1.1]). Suppose that

\[ \int_{t-r}^{\sigma(t)} p(\eta) \Delta \eta \leq \frac{3}{2} + \frac{\min_{t \in \mathbb{T}} \mu(t)}{\max_{t \in \mathbb{T}} \mu(t) + r} \quad \text{for all } t \in [\beta + r, \infty)_{\tau} := [\beta + r, \infty) \cap \mathbb{T} \]

holds. Then the zero solution of (1) is uniformly stable.

Theorem 0.2 ([1, Theorem 1.2]). Suppose that there exists a positive constant \( c \) such that

\[ \int_{t-r}^{\sigma(t)} p(\eta) \Delta \eta \leq c + \frac{\min_{t \in \mathbb{T}} \mu(t)}{\max_{t \in \mathbb{T}} \mu(t) + r} \quad \text{for all } t \in [\beta + r, \infty)_{\tau} \]

and

\[ \int_{\beta}^{\infty} p(\eta) \Delta \eta = \infty. \]

Then the zero solution of (1) is uniformly asymptotically stable.

Note that, when \( t \in \mathbb{T} \), \( t - r \) may not be in \( \mathbb{T} \). For instance, let \( \mathbb{T} = \bigcup_{k=0}^{\infty} [2k, 2k + 1] \) and

\[ \tau(t) = \begin{cases} t - 2k, & t \in (2k, 2k + 1) \\ 1, & t = 2k \end{cases} \]
for all $k \in \mathbb{Z}$. It is easy to see that $r = 1$, but $t - 1$ is not in $T$ for all $t \in T$; i.e., $t = 5/2 \in T$ but $t - 1 = 5/2 - 1 = 3/2 \notin T$.

Now, we correct and restate the theorems given above. First, we rewrite (1) as follows:

\[ x^3(t) + p(t)x(\tau(t)) = 0 \quad \text{on } [t_0, \infty)_{\tau}, \tag{2} \]

where $t_0 \in T$, $\sup T = \infty$, $p \in C_{rd}(T, \mathbb{R}^+)$ and the delay function $\tau : T \to T$ is an increasing function which satisfies $\lim_{t \to \infty} \tau(t) = \infty$ and $\tau(t) < t$ for all $t \in T$. For convenience, we define

\[
\lambda(t) := \inf_{s \in [t, \infty)_{\tau}} \sup_{s \in [t, \infty)_{\tau}} \mu(s) + \sup_{s \in [t, \infty)_{\tau}} [s - \tau(s)]
\]

on $[t_0, \infty)_{\tau}$.

**Correction of [1, Theorem 1.1].** Suppose that

\[
\limsup_{t \to \infty} [t - \tau(t)] < \infty
\]

holds. If

\[
\int_{\tau(t)}^{\sigma(t)} p(\eta) \Delta \eta \leq \frac{3}{2} + \lambda(t_0) \quad \text{for all } t \in [\tau^{-1}(t_0), \infty)_{\tau}
\]

holds, then the zero solution of (2) is uniformly stable on $[t_0, \infty)_{\tau}$.

**Correction of [1, Theorem 1.2].** Suppose that (3) holds and there exists a constant $c > 0$ satisfying

\[
\int_{\tau(t)}^{\sigma(t)} p(\eta) \Delta \eta \leq c < \frac{3}{2} + \lambda(t_0) \quad \text{on } [\tau^{-1}(t_0), \infty)_{\tau}.
\]

If

\[
\int_{\tau(t)}^{\infty} p(\eta) \Delta \eta = \infty
\]

holds, then the zero solution of (2) is uniformly asymptotically stable on $[t_0, \infty)_{\tau}$.

Proofs of the above corrections follow similarly to the proofs in [1].

**References**