Image Segmentation Based on Adaptive Fuzzy-C-Means Clustering

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Abstract

The clustering method “Fuzzy-C-Means” (FCM) is widely used in image segmentation. However, the major drawback of this method is its sensitivity to the noise. In this paper, we propose a variant of this method which aims at resolving this problem. Our approach is based on an adaptive distance which is calculated according to the spatial position of the pixel in the image. The obtained results have shown a significant improvement of our approach performance compared to the standard version of the FCM, especially regarding the robustness face to noise and the accuracy of the edges between regions.

1. Introduction

Image segmentation constitutes an important step and an essential process of image analysis. Fuzzy-C-Means (FCM) [2] is one of the most popular unsupervised fuzzy clustering techniques that are applied with success in image segmentation. Although the original FCM algorithm yields good results for segmenting noise free images, it fails to segment images corrupted by noise or containing inaccurate edges. This sensitivity is essentially due to the absence of utilization of the information on the spatial position of pixels to be classified. Several authors tried to overcome this drawback by the integration of spatial information. Chuang [4] proposed a novel fuzzy clustering algorithm that uses a spatial membership degree representing the summation of the membership degree in the neighbourhood of each pixel. Tolias[5] developed a Sugeno type rule based system that imposes spatial constraint by modifying the membership degree of clustering results obtained after FCM algorithm.

In our paper we propose a novel version of FCM that integrates the spatial information. The novelty concerns essentially the way of calculating the distance of similarity between the pixels of the image and the centers of the classes. The rest of this paper is organized as follows. Sections 2 and 3 present respectively the principle and the limits of the conventional FCM algorithm. In the following section we present our fuzzy clustering approach algorithm that incorporates a spatial constraint for image segmentation. Our segmentation method is tested on synthetic and MRI images. The results are illustrated and discussed in section 5. Finally, in section 6 we conclude the paper.

2. Fuzzy C Means Clustering

Fuzzy-C-Means (FCM) clustering was developed by Bezdek [2]. It can be described as follows: Let \( X = \{x_1, x_2, \ldots, x_n\} \) denoted a set of \( n \) objects to be partitioned into \( C \) clusters, where each \( x_j \) has \( d \) features. The FCM algorithm minimizes the objective function defined as follows:

\[
J = \sum_{i=1}^{C} \sum_{j=1}^{n} (u_{ij})^m D(x_j, v_i)
\]

(1)

where:

- \( u_{ij} \) represents the membership degree of \( j^{th} \) object in the \( i^{th} \) cluster,
- \( v_i \) represents the \( i^{th} \) cluster center,
- \( D \) represents a distance metric (generally the square of Euclidian distance) that measures the similarity between an object and a cluster center,
- \( m \geq 1 \) the degree of fuzzyfication.

The membership degree of \( x_j \) to the \( i^{th} \) cluster is determined by calculating the gradient of \( J \) with respect to \( u_{ij} \). Thus, these membership degrees are given by Equation 2:

\[
u_{ij} = \left( \sum_{k=1}^{C} \left( \frac{D(x_j, v_{ki})}{D(x_j, v_{ki})} \right)^{\frac{1}{m-1}} \right)^{-1}
\]

(2)
The cluster centers $v_i$, $i:1..C$ are determined by calculating the gradient of $J$ with respect to $v_i$. These centers are given by Equation 3:

$$v_i = \frac{\sum_{j=1}^{n} (u_{ij})^m x_j}{\sum_{j=1}^{n} (u_{ij})^m} \quad (3)$$

The FCM algorithm can be summarized in the following steps:

**Step 1:** Fix the cluster number and initialize the centers by random points from data set.

**Step 2:** Update the membership degrees by using Equation 2.

**Step 3:** Update centers using Equation 3.

**Step 4:** Repeat steps 2 and 3 until convergence.

The convergence of this algorithm will be reached when the change in membership values is less than a given threshold.

3. Limits of FCM

Figure 1.a shows a grey level synthetic image formed by two regions: black region (0) and white region (255) that includes a black noisy pixel (0). The application of standard FCM (using the grey level as a single feature of pixels) on this image yields to a good segmentation of pixels inside regions and pixels of edges. However, it provokes a bad clustering of noisy pixel of white region. This clustering drawback is essentially due to the only use of the intrinsic feature of pixel to be classified (grey level) without taking into account spatial information. This information was proved to be very important in the context of segmentation.

To overcome this limitation, one of solutions consists to integrate the neighborhood effect of pixel to be classified. There are several statistic estimators to accomplish this effect. In this work, we have chosen the spatial feature: arithmetic means estimator denoted $\mu$. Figure 1.b represents the image of the means obtained by replacing the grey level (GL) of the pixels of the image of Figure 1.a with the means of the GL of their neighborhood calculated on a window of size 3x3. The application of the FCM on this image engenders a good clustering of pixels inside regions as well as the noisy pixel. But it produces a degradation of edges between regions. This result is essentially due to the smoothing effect of the spatial feature used in the clustering processes. Table 1 shows the advantages and the inconveniences of using the GL and the spatial feature for the clustering of noise and edges.

4. Proposed Method

The complementarity of the grey level feature and the spatial feature as regards the FCM clustering can let envisage a joint use of these two features in image segmentation. In this section we present a new version of FCM called Adaptive Distance based FCM (ADFCM) which takes the advantages of both features, while avoiding their drawbacks by using the one or the other in an adaptive way according to the spatial configuration of each pixel.

4.1. Considering Spatial Configurations

4.1.1. Presenting Spatial Configurations

In our work, we distinguish four possible spatial configurations for pixels demanding each a specific choice of the clustering criterion (cf. Figure 2). These configurations are: Pixel belonging to a Region (PR), Pixel belonging to an Edge (PE), Noisy Pixel (NP) and Neighbour of a Noisy pixel (NN).

4.1.2 Characterizing Spatial Configurations

Formally, the spatial configurations are characterized by two statistical descriptors of decision that are presented as follows:

$\sigma(x_j) = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - \mu(x_j))^2} \quad (4)$

Table 1: Advantages and drawbacks of using grey level and spatial feature in the clustering process.

<table>
<thead>
<tr>
<th></th>
<th>Grey level</th>
<th>Spatial feature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noise</td>
<td>Bad clustering</td>
<td>Good clustering</td>
</tr>
<tr>
<td>Edges</td>
<td>Good clustering</td>
<td>Bad clustering</td>
</tr>
</tbody>
</table>

Figure 2: Spatial configurations of pixels

Figure 1: (a) Grey levels of image (b) The means of grey levels of image (a)
The $knn$ which represents the number of the closest neighbours in terms of grey levels with regard to the considered pixel. The $knn$ is defined as follows:

$$knn(x_j) = \text{Card}\{x_p \in \text{Neighborhood}(x_j) \mid |x_p-x_j|<S\}$$ (5)

where $S$ designates a threshold which is generally chosen in an empirical way.

From these two features we can characterize the various possible spatial configurations of pixels. In case of a PR, the standard deviation ($\sigma$) is generally low, it is null for the constant regions. However, the $\sigma$ becomes high for the PE, NP and NN. The distinction between these three configurations is made by using the $knn$ feature. This number is generally low for a NP, medium for a PE and high for a NN.

### 4.1.3. Selecting Clustering Criterion

The clustering of a PR or a NP has to privilege the spatial feature (Spatial) because the decision must be taken on the basis of the information of its neighborhood. On the other hand the clustering of a PE or a NN has to privilege the grey level of the pixel respectively to preserve contours and to avoid the influence of the noise. The choice of the criterion of clustering and the characterizations of the spatial configurations are summarized in Table 2.

**Table 2: Spatial configurations, their characterizations according to both descriptors: $\sigma$ and $knn$ and their privileged classification feature.**

<table>
<thead>
<tr>
<th>Cases of spatial configurations</th>
<th>Characteristic $\sigma$</th>
<th>Privileged feature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pixel of region (PR)</td>
<td>Low</td>
<td>Spatial feature</td>
</tr>
<tr>
<td>Pixel of edges (PE)</td>
<td>High</td>
<td>Grey level</td>
</tr>
<tr>
<td>Noisy pixel (NP)</td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td>Nearby pixel of a noise (NN)</td>
<td>High</td>
<td>High</td>
</tr>
</tbody>
</table>

### 4.2. Introducing a New Similarity Distance

The standard FCM uses generally to measure the similarity between an object $x_j$ and a class given by its center $v_i$ a distance which grants the same importance for the features taken into account in the clustering process. To introduce the adaptive effect for the features selection, we propose to use a dynamic and weighted distance derived from the Euclidian distance. This new distance is given by Equation 6.

$$D(x_j, v_i) = (1 - p_j) \left( x_j^{GL} - v_i^{GL} \right)^2 + p_j \left( x_j^{Spatial} - v_i^{Spatial} \right)^2$$ (6)

$D$ is a bi-dimensional distance based on both features GL and Spatial. In this equation, the weight $p_j$ allows to control the importance of each feature for the pixel $x_j$ clustering. So if $p_j$ is high then we privilege the spatial feature otherwise we privilege the grey level. The term $p_j$ must be calculated for each pixel to be classified according to its spatial configuration in the image. From the configurations presented in Table 2, we can deduce that the weight $p_j$ must be maximized (tend to 1) when the pixel to be classified is a PR or a NP, because the decision of its membership in the various classes must be taken only on the basis of the spatial feature. However, this weight $p_j$ must be minimized (tend to 0) in case of a PE or a NN, because the grey level in these cases will constitute a good clustering criterion.

#### 4.3. Estimation of the Spatial Weight

The choice of the spatial weight $p_j$ is very important for calculating the new distance (6). We propose in this paragraph a fuzzy method for the estimation of this weight. This method uses a fuzzy inference system possessing as entries two decision linguistic variables $\sigma$ and $knn$ to calculate the output linguistic variable $p$. This system is based on four inferences rules deduced from the relationship existing between the spatial configuration of a given pixel and the choice of $p$. The linguistic rules are defined as follows:

- $R_1$: IF $\sigma$ is Low THEN $p$ is High,
- $R_2$: IF $\sigma$ is High AND $knn$ is Low THEN $p$ is Low,
- $R_3$: IF $\sigma$ is High AND $knn$ is High THEN $p$ is High,
- $R_4$: IF $\sigma$ is High AND $knn$ is Medium THEN $p$ is Low.

The membership functions of variables $\sigma$, $knn$ and $p$ used in our system are represented in the Figure 3.

![Figure 3: Membership functions used for the estimation of $p$.](Image)

The thresholds used in these curves ($T_\sigma$, $T_1$, $T_2$, $T_3$ and $T_4$) are fixed in an empirical way.

#### 5. Experiments and Results

In this section, we present the results of the application of ADFCM algorithm. The performance of this algorithm is compared to the standard version of the FCM. Both techniques are tested on "Panda" synthetic image corrupted by a mixture of Gaussian and impulsive noise, and an MRI cerebral image corrupted by 5% of Gaussian noise. These techniques
are experimented in the same conditions (a factor of fuzzification \( m = 2 \) and a convergence error \( = 0.001 \)). The ADFCM uses as spatial feature the means \( \mu \) calculated on an analysis window of size 3x3.

Figure 4.a shows Panda image containing three classes of regions perfectly identified. Figure 4.b shows the result of applying the standard FCM to the original image using as a clustering criterion the grey levels. This result clearly illustrates the limitations of this method for the classification of noisy pixels. However, the application of the FCM based on the means feature can resolve the problem of noise as shown in Figure 4.c. Conversely, it engenders inaccurate edges segmentation (tree branches). The application of ADFCM (with \( T = 55 \)) yields the segmentation shown in Figure 4.d. The three classes are correctly detected. This result confirms the good performance of the ADFCM compared to the standard FCM. Indeed, by using the adaptive distance, the ADFCM has achieved a compromise that allowed the reduction of noise while producing accurate edges. The visual result is supported by the statistics shown in Table 3 which gives the number of misclassified pixels inside and at edges of regions for each of the three classes that make up the image "Panda".

Cerebral image segmentation consists in bounding three cerebral structures: grey matter (GM), white matter (WM) and the cerebrospinal fluid (CSF). Our tests are realized on the cerebral image of Figure 5.a. The application of the FCM based on the GL feature on this image gives noisy and overlapped classes particularly between both classes GM and WM (cf. Figure 4.b). The use of the FCM based on the spatial feature provokes a degradation of the obtained edges (cf. Figure 4.c). Whereas the use of the ADFCM for a threshold equal to 15 help enormously to reduce the noisy pixels while obtaining good identified regions and having continuous edges that are closer to the reality (cf. Figure 5.d).

### 6. Conclusion

In this paper we have proposed a novel version of FCM based on dynamic and weighted similarity distance. Our approach is tested on synthetic and MRI cerebral images. The obtained results have shown a significant improvement of our approach performance compared to the standard FCM. The robustness face to noise and the accuracy of the edges between regions have been shown. However, the choice of the threshold \( T \) is strongly dependent on used images. This problem hasn’t been addressed in this paper and remains as further steps of research.

### References


### Table 3: Misclassified pixels number inside and on the edges of regions of MRI cerebral image for the three tested techniques.

<table>
<thead>
<tr>
<th>Classe</th>
<th>Region</th>
<th>FCM (GL)</th>
<th>FCM (( \mu ))</th>
<th>ADFCM (( T = 55 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classe1</td>
<td>20</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Classe2</td>
<td>2</td>
<td>464</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>Classe3</td>
<td>689</td>
<td>10</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Classe3</td>
<td>169</td>
<td>664</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>948</td>
<td>1146</td>
<td>20</td>
<td>2301</td>
</tr>
</tbody>
</table>

**Figure 4:** Segmentation results using FCM (GL), FCM (\( \mu \)) and ADFCM (\( T = 55 \)) algorithms for Panda synthetic image.

**Figure 5:** Segmentation results using FCM (GL), FCM (\( \mu \)) and ADFCM (\( T = 15 \)) algorithms for MRI cerebral image.