Computer aided contouring operation for traveling wire electric discharge machining (EDM)

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Abstract

This paper develops a computational method for numerical control (NC) of traveling wire electric discharge machining (EDM) operation from geometric representation of a desired cut profile in terms of its contours. Normalized arc length parameterization of the contour curves is used to represent the cut profile and a subdivision algorithm is developed together with kinematic analysis to generate the required motions of the machine tool axes. In generating the tool motions for cutting sections with high curvatures such as corners with small radii, a geometric path lifting method is presented that increases the machining gap and prevents gauging or wire breakage.

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1. Introduction

Wire EDM is a process of metal cutting where a special traveling wire is used as the cutting tool and metal cutting is achieved through spark erosion. A functional diagram of an EDM system is shown in Fig. 1. Sparks are generated between the workpiece and the traveling wire which acts as the tool electrode. The workpiece is connected to a DC power and immersed in a dielectric fluid. The wire is made of thin copper (usually 0.1–0.3 mm in diameter) and is drawn by a wire feed through guides or wire heads and is kept in tension using a mechanical tensioning device. The tension in the wire reduces the potential for taper and keeps the wire as a straight line. During the operation of the machine tool, a pump provides filtered dielectric fluid to keep the workpiece and the wire surrounded by fluid. Time pulses of electric energy are sent through the wire vaporizing the dielectric fluid at the point of discharge blowing off tiny pieces of the workpiece material as well as the wire at the area of the cut. Because the wire is drawn by the wire feed at each time instant a new section of the wire acts as the electrode so that old sections of the wire are not involved in the cutting operation and instead they are gathered for the scrap. The gap between the wire and the workpiece is maintained by the control system to produce a smooth cut. The dielectric fluid serves as an insulator until the electric potential is sufficiently high in the gap for the spark to be generated. The same fluid carries the debris that is removed from the workpiece and the wire away from the machining gap. If the electric potential is too high, then the sparks can cause over erosion of the workpiece (gauging) or breakage of the wire. In this paper, a method is presented that can adjust the gap using geometric information from the desired cut profile to avoid gauging or wire breakage.

A typical kinematic structure for a traveling wire EDM system is shown in Fig. 2. The figure shows the traveling wire, the wire guiding head, a wire guiding rack, and the machining table. The wire guiding rack is stationary and provides a fixed point about which the orientation of the wire is changed due to the motion of the wire guiding head. The workpiece is fixture to the machining table. Although the machine tool has a total of five degrees of freedom, the tool or the wire only moves with four degrees of freedom relative to the workpiece. This is because the wire guiding head moves such that one of its axes is used to adjust the length of the wire. In most applications, the remaining four degrees of freedom of the machine tool are used in a coordinated fashion to produce a cut profile, which is ideally a ruled surface.

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Contouring operation, discussed in this paper, is the computational process of generating this cut profile based on specification of only two boundary (contour) curves. The two boundary (contour) curves are specified on the raw material or the workpiece and the EDM system is to cut a surface on it. Since the cutting tool is a thin wire, it can be approximated by a line and the resulting cut profile is, ideally, a ruled surface. Ravani and Wang [1] used line geometry for representation of cut profiles in EDM and developed methods for design of Bezier and B spline ruled surfaces using line segments as control elements. Here we consider tool motion generation in the contouring operation where a desired cut profile is only specified in terms of its two boundary curves. Zewang et al. [2] studied contouring operations for NC machining with wire EDM and used a method to model the ruled surface cut profile by relating the parameters of the two contours or boundary curves in the parameter domain. Ravani and Chen [3] have shown that representation of a ruled surface in terms of its two boundary curves is not unique but depends on the relationship between the parameterization of the two boundary profiles. Here we introduce a relationship between the parameters of the two boundary or contour curves based on normalized arc length which results in a unique and desirable representation of the ruled surface definition of the cut profile for EDM applications. This limits the shape of the resulting ruled surface but eases up the task of tool motion generation. It is also different from the method of Zewang et al. [2] in that it is not dependant on the variations of the parameters of the two boundary curves resulting in a more uniform distribution of the rulings of the ruled surface which is important in machining with wire cut EDM. In addition, a subdivision method is developed which is in terms of segmental approximation of the contour curves and takes advantage of the arc length parameterization. Kinematics analysis of the EDM system is then used to

![Functional diagram of a wire EDM system](Fig.1)

![Typical kinematic structure of a wire cut EDM system](Fig.2)
generate the required motion of the machine tool axes (NC program) in terms of the desired incremental tool trajectory in the Cartesian space represented in terms of the subdivided cut profile.

In the case of generating the tool motions for cutting areas with rather high curvatures such as corners with small radii, a path lifting method is presented which increases the gap between the wire and the workpiece avoiding gauging or breakage of the wire. Inaccuracies due to gauging have been inevitable in cutting small radii (<0.1 mm) and sharp corners (<135°) using wire EDM when no correction is taken. Several investigations have managed to reduce the inaccuracies by changing the machining parameters [4] and by using adapting and optimization techniques [5]. In this paper, we develop a geometric method based on a path-lifting operation that can ideally improve the machining accuracy in cutting small radii. Other works in modeling contouring operations in wire EDM includes that of Liu and Sterling [6] in which a solid modeling approach was used together with a swept volume representation of the wire movement.

2. Modeling of cut profiles for traveling wire EDM

In this section, we will discuss the problem of modeling the cut profiles in contouring operation for traveling wire EDM. In general the shape of a ruled patch is determined by the shape of its two boundary curves and the parameterization of the ruling segments [3]. In the contouring operation, the shape of the desired cut profile is specified in terms of the two boundary curves or contours. The correspondence between the parameters of the two boundary curves which determines the parameterization of the ruling segments is, however, not usually specified. This means that a large number of cut profiles can be machined that would satisfy the desired specification of the two boundary curves or contours of the cut profile.

For simplicity in discussion, we denote the two boundary curves as \( r_1(u_1) \) and \( r_2(u_2) \), respectively, \( u_1, u_2 \in [0, 1] \). We want to generate the ruled surface patch whose boundary curves are \( r_1(u_1) \) and \( r_2(u_2) \). The two parameters must have a functional correspondence such that a parameterization for the generating ruling can be established. The parameters \( u_1 \) and \( u_2 \) can, therefore, be expressed as parameterized functions of another parameter \( u, u \in [0, 1] \). In other words, the parameter \( u_2 \) should be a function (here we only consider single-valued function) of the parameter \( u_1 \) and vice versa. Let us write the functional correspondence between \( u_1 \) and \( u_2 \) as \( F \) or

\[
F : u_1 \rightarrow u_2
\]

If \( u_1 = f(u) \) then \( u_2 = F(u_1) = F(f(u)) \). Therefore, the two curves can be re-parameterized in terms of \( u \) as

\[
r_1(u_1) = r_1(f(u)) = r_1^*(u)
\]

and

\[
r_2(u_2) = r_2(F(f(u))) = r_2^*(u)
\]

respectively. The correspondence between the parameters of the two boundary curves determines the parameterization of the generating ruling.

In generating the toolpath of the wire cut EDM, we have to establish a fixed correspondence between the two boundary curves. Here, we introduce the normalized arc length parameterization for both boundary curves. This parameterization is defined as

\[
u = s/L
\]

where \( s \) is the accumulating arc length from one end of the curve to the datum point and \( L \) is total arc length of the curve. The parameter \( u \) ranges between 0 and 1 and is a measure of the relative position of a point on the curve, regardless of the curve’s length. Such a correspondence is most appropriate for our application, since the arc length of a curve \( r(f) \) is defined as

\[
s(f) = \int_0^f |dr/d\phi| \, d\phi
\]

As long as \( |dr/d\phi| \neq 0 \) (i.e. a smooth curve), we can invert the function \( s(\phi) \) to yield \( \phi(s) \). This means that we can now re-parameterize \( r_1^*(u_1) \) and \( r_2^*(u_2) \) as

\[
r_1(u_1) = r_1^*(u)
\]

and

\[
r_2(u_2) = r_2^*(u)
\]

respectively, such that \( u \in [0, 1] \). After the re-parameterization, we have the new representations of the two curves as \( r_1^*(u) \) and \( r_2^*(u) \) and they define the two boundary curves of a ruled surface patch.

Once the correspondence between the parameters of the two contour curves are established, we can construct the ruled patch since it is defined as a one-parameter family of line segments over a finite interval. We can take the two corresponding points \( r_1^*(\tilde{u}) \) and \( r_2^*(\tilde{u}) \), \( \tilde{u} \) being a parameter value, of the two curves at two end points of a line segment \( P(v) \). In this case, we write the parametric equation of \( P(v) \) as

\[
P(v) = \begin{bmatrix} v \\ 1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}
\]

(2)

where \( v \in [0, 1] \) and \( P_1 \) and \( P_2 \) are the two end points of \( P(v) \), then the ruled patch can be written as

\[
P(u, v) = \begin{bmatrix} v \\ 1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} r_1^*(u) \\ r_2^*(u) \end{bmatrix}
\]

(3)

Alternatively, we can represent a ruled patch in terms of one of the boundary curves, say \( r_1^*(u) \), and its generating ruling
\[ I(u) = r_2(u) - r_1(u). \]

Both such representations are in terms of the new parameterization based on the normalized arc length. Using this new parameterization results in a more even transition from one ruling of the surface to the next ruling. This smoother transition would make the method well suited for tool motion generation in wire cut EDM. Fig. 3 shows the distribution of the generating ruling for a cut profile using this method (Fig. 3B) as compared to the ruling generated (Fig. 3A) for the same definition of contour curves but using a uniform correspondence between the two contour curves in the parameter domain [2]. Comparing these two figures, it is clear that the method of generating the cut profile presented here results in a smoother transition between the generators of the surface. This means that the wire orientation does not have to change abruptly in some areas of the cut when compared to using the tradition method that uses uniform parameterization.

The machining of the ruled patch requires determination of tool motion of the wire cut EDM system according to the toolpath that will produce the profile. This is presented in Section 3.

3. Tool path generation

In tool path generation for wire cut EDM, the relative motion between the wire guiding head and the machining table of the EDM system should be determined as a function of the desired ruled generator of the cut profile. This requires developing the kinematic equation relating the trajectory of the wire to the motion of the machine tool axes.

3.1. Kinematics of the five-axis EDM system

In a typical five-axis wire cut EDM system, the wire guiding head has three axes of movement and the machining table moves in the two linear directions. The wire guiding rack is stationary and provides a fixed point about which the orientation of the wire is changed due to the motion of the wire guiding head. The wire guiding head moves such that its axis of motion (in this case the Z axis) is used to adjust the length of the wire. The tool or the wire, therefore, only moves with four degrees of freedom with respect to the workpiece. We shall refer to the fixed point of the wire guiding rack as the base of this rack. We let \( U, V, \) and \( Z \) indicate the motion directions for the head and \( X \) and \( Y \) represent the directions of the motion of the machining table.

We attach a coordinate system \( \sigma_1 \) fixed to the machining table and measure its relationship to the world coordinate system \( \Sigma \) by the motion variables \( d_1 \) and \( d_2 \) (Fig. 4). The fixed coordinate system \( \Sigma \) is attached such that at the initial position of the machining table the two coordinate systems \( \sigma_1 \) and \( \Sigma \) are coincident. The motion variables \( d_1 \) and \( d_2 \) are then the machine tool parameters specifying the linear displacements of the machining table in \( X \) and \( Y \) directions.

If we attach a coordinate system \( \sigma_2 \) to the wire guiding head (Fig. 5), then the three parameters \( d_3, d_4 \) and \( d_5 \) can be used to describe its kinematic relationship with respect to the world coordinate system (Fig. 5). For most of the existing wire cut EDM systems, the initial position of \( \sigma_2 \) does not coincide with \( \Sigma \) defined above. We shall refer to the initial position of the wire guiding head with respect to the world coordinate system as \( a = [a_1, a_2, a_3]^T \).

In the wire cut EDM operation, simultaneous motion of the machining table and the wire guiding head produces a trajectory for the wire relative to the workpiece (fixture on the machining table) which is the desired shape. As pointed out earlier, this trajectory has only four degrees of freedom.

The appropriate kinematic equations for the EDM system should relate the motion of the tool (or the cutting wire) to that of the machining table. Using the \( 4 \times 4 \) transformation
matrices, a position vector $\mathbf{q}$, written in the coordinate system $\sigma_1$ with respect to the world coordinate system $\Sigma$ can be represented as

$$\begin{bmatrix} \mathbf{Q} \\ 1 \end{bmatrix} = \mathbf{T}_1 \begin{bmatrix} \mathbf{q}_1 \\ 1 \end{bmatrix}$$  \hspace{1cm} (5)$$

where the matrix $\mathbf{T}_1$ written in terms of the machine tool parameters is

$$\mathbf{T}_1 = \begin{bmatrix} 1 & 0 & 0 & d_1 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The vectors $\mathbf{Q}$ and $\mathbf{q}_1$ represent a position vector expressed in the coordinate systems $\Sigma$ and $\sigma_1$, respectively. The position vector in the wire guiding head coordinate system $\sigma_2$ relative to the world coordinate system $\Sigma$ can be obtained in a similar manner by the following transformation

$$\begin{bmatrix} \mathbf{Q} \\ 1 \end{bmatrix} = \mathbf{T}_2 \begin{bmatrix} \mathbf{q}_2 \\ 1 \end{bmatrix}$$  \hspace{1cm} (6)$$

where the matrix $\mathbf{T}_2$ written in terms of the machine tool parameters is given by

$$\mathbf{T}_2 = \begin{bmatrix} 1 & 0 & 0 & a_1 + d_1 \\ 0 & 1 & 0 & a_2 + d_4 \\ 0 & 0 & 1 & a_3 + d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and $\mathbf{q}_2$ is the position vector expressed in the coordinate system $\sigma_2$.

The kinematic equations describing a position of the tool (the cutting wire) in the wire guiding head coordinate system relative to the coordinate system of the machining table are now obtained by substituting from Eq. (5) to Eq. (6), namely

$$\begin{bmatrix} \mathbf{q}_1 \\ 1 \end{bmatrix} = \mathbf{T}_2^{-1}\mathbf{T}_1 \begin{bmatrix} \mathbf{q}_2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & a_1 - d_1 + d_3 \\ 0 & 1 & 0 & a_2 - d_2 + d_4 \\ 0 & 0 & 1 & a_3 + d_5 \end{bmatrix} \begin{bmatrix} \mathbf{q}_2 \\ 1 \end{bmatrix}$$  \hspace{1cm} (7)$$

3.2. Machine tool motion

Tool motion generation in numerical control (NC) machining with wire cut EDM involves the determination of the machine tool parameters in terms of a tool path that would produce the desired cut profile. At one instant, the cutting wire appearing in the machining table coordinate system $\sigma_1$ should be identical to the ruling of the designed shape expressed in the same coordinate system. By matching these two representations, we are able to obtain the expressions of the machine tool parameters $d_1, d_2, d_3, d_4$, and $d_5$.

To illustrate the development, let us consider a ruled surface patch $\mathbf{R}(u, v), u, v \in [0, 1]$, to be machined. The patch $\mathbf{R}(u, v)$ is designed by using the methods we presented in the previous section with reference to a coordinate system which we will refer to as the workpiece coordinate system. A set of data of $\mathbf{R}_1(u)$ and $\mathbf{R}_2(u)$ are generated as $u$ varies between zero and one. Let the maximum and minimum $x, y$, and $z$ components of the boundary curves be $x_{\text{max}}, x_{\text{min}}, y_{\text{max}}, y_{\text{min}}$, and $z_{\text{max}}, z_{\text{min}}$, respectively. We then choose the size of a workpiece according to these quantities. In other words, if we attach the workpiece coordinate system to a workpiece then the dimensions of the workpiece should be able to cover these extreme values.

When undergoing machining, the workpiece is fixtured on the machining table such that, for most cases, the workpiece coordinate system is not coincident with the machining table coordinate system $\sigma_1$. The position vector in the workpiece coordinate system is therefore different from that expressed in the machining table coordinate system $\sigma_1$. In other words, the position vector $\mathbf{r}$ of the designed ruled surface (in the workpiece coordinate system) appears as the position vector $\mathbf{q}$ in the machining table coordinate system $\sigma_1$, i.e.

$$\begin{bmatrix} \mathbf{q}' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & c_1 \\ 0 & 1 & 0 & c_2 \\ 0 & 0 & 1 & c_3 \end{bmatrix} \begin{bmatrix} \mathbf{r} \\ 1 \end{bmatrix}$$  \hspace{1cm} (8)$$
where \( \mathbf{c} = [c_1, c_2, c_3]^T \) is the position vector of the origin of the workpiece coordinate system as measured in \( \sigma_r \), \( \mathbf{r} \) is the position vector of the designed ruled patch generated in the workpiece coordinate system.

From the configuration of the EDM system, we have the position vector \( \mathbf{b} = [b_1, b_2, b_3]^T \), which is the base of the wire guiding rack with reference to the initial position of the wire guiding head. The vector determined by the cutting wire (i.e. its initial point is at the base of the wire guiding rack and the terminal point is at the wire guiding head), expressed in the head coordinate system \( \sigma_2 \), is \( \mathbf{q}_b = [d_3 - b_1, d_4 - b_2, d_5 - b_3]^T \). Assume that this vector intersects with the machining table at a point \( \mathbf{q}_t \) while the workpiece is being cut. Since the table moves horizontally only, its height keeps constant and the z-component of \( \mathbf{q}_b, z_b \), is therefore fixed. Thus \( \mathbf{q}_t \) (in \( \sigma_2 \)) can be determined by

\[
\mathbf{q}_t = \mathbf{b} + \mathbf{q}_b(z_t - b_3)(d_5 - b_3)
\]  

(9)

This intersection point can be expressed in the machining table coordinate system \( \sigma \) using Eq. (7)

\[
\mathbf{Q}_t = \begin{bmatrix} 1 & 0 & 0 & a_1 - d_1 + d_3 \\ 0 & 1 & 0 & a_2 - d_2 + d_4 \\ 0 & 0 & 1 & a_3 + d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{q}_t \\ 1 \end{bmatrix}
\]

(10)

On the other hand, one of the boundary curves, say \( \mathbf{r}_2^* = [r_{21}^*, r_{22}^*, r_{23}^*]^T \), of the designed profile is placed on the machining table and this point should be the same as \( \mathbf{Q}_t \). Therefore, we want \( \mathbf{Q}_t \) to be equal to the position vector of \( \mathbf{q}_2^* \) which is obtained from Eq. (8) if we substitute \( \mathbf{r}_2^* \) for \( \mathbf{r} \).

Equating \( \mathbf{Q}_t \) and \( \mathbf{q}_2^* \), we obtain the following equations

\[
\begin{bmatrix}
b_1 + (d_1 - b_1)(z_t - b_1)(d_5 - b_3) + a_1 - d_1 + d_3 \\
b_2 + (d_2 - b_2)(z_t - b_3)(d_5 - b_3) + a_2 - d_2 + d_4 \\
z_t + a_3 + d_5
\end{bmatrix}
= \begin{bmatrix}
r_{21}^* + c_1 \\
r_{22}^* + c_2 \\
r_{23}^* + c_3
\end{bmatrix}
\]

(11)

One more equation can be obtained from the fact that the vector \( \mathbf{q}_b \) is parallel to \( \mathbf{r}_1^* - \mathbf{r}_2^* \) when they are both described in the table coordinate system \( \sigma_1 \). Therefore we have

\[
\begin{bmatrix}
a_1 - d_1 + d_3 + d_5 - b_1 \\
a_2 - d_2 + d_4 + d_4 - b_2 \\
a_3 + d_5 + d_5 - b_3
\end{bmatrix} \times \begin{bmatrix}
(r_{11}^* + c_1) - (r_{21}^* + c_1) \\
r_{12}^* + c_2 - (r_{22}^* + c_2) \\
(r_{13}^* + c_3) - (r_{23}^* + c_3)
\end{bmatrix} = 0
\]

which yields

\[
\begin{bmatrix}
(a_2 - d_2 + 2d_4 - b_2)(r_{13}^* - r_{23}^*) \\
(a_3 + 2d_5 - b_3)(r_{11}^* - r_{21}^*) \\
(a_1 - d_1 + 2d_3 - b_1)(r_{12}^* - r_{22}^*)
\end{bmatrix}
= \begin{bmatrix}
(a_3 + 2d_5 - b_3)(r_{12}^* - r_{22}^*) \\
(a_1 - d_1 + 2d_3 - b_1)(r_{13}^* - r_{23}^*) \\
(a_2 - d_2 + 2d_4 - b_2)(r_{11}^* - r_{21}^*)
\end{bmatrix}
\]

(12)

By solving Eqs. (11) and (12), the equations governing the motion between the machining table (parameters \( d_1, d_2 \)) and the wire guiding head (parameters \( d_1, d_2, d_3 \)) can be obtained as follows

\[
d_1 = [-2r_{11} - 2r_{22}^*](-(a_1 - a_3 - a_3b_1 - a_1b_3 + (a_3 + b_3)c_1 + (a_1 + b_1 - c_1)c_3 + (a_3 + b_3 - c_3)r_{21}^*) + (a_1 + b_1 - c_1 - r_{21}^*)r_{22}^* + (-a_1 - 2b_1 + c_1 + r_{21}^*)z_t) - (a_3 + 2b_3 - c_3 - r_{23}^*)((-3a_1 + b_3 + 2c_3)r_{11}^* + (a_1 - b_1)r_{12}^* + (3a_3 - b_3 - 2c_3)r_{21}^*) + (-a_1 + b_1 + 2r_{11} - 2r_{22}^*)r_{23}^* + (-2r_{11}^* + 2r_{22}^*)z_t)]/(r_{11}^* + r_{22}^*)(-a_3 + c_3 + r_{23}^* - 2z_t)
\]

(13)

\[
d_2 = [-2r_{13} - 2r_{22}^*](-(a_2 - a_3 - a_3b_1 - a_1b_3 + (a_3 + b_3)c_2 + (a_2 + b_2 - c_2 - r_{22}^*)r_{23}^* + (-a_2 - 2b_2 + c_2 + r_{22}^*)z_t) - (a_3 + 2b_3 - c_3 - r_{22}^*)((-3a_1 + b_3 + 2c_3)r_{12}^* + (a_2 - b_2)r_{12}^* + (3a_3 - b_3 - 2c_3)r_{22}^*) + (-a_2 + b_2 + 2r_{12} - 2r_{22}^*)r_{23}^* + (-2r_{12}^* + 2r_{22}^*)z_t)]/(r_{13}^* + r_{23}^*)(-a_3 + c_3 + r_{23}^* - 2z_t)
\]

(14)

\[
d_3 = [-r_{13} - r_{22}^*](-(a_3 + a_3b_1 - a_1b_3 + (a_3 + b_3)c_1 + (a_1 + b_1 - c_1)c_3 + (a_3 + b_3 - c_3)r_{21}^*) + (a_3 + 2b_3 - c_3 - r_{23}^*)((-3a_1 + b_3 + 2c_3)r_{11}^* + (a_1 - b_1)r_{13}^* + (3a_3 - b_3 - 2c_3)r_{21}^*) + (-a_1 + b_1 + 2r_{11} - 2r_{22}^*)r_{23}^* + (-2r_{11}^* + 2r_{22}^*)z_t)]/(r_{13}^* + r_{22}^*)(-a_3 + c_3 + r_{23}^* - 2z_t)
\]

(15)
perform a simple flatness test, we use a convex triangular toolpath such that few larger subpatches are generated in the accuracy. It is however more efficient if we can produce the can approximate the desired cut profile to an acceptable in turn can define the sequence of tool movements. As long parameter values to specify a collection of subpatches which subpatches that approximate the desired profile of a ruled patch. In general, one may use small increments of the parameter values to specify a collection of subpatches which.The ruled surface patch \( \mathbf{r}(u,v) \) is therefore constructed under the simultaneous motion of the wire guiding head and the machining table.

4. Incremental toolpath approximation

Toolpath generation for a commercially available wire cut EDM system requires determination of a sequence of tool movements that cuts the workpiece into a number of subpatches that approximate the desired profile of a ruled patch. In general, one may use small increments of the parameter values to specify a collection of subpatches which in turn can define the sequence of tool movements. As long as the increments are sufficiently small, the resulting shape can approximate the desired cut profile to an acceptable accuracy. It is however more efficient if we can produce the toolpath such that few larger subpatches are generated in the flat regions of a ruled surface while many smaller subpatches are generated in the curved regions. In order to achieve this, a subdivision algorithm is presented here for such an incremental approximation of a ruled surface defined by its two contours.

Consider a step of tool motion that cuts the ruled patch at a subpatch. Let us denote the flatness tolerance as \( \delta \). One of the boundaries of the subpatch is as shown in Fig. 6, where \( \mathbf{r}_a \) and \( \mathbf{r}_b \) are the datum positions on the toolpath. A flatness test can be used based on the ratio of arc length over chord length between two points of a curve segment. In order to perform a simple flatness test, we use a convex triangular approximation to the arc length and define an approximate ratio \( \mu \), as follows (Fig. 6)

\[
\mu = \frac{2\sqrt{b^2 + (c/2)^2}}{c}
\]  

where \( c \) is the chord length between the two datum points. The arc length between the two point can be obtained by using Eq. (1). Let us denote the ratio of arc length over chord length of the boundary of one step as \( \nu \). If \( \nu \approx \mu \) then the subdivision is not acceptable and further subdivision is required. On the other hand, if \( \nu < \mu \) then we have reached an acceptable level of incremental linear approximation and no further subdivision is necessary.

The subdivision method presented here involves subdivision on both boundary curves of the ruled patch such that the resulting pairs of curve segments are within a flatness tolerance. These pairs of curve segments are then used to determine the nearly flat subpatches by applying Eqs. (3) and (4)

Let the two boundary curves be \( \mathbf{r}_1^*(u) \) and \( \mathbf{r}_2^*(u) \), respectively. The algorithm to subdivide them along the parameter values \( u_i, i = 0, 1, 2, \ldots \) is as follows:

Algorithm BS: Boundary Subdivision

1. Start with \( u_i = 0, i = 0 \).
2. Determine the value \( x \) between \( u_i \) and 1 by using the following recursive formula

\[
x_j = x_{j-1} + (x_j - x_{j-1})u_i/v_1, \quad j = 1, 2, \ldots
\]

where \( \mu_1 \) and \( v_1 \) are determined from the profile between \( \mathbf{r}_1^*(u_i) \) and \( \mathbf{r}_1^*(x_{j-1}) \). The \( x_j \) that makes the flatness between \( \mathbf{r}_1^*(u_i) \) and \( \mathbf{r}_1^*(x_j) \) pass the test is the desired \( x \).
3. Determine the value \( y \) between \( u_i \) and 1 by using the following recursive formula

\[
y_k = y_{k-1} + (y_k - y_{k-1})u_i/v_2, \quad k = 1, 2, \ldots
\]

where \( \mu_2 \) and \( v_2 \) are determined from the profile between \( \mathbf{r}_2^*(u_i) \) and \( \mathbf{r}_2^*(y_{k-1}) \). The \( y_k \) that makes the flatness between \( \mathbf{r}_2^*(u_i) \) and \( \mathbf{r}_2^*(y_k) \) pass the test is the desired \( y \).
4. Increment \( i \) by one.
5. Replace \( u_i \) with the smaller one between \( x \) and \( y \); that is \( u_i = \min(x, y) \).
6. Repeat step 2 through step 5 until \( u_i = 1 \).

The step length parameters \( u_i \)'s subdivide the two curves into two sets of equal-numbered straight line segments which approximate the two boundary curves to within a tolerance. Each corresponding pair of points \( \mathbf{r}_1^*(u_i) \) and \( \mathbf{r}_2^*(u_i) \) defines one of the boundary lines of a subpatch and the union of those subpatches approximates the original ruled surface. Fig. 7 shows the results of the subdivision. After the subdivision is completed, a toolpath can be specified such that it passes through the sequence of those pairs of points \( \mathbf{r}_1^*(u_i) \) and \( \mathbf{r}_2^*(u_i), i = 0, 1, 2, \ldots \)
5. A path lifting algorithm to avoid gauging

In NC machining with wire EDM, the sparks generated in the gap between the wire and the workpiece can cause over erosion of the workpiece. This reduces the machining accuracy by producing minor gauging of the cut profile. Gauging occurs when the machining parameters such as the gap distance and the voltage are pre-set and the travelling wire is cutting sharp corners and areas with small radii. One way to avoid such a situation is to use adaptive methods to change the machining parameters [5,7]. This type of approach, however, usually requires use of specialized controllers and cannot be implemented on a commercially available EDM system unless one overrides the control system of the machine tool. In this paper, we present a toolpath lifting algorithm that increases the gap between the wire and the workpiece in such critical areas at the geometric design stage. In this fashion, the electric potential in the gap reduces to that of the non-critical areas and gauging is avoided during actual machining.

The subdivision algorithm can be used to detect the regions with small radii of curvature and a revised path will be defined based on the tool lifting algorithm. The revised path becomes the new boundary that defines the profile of the ruled patch.

Let σ denote the smallest acceptable radius of curvature for cutting with an EDM system. The acceptable value for this parameter varies for different EDM systems and depends on many factors including workpiece materials and the type of wire. The curvature \( \kappa(u) \) of a curve \( \mathbf{r}(u) \) is defined as

\[
\kappa = \frac{|| \mathbf{r} \times \mathbf{r}' ||}{|| \mathbf{r}' ||^3}
\]

where dots indicate the derivatives with respect to the parameter \( u \). Therefore, if \( 1/\kappa \geq \sigma \) then no path correction is needed. We can determine the locations where \( 1/\kappa < \sigma \) and lift the toolpath at such locations. The subdivision algorithm can also be used for locating the critical regions with small radii. The subdivision algorithm results in a cluster of segments some of which need to be modified to avoid gauging. We shall refer to such segments as critical segments.

The curve regions with small radius of curvature are those composed of critical segments. We need to lift the toolpath in these areas. The lifted-path can be regarded as an independent curve segment that replaces the region with the small radius of curvature. The boundary curve then becomes a composite curve consisting of some lifted-paths and parts of the original curve. Suppose, for example, a region with small radius of curvature starting point \( r_1^t(u_a) \) and ending point \( r_1^t(u_e) \) of one of the boundary curves. We will separate the original curve \( r_1^t(u) \), \( u \in [0, 1] \) into three portions:

\[
r_1^t(u_a), \quad u_a \in [0, u_a]; \quad r_1^t(u_b), \quad u_b \in [u_a, u_e]; \quad r_1^t(u_e), \quad u_e \in [u_e, 1].
\]

Due to this separation, the other boundary curve \( r_2^t(u) \) needs to be also subdivided at the same parameter values. As a result, we will have three pairs of corresponding curve segments that define three profiles and their union determines the ruled patch to be machined. In other words, we will define the profiles for the pairs of curves \( r_1^t(u_a) \) and \( r_2^t(u_a) \), \( r_1^t(u_b) \) and \( r_2^t(u_b) \), and \( r_1^t(u_e) \) and \( r_2^t(u_e) \). The tool path is, therefore, defined by three independent subprofiles. The pair containing the region of curve requiring path-lifting needs special treatment, as will be discussed in the subsequent part of this section. For the other two pairs, we require the following transformation on the parameters \( u_a \) and \( u_e \) prior to the modeling and toolpath generation:

\[
t_a = \frac{u_a}{u_s}, \quad t_e = \frac{u_e - u_s}{1 - u_e}
\]

The two pairs are then transformed into \( r_1^t(t_a) \) and \( r_2^t(t_a) \), \( t_a \in [0, 1] \), and \( r_1^t(t_e) \) and \( r_2^t(t_e) \), \( t_e \in [0, 1] \). We can now...
apply the methods presented in the previous sections on these two pairs to model the two sub-profiles and generate the toolpath.

For the pairs containing the regions of curve requiring path-lifting, we will define new curve segments that replace original portions of the contour curves. In general, the lifted-path should meet the following requirements: (1) its starting point and end point are on the original curve, (2) it should be elevated with respect to the starting and end points based on the needed gap distance. Since the two end points of a lifted path will usually be close to one another, in this paper, we will use a conic section to represent the lifted path.

Using the rational representation of conics [11], the new lifted-path is characterized by the three control points and the weights (shape parameters). Rational form of a conic segment is as follows

\[
x(t_b) = \frac{\sum_{i=0}^{2} w_i b_i(t_b) x_i}{\sum_{i=0}^{2} w_i b_i(t_b)} \quad t_b \in [0, 1]
\]

where \(w_i\)'s (positive real numbers) are the weights associated with the control points \(x_i\), and \(b_i(t_b)\) are the Bézier/Bernstein basis functions. In order to manipulate the shape of the conic section in a manner that would geometrically relate to the needed adjustment of the machining gap, we will apply the so-called \(\rho\)-conic parameterization [8]. If we take \(\bar{x} = x(1/2)\) (the so called shoulder point) and \(x_m = (x_0 + x_2)/2\), then \(\bar{x}\) is the intersection of the line \(x_1x_m\) with the conic. The value \(\rho\) is determined by

\[
\rho = \frac{\bar{x} - x_m}{x_1 - x_m}
\]

The geometric interpretation of the parameter \(\rho\) is given in Fig. 8.

The resulting conic can represent the lifted path, if one chooses the control points and the \(\rho\) value appropriately. The control points \(x_0\) and \(x_2\) are chosen as the starting point \(r_1^r(u_s)\) and the end point \(r_1^r(u_e)\) of the lifted-path, respectively. The control point \(x_1\) will be chosen along the line defined by the two points \((r_1^r(u_s) + r_1^r(u_e))/2\) and \((r_1^r(u_s + u_e))/2\) such that, with the suitable \(\rho\) parameter, the resulting conic gives the desired elevation based on the needed adjustment for the machining gap. The use of \(\rho\)-conic parameterization allows for the specification of the lifting distance in accordance with the desired adjustment for the gap between the wire and the workpiece. This approach enables us to define the lifted-path from the CAGD point of view. In this manner we can obtain a new curve segment that serves as a revised toolpath. Such a path and its corresponding part, \(r_2^e(u_b)\), of the other boundary curve constitute an independent pair of boundary curves. After we transform the parameter \(u_b\) for \(r_2^e(u_b)\) to be

\[
t_b = \frac{u_b - u_s}{u_e - u_s}
\]

the pair becomes \(x(t_b)\) and \(r_2^e(t_b), \ t_b \in [0, 1]\). The modeling of profile and the generation of toolpath can then be carried out in the fashion described before. Fig. 9 shows an example of a lifted path for a cut profile near an area of small radii of curvature. The lifted path generated provides additional gap between the cutting wire and the workpiece that minimizes the potential for gauging or wire breakage resulting in a smoother cut and continuation of the process.

Fig. 8. The definition of \(\rho\) parameterization.

Fig. 9. An example of the ruled patch whose toolpath is lifted.
6. Conclusions

In this paper, we have presented tool motion generation for wire cut EDM using a rather simple surface modeling scheme based on boundary profiles or contours. The methods presented provide several improvements over the existing techniques both on the modeling as well as the computational aspects of tool motion generation.

The tool motion generated is in terms of the motion of the centerline of the wire and neglects the effects of the wire thickness as well as the gap distance on modeling the cut profile. In order to include such effects, one has to use an offset of the cut profile to generate the incremental path of the machine tool. Methods for generating offsets of ruled surfaces have been developed in the past [9,10]. The offsets of the cut profiles should be used together with the algorithms presented in this paper to generate the tool motion of the wire cut EDM. It is hoped that this paper would enhance further applications of computational geometry in manufacturing and would contribute to CAD/CAM integration for wire cut EDM operation.

References


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