APPLICATION OF MVDR BEAMFORMING TO REJECT TURBULENCE NOISE IN A DUCT

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ABSTRACT

In active noise control (ANC) on air duct/fan systems it is desirable to achieve noise cancellation using the shortest possible duct length. A barrier to this goal is the presence of air turbulence at the duct input due to the fan driving the system. Turbulent pressure fluctuations contaminate measurement of the acoustic noise in the duct, resulting in degraded noise cancellation performance. In this paper we investigate the use of adaptive array processing techniques to suppress the effects of turbulence. Based upon spatial correlation and temporal spectral characteristics of turbulence, we propose a hybrid generalized sidelobe canceller (GSC) which is constrained to adapt only in the frequency range where both acoustic noise and turbulent noise contribute correlated components to the multivariate input covariance. The effectiveness of this method is demonstrated using actual experimental data.

1. INTRODUCTION

In this paper we discuss the use of minimum variance distortionless response (MVDR) beamforming for separating acoustic noise from pressure fluctuations due to turbulent flow in an air duct. An ANC system is illustrated in Fig. 1. A reference input is passed to an adaptive controller that drives an actuator (i.e., audio speaker) to cancel acoustic waves travelling along the air duct. The reference signal originates from a region near the fan discharge where there is significant turbulence. The controller error signal originates from a region further away from the fan discharge, past the actuator. Here turbulence effects are insignificant. Turbulence at the reference input contaminates measurement of the acoustic noise in the duct and, consequently, degrades system performance.

Previous efforts at reducing turbulent pressure fluctuations in ducts have employed anti-turbulence microphone probes [1]. Use of adaptive techniques for reducing turbulent boundary layer pressure fluctuations in mobile sonar arrays is reported by Owsley and Fay [2]. While our application is similar in some respects to that of [2], there are several important differences. First of all, in our application turbulence is generated by a fan and is not a boundary layer phenomenon. Hence, the turbulent pressure fluctuations are expected to be of a different character. Second, we pursue a broadband beamforming approach, in contrast to the narrowband methods of [2]. Lastly, the performance predictions in [2] are based on simulations and not on actual experimental data.

The metric used to evaluate performance is the coherence between the beamformer output and the acoustic signal measured far down stream in the duct out of the turbulent regime. Results are interpreted in terms of the eigenstructure of the estimated covariance matrix. This analysis identifies the impact of array geometry and turbulence characteristics on performance and suggests use of a hybrid adaptive-nonadaptive beamformer. We demonstrate coherence improvement in frequency bands where the turbulence is spatially correlated. We also demonstrate the effectiveness of a simple power based array calibration method.

2. SYSTEM AND APPLICATION OF MVDR PROCESSING

Anti-turbulence microphone probes discriminate against turbulence most effectively when positioned parallel to the air flow [1]. For this reason we chose a similar configuration for the microphone arrays. A linear array of 9 microphones was placed parallel to the air flow near the fan, a position where turbulence is prevalent in this rectangular duct. A linear array of 4 microphones was similarly placed further down the duct where the effects of turbulence are insignificant. The setup is shown in Fig. 2.

As pointed out in [1] pressure fluctuations associated
with a propagating acoustic wave will travel near the speed of sound, \( v_s \) (1100 ft/s), whereas pressure fluctuations due to turbulence will travel more on the order of the mean flow rate, \( v_f \) (approximately 33 ft/s in this duct). We model the propagation of energy as a function of velocity using a phase vector

\[
d(v, \omega) = \begin{bmatrix}
\sigma_0(\omega) \\
\sigma_1(\omega) e^{-j \frac{k}{2} d} \\
\sigma_2(\omega) e^{-j \frac{3k}{2} d} \\
\vdots \\
\sigma_{N-1}(\omega) e^{-j \frac{(N-1)k}{2} d}
\end{bmatrix}^T,
\]

(1)

where \( v \) is the propagation velocity, \( \omega \) the temporal frequency, \( N \) the number of microphones, \( \sigma_i(\omega) \) the frequency response of microphone \( i \) and \( d \) the microphone spacing. This model assumes the array is short enough so that the turbulence does not decorrelate across the aperture.

The beamformer weights are constrained to pass signals propagating at the speed of sound with unit gain, i.e.,

\[
w^T d(v_s, \omega) = 1.
\]

(2)

A single linear constraint is sufficient to control the acoustic wave response over the entire frequency band due to the relatively small transit time of the acoustic sound across the array. More explicitly, \( v_s \) is large enough relative to \( d \) such that the exponents of (1) are approximately zero for all \( \omega \) and \( i \). Thus

\[
d(v_s, \omega) \approx \begin{bmatrix}
\sigma_0(\omega) \\
\sigma_1(\omega) \\
\sigma_2(\omega) \\
\vdots \\
\sigma_{N-1}(\omega)
\end{bmatrix}^T.
\]

(3)

This constraint results in an MVDR solution for the weight vector.

The \( \{ \sigma_i(\omega) \} \) of (3) represent the gain and phase response of the microphones, which is unknown. Since calibration data is not available, we adopt the simple approach of estimating the broadband microphone gains from the power received at each microphone. We ignore the phase response and set

\[
d(v_s, \omega) = \sqrt{\text{diag}(R_x)}.
\]

where \( R_x = E(xx^T) \) is the data covariance matrix and \( x \) is the data at the reference array. With this approach \( d(v_s, \omega) \) is independent of frequency as is the constraint (2). As the gains are approximately the same, \( d(v_s, \omega) \) is close to a constant times the all ones vector.

The MVDR weights are obtained by minimizing output power

\[
\min_w w^T R_x w
\]

subject to (2).

### 3. DATA ANALYSIS

Near the fan, the power spectrum of the microphone output is dominated by very low frequency noise (see Fig. 3). Cancellation is difficult at very low frequencies due to loudspeaker limitations. Furthermore, at low frequencies the turbulence phase vector and acoustic noise phase vector are nearly identical, so spatial filtering cannot be used to separate them. To mitigate these difficulties, we applied a 5th-order, 60 Hz highpass Butterworth filter to data prior to processing. The results are depicted in Fig. 4 and compared to the coherence obtained by simply averaging the microphone outputs. \( R_x \) was estimated using 60,000 snapshots. The simple calibration method used here results in dramatic improvements in coherence. The MVDR method offers significant improvements in coherence relative to averaging between 50 and 100 Hz. However, above 100 Hz the MVDR performance is worse than that of averaging.

To understand these performance characteristics we examined the eigenstructure of the data covariance matrix as a function of frequency. The data was highpass filtered using successively higher cut-on frequencies. This gives an indication as to how the eigenstructure changes as a function of frequency because of the rapid decrease in signal energy as frequency increases. The results are depicted in Table 1 with eigenvalues \( \lambda_i \) ordered \( \lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_9 \). \( e_i \) is the \( i \)th eigenvector. Column two presents eigenvalues that are 10% of the largest eigenvalue or greater. This serves as an approximate indication of the covariance matrix rank. In column three is a plot of the eigenvectors associated with the first three largest eigenvalues.

We have identified three different regimes of covariance properties as a function of frequency and have chosen four frequency bands exemplifying these properties.

1. With no highpass filtering low frequency characteristics are dominant. The covariance is essentially rank one, and the most significant eigenvector corresponds to \( d(v_s, \omega) \). This suggests that both acoustic and turbulent
signal energy lies mainly along the constraint, \( d(v_a, \omega) \).
Hence, with our array configuration it is not possible to suppress turbulence effects at low frequencies.

2. Filtering out the low frequency energy distributes spatial components along directions other than the constraint direction. With a 60 Hz cut-on frequency the covariance has two dominant modes along with some lesser modes. This scenario is suited to beamforming since there is spatial correlation in modes other than the constraint.

3. With a 150 Hz cut-on frequency there are many significant modes suggesting that the turbulence is nearly spatially white. This scenario is well-suited to simple averaging of microphone inputs.

4. With a 300 Hz cut-on frequency the covariance is again rank one, but for reasons other than in Item 1. Here there is no turbulent energy and neither MVDR nor simple averaging will improve coherence.

These results suggest the following. The greatest potential for performance improvement via adaptive techniques lies in the middle frequencies (60-150 Hz). The lack of spatial correlation at higher frequencies (150-300 Hz) indicates averaging is a good choice. In theory, the MVDR weights will reduce to averaging when the noise is white, however, in practice calibration errors and covariance matrix estimation errors will cause the adaptive solution to deviate from the averaging solution. This is most likely the cause of the performance loss observed in Figure 4.

4. HYBRID GENERALIZED SIDELOBE CANCELLER

In order to prevent adaptive performance loss at higher frequencies, we propose the hybrid generalized sidelobe canceller illustrated in Fig. 5 to implement an MVDR solution. The temporal bandpass filter (60-150 Hz, 10th-order Butterworth) prevents adaptation above 150 Hz. To compen-

\[
w_\omega = d(v_a, \omega) (d(v_a, \omega)^H d(v_a, \omega))^{-1},
\]

\( C_n \) is a matrix with columns orthogonal to \( d(v_a, \omega) \) and the adaptive weight vector is

\[
w_n = R_s^{-1} r_{xy_n}
\]

with \( R_s = E[zz^H] \), \( r_{xy_n} = E[xyz] \). The results of processing using the hybrid GSC are shown in Fig. 6. The performance now equals or exceeds that of averaging at nearly all frequencies.

In principle, increasing the array aperture will allow coherence improvement for frequencies less than 60 Hz. This is a desirable result. However, the turbulence tends to decorrelate with increasing distance, an effect that reduces the beamformer’s ability to achieve cancellation and thus may limit the effectiveness of an increased aperture.

5. REFERENCES


<table>
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<th>Filter Specs</th>
<th>Significant Eigenvalues</th>
<th>Significant Eigenvectors</th>
<th>Turbulence Conjectures</th>
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<td>$\lambda_1/\lambda_5$</td>
<td><img src="image4" alt="Graph" /></td>
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Table 1: Eigendecomposition for the covariance matrix of the rectangular duct input array data. The solid line is $e_1$; the dash/dot line is $e_2$; and the dotted line is $e_3$.

Figure 5: Hybrid sidelobe canceller that adapts to correlated turbulence, but is nonadaptive in frequency bands with spatially uncorrelated noise.

Figure 6: Coherence results. Dotted line is averaging technique. Solid line is hybrid GSC method.