Fuzzy graphs modelling for HazMat telegeomonitoring

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Abstract

Telegeomonitoring system development combines two heterogeneous technologies: the geographical information systems technology (GIS) and telecommunications technology. In this paper, we give the system components for telegeomonitoring transportation of hazardous materials. The telegeomonitoring system uses GIS to capture civil infrastructure (urban network, land use, industries, etc.) and decision support systems technology to allow risks analysis and evaluate routing strategies that minimize transportation risk. Routing algorithms are to this effect adapted to graphs of the fuzzy risk. A new algebraic structure is proposed to solve a path-finding problem in a fuzzy graph. This algebraic structure is adapted precisely to solve the problem of the $K$-best fuzzy shortest paths. The approach that we proposed, consists of defining generic structures of operator's traversal problem in fuzzy graphs. The principal contribution of our approach is to build adequate structures of path algebra to solve the problem of graph traversal in a fuzzy graph without negative circuits. Foundations of the system studied in this work will be able to be transposed to other fields of transportation. © 2005 Elsevier B.V. All rights reserved.

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1. Introduction

This paper describes a fuzzy routing algorithm integrated in a telegeomatic system designed for monitoring in real-time the transportation of hazardous materials. The telegeomatic system consists of integration of spatial decision support system (SDSS) with new information and communication technologies, in a reactive context. With regard to decision aspect, the fuzzy modelling and in particular the fuzzy routing techniques will be described in this work. The fuzzy path-finding is important for risk graphs. The fuzzy technology exhibits solutions that often contribute to the decision making as far as complex problems are concerned. Some orientations will also be discussed to integrate real-time intelligent systems capabilities.

Recent research led by Boulmakoul and his team [3–5], on the monitoring of HazMat
transportation, contributed to the emergence of a new concept “telegeomatic” [16]. The telegeomatic technology is a federation of two fundamental fields, the spatial decision support system in real-time context and communication processes integrating new information and communication technologies (Internet, mobile and wireless applications). Our first works were destined to the conception and the development of a monitoring system for hazardous materials transportation, using the global positioning system technology (GPS). A prototype of the system has been achieved, including a HazMat multimedia data base opened on the internet and a routing system based on graphs modelling [1,2].

The goal of this paper is to define a generic structure based on telegeomatic technology to strengthen tasks of the HazMat telegeomonitoring system. The architecture of this organization calls on Groupware’s concepts. In this work we introduce a new development concerning the integration of spatial networks operators (network analysis, fuzzy path operators, etc.). The decisional aspect often requires complex analyses, therefore the contribution of the fuzzy modelling is asked at this level.

The paper is organized as follows. In Section 2, we provide conceptual definitions of the telegeomatic systems and we give the foundation of the telegeomatic transportation systems. In this section, a high level object description is developed to introduce essential definitions that denote the telegeomonitoring system. Data management and the data collection processes are put forward in this section. In the Section 3, we introduce the HazMat transportation risk modelling. A fuzzy approach to evaluate the risk is proposed. Section 4 outlines developed works devoted to paths operators and to the fuzzy modelling for hazardous materials transportation routing. In this section a new path algebra structure \((\Omega_k, \oplus, \otimes, \alpha, e)\) is proposed to solve the path-finding problem in a fuzzy graph. This algebraic structure is adapted precisely to solve the problem of the \(K\)-best fuzzy shortest path. The fuzzy routing algorithm is used to find the path (or paths) that minimizes risks of HazMat transportation. Section 5 gives the conclusion and perspectives to drive investigation in future.

2. Telegeomatic concept

A telegeomatic system is a federation of several components as geographical information system (GIS), decision support system (DSS) and open communication system (OCS) with real-time capabilities.

The main objective of a telegeomonitoring system is to monitor dynamic processes in real-time. To satisfy the objective of such a system, it must be supported by a multitask operating system permitting real-time primitives. It must also assure the opening on the internet and the collection of data provided by the external devices (see Fig. 1). Exchanges between a telegeomatic system and the process to monitor are submitted to strict time constraints, imposed by the process dynamics. Aspects bound to the development of a telegeomatic system are complex because of the integration of several components. Other constraints bound to the systems interoperability must be assured in the development of telegeomatic systems.

2.1. Telegeomatic transportation systems

Geographical information system for transportation (GIS-T) represents a class of GIS applications. In the same spirit, telegeomatic transportation systems characterize a category of telegeomatic systems. Environmental monitoring in the field of transportation (HazMat monitoring,
pollution monitoring) focuses on geographical data organized with spatial representation. For this kind of problems, intensive use of telecommunications and positioning systems is highly important. Other fields of transportation are concerned by the telegeomatic: fleet management (vehicles, boats, aircraft, etc.), dynamic guidance of vehicles, etc. (see Fig. 2).

2.2. Groupware organization

The Groupware [11] is a set of work partner methods technologies that permits the sharing of information on a numeric support for a hired group in a collaborative work through electronic communication. In the case of telegeomatic transportation systems for HazMat monitoring, the federation of Groupwares is given in the following (see Fig. 3):

- **GIS-GW**, concerns a work group around tasks relative to spatial data manipulations and GIS administration.
- **Traffic data-GW**, it is a grouping of activities of preparation and selection of traffic data (choice of sensors, techniques of data collection, etc.).

- **HazMat-GW**, this grouping includes the management and the administration of HazMat database, and all operations of surveillance and routing of vehicles transporting these products. This node corresponds to the activity of the urban zone HazMat monitoring. This Groupware provides powerful tools for analyzing spatial and temporal characteristics of road traffic impacts.
To support decision-making process in the context of road traffic monitoring, this group provides: storing, retrieving and analyzing historical events and accidents, traffic and weather related information, manipulating data for effective analysis, visualizing on a digitized map traffic flow information, as well as any other relevant information.

2.3. Architecture foundation

The objective of this section is to describe telegeomatic transportation system technical architecture for HazMat monitoring. This architecture shows two fundamental control types entities (see Figs. 4 and 5):

- The Central Control Center (CCC) is a grouping node (only one CCC for a zone to control),
- The Hierarchical Control Center (HCC) that can be several. Each hierarchical node belong to a geographical zone fastened to a geographical region under the control of a CCC.

![Hierarchical nodes network architecture](image)

![Central node of the architecture](image)

Obviously, this decomposition allows a controlled communication flow management. The CCC nodes are distributed agents for hazardous materials transportation supervision. The CCC is the main node that centralizes staleness, and lines itineraries provided by technical routing. The totality of nodes being federated by an intranet protected network and opened toward the internet. Private network system guarantees authoritative control services and supervision, and extranet system allows professional users access to services given by nodes controllers.

2.4. Communication process and data management

The client/server mechanism is used in telegeometric monitoring context. A server process at the side of data server and several processes at the level of interfaces with data sources. The main use reasons of this paradigm is its great flexibility for specific development data collection from sensors.

This practice insures communications programs modularity. As for the other communication forms CORBA/ACTIVEX support, offers guarantees of opening and interoperability between software components. Fig. 6 illustrates the two levels of data collection process.
2.5. Telegeomonitoring object components

The high level meta-object describing the telegeomonitoring structure shows the fundamental objects components:

- GIS engine, that is specialized in managing spatial information.
- Data server (data engine) whose role is to provide values to the application objects attributes. These data being stemming from several data sources such as sensors, GPS, remote databases, files, remote simulators, etc.
- Database engine, that represents the textual DBMS aspects. It operates with GIS engine to complete GIS functions.
- Real-time operating system, the real-time is a relative notion for the time required by a system to react to an outside event while giving a correct answer. One often speaks of cycle, that corresponds to interval of time separating two meaningful events. System answer time to events must be lower to the cycle. Because of the very large number of functions that must be executed in parallel, and the asynchronous nature of applications, real-time systems require traditionally the following abilities: distributed multitask model, large capacity of processing to guarantee the execution time of certain critical procedures, deterministic response time, scheduling ensuring a rigorous task states control and the execution of important task, many tools of communication and synchronization between tasks, explicit time management under the shape of clocks, timers, the respect of constraints integration, etc.

3. Risk modelling

In the case of accident of hazardous materials transportation, the impacts can reach considerable dimensions (environment, infrastructure, economy, ...).

A good monitoring system for HazMat transportation on a network must integrate procedures of potential risks prevention related to transportation of hazardous materials. This risk depends on the type of transported product and all the targets which can be touched according to the considered dimensions. The US Department of Transportation guide, published by the Federal Highway Administration gives procedures for risk calculation per route segment. These calculations are essential for routing algorithms that identify minimum risk routes, where risk is established as being the probability product of having an accident by the consequences in term of cost, which can be expressed on the route segment in question and on its vicinity:

$$\text{Risk} = \frac{\text{accident probability}}{\text{accident consequences}}$$

Two remarks are to be formulated relatively about this procedure of risk calculation. They underline limits of risk calculation according to this classical and probabilistic method:

1. The above calculations suppose the existence of probabilities of accidents occurrence on a route sections. However, information is often insufficient to allow this calculation. In certain cases, the data results absence in taking null probabilities, by considering that the sections in question are invulnerable.
2. It may be very difficult to give a precise evaluation of an accident consequences in term of cost. For example, how can one quantify, with
high accuracy, cost component of an environmental impact?

Since those data can’t be precisely known, we have presented a fuzzy approach, which uses fuzzy data, to model risk on route sections. This risk fuzzification will allow use of results concerning path-finding problems in fuzzy graphs, described in Section 4.

3.1. Risk modelling using fuzzy set theory

Our approach models the concept of accident risk on each link of the transportation network by taking into account vulnerability of the link in question and cost generated in the event of accident on this link with respect to the various impacts considered. Examples of such impacts are given in the next section.

In our approach, the concept of vulnerability of a link replaces and generalizes that of the probability of having an accident on an link in the traditional method. It is evaluated by considering not only accidents data on a given link (such data are not always available) but more general information concerning the link in question and its vicinity with respect to a given impact. The generated consequences cost in the event of accident on a link is estimated in components relating to the considered impacts. Our method is thus to be more general than the traditional method since it makes it possible to take into account more factors about risk modelling. Moreover, the introduction of all these parameters will be done in such a manner which will simplify the complexity of probability calculation in the classical method since the level of vulnerability and the eventual accident cost on a link with respect to a given impact are taken as being fuzzy quantities. These fuzzy quantities are obtained by asking the actors (experts and decision makers) in the management system of the involved network to assign, with a degree of plausibility, qualitative evaluations to theses various parameters. Thus, the complexity of probability calculations will be replaced by a human judgment which allows to integrate into our model the experience of the transportation network actors. On each link the risk by impact is then taken as being vulnerability product and cost components evaluated for the considered impact. The overall risk force on each link is calculated by the application of an adequate fuzzy aggregation operator on the previous fuzzy parameters corresponding to the various impacts taken into account.

The approach that we propose for risk modelling on each link of a transportation network consists of the following major components:

- Impacts selection.
- Quantifying vulnerability and cost of an link with respect to each impact.
- Evaluating risk of a link relatively to a given impact.

3.1.1. Impacts selection

Numerous impacts can be proposed to account for modelling accident risk on links of a transportation network. These impacts can be classified into categories according to the considered dimensions. Table 1 shows examples of such dimensions and impacts.

3.1.2. Quantifying vulnerability/cost of a link with respect to each impact

After having fixed various impacts to be considered in accident risk modelling on links, the next step is to rate vulnerability and cost of a link with respect to each of these impacts. In real life transportation network theses rating data can’t be precise. A decision maker may encounter difficulty in quantifying what should be the consequence in term of cost on a link for the environmental impact population for example. Thus, these rating data are subjective and depend on decision makers judgments. Different rating systems may be used. They

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<tr>
<th>Table 1</th>
<th>Risk components</th>
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<td>Environmental components</td>
<td>Population</td>
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<td>Protected areas</td>
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<td>Hydrography and hydrology</td>
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<td>Land use</td>
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<td>Infrastructure components</td>
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<td>Traffic conditions</td>
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<td>Economic components</td>
<td>Transport costs, factories</td>
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are based on fuzzy sets, fuzzy or linguistic terms. Each linguistic term can be represented by the approximate reasoning of fuzzy set theory. Several standard conversion scales are proposed in to systematically convert linguistic terms to their corresponding fuzzy numbers for eventual fuzzy arithmetic operations. In these referenced works, the authors consider trapezoidal fuzzy numbers to capture vagueness of those linguistic assessments since they are easy to use and easy to interpret.

Here, we propose fuzzy sets on ordinal set of integer $S = \{0 \text{ (very low)}, \ 1 \text{ (low)}, \ 2 \text{ (medium)}, \ 3 \text{ (high)}, \ 4 \text{ very high}\}$ to account for ambiguities involved in the evaluation of vulnerability and cost of a link with respect to each of the considered impacts.

3.1.3. Evaluating link risk with respect to a given impact

We note $I = \{\text{imp}_1, \text{imp}_2, \ldots, \text{imp}_n\}$ the considered impacts set. For each link $u$ of the graph structure modelling the transportation network, let us denote:

- $\varphi_1(u,\text{imp}_i)$, the fuzzy quantity obtained as described in the section above and which represents the vulnerability assessment for link $u$ with respect to the impact $i$.
- $\varphi_2(u,\text{imp}_i)$, the fuzzy quantity representing the evaluation cost for link $u$ with respect to the impact $i$.

We define the risk degree relating to an impact of $I$ for an link $u$ as a fuzzy quantity noted $\varphi_3(u,\text{imp}_i)$, and calculated on the basis of the fuzzy quantities $\varphi_1(u,\text{imp}_i)$, and $\varphi_2(u,\text{imp}_i)$, according to the following formula:

$$\varphi_3(u,\text{imp}_i) = \varphi_1(u,\text{imp}_i) \otimes \varphi_2(u,\text{imp}_i)$$

for $i = 1, \ldots, n$, where $\otimes$ is the fuzzy multiplication.

After having evaluated in the previous paragraph, $(\varphi_3(u,\text{imp}_1), \ldots, \varphi_3(u,\text{imp}_n))$, the components risk on a link with respect to each of the n impacts of $I$, this section will present how to integrate these fuzzy quantities to determine the overall risk force on a link. There are many methods to aggregate decision maker fuzzy assessments. A broad classification of such methods which are frequently cited can be found in [22]. As we face here a problem of weighted aggregation, our proposition is based on Yager’s work on weighted aggregation [20].

To include weights in the aggregation procedure, for each link $u$ we transform the quantities $\hat{\varphi}_3(u,\text{imp}_i)$ into $\varphi_3(u,\text{imp}_i)$ using the following transformation function $g$ based on the probabilistic $t$-norm as follows:

$$\varphi_3(u,\text{imp}_i) = g(w_i, \varphi_3(u,\text{imp}_i))$$

Then we apply the max aggregation to obtain the overall risk force on link $u$.

As discussed by Yager for the max aggregation operator, since it is large values that play the most important role in aggregation, we desire to transform the least important elements into small values and thus have them not play a significant role in the max aggregation. The transformation function $g$ could be any $t$-norm for max aggregation.

4. Paths operators and fuzzy modelling

The fuzzy shortest path-finding problem from a specified source node to the other nodes appears in several applications. In transportation systems area, their corresponding networks use fuzzy information on the links, assumed to represent transportation time or economic cost than traffic flow, etc. These information are soft and would be well presented by fuzzy numbers or fuzzy set based on fuzzy set theory [13,21]. The first developed works to solve the problem of the fuzzy shortest path have been initiated for the first time by Dubois and Prade [8]. Nevertheless, if the research of the shortest path length in a fuzzy graph is feasible, generally this path doesn’t correspond to a real path in the considered fuzzy graph. This exception is explained by the particular behavior of the generalized min and max operators for the fuzzy numbers.

Dubois and Prade [8] comment the solution of the classical fuzzy shortest path problem through use of extended sum, and extended min and max. To solve the problem Floyd’s algorithm and Ford’s algorithm are applied. Unfortunately, this approach, even though it can determine the length
of a fuzzy shortest path, it cannot find a fuzzy path which corresponds to this length in the fuzzy graph. This failure is a consequence of classical operators extension min and max, according to the extension principle. From that principle, extended min or max of several fuzzy numbers may not be one of those numbers.

Some approaches based on the concept of z-cut [6,7,14,18] and other models based on the parametric orders [9] or relation order [19] did permit to reuse the classic methods in various fuzzy graphs applications on operations research field.

With regard to the based methods on parametric or relation orders, a work proposed by Furukawa [9] uses a modified Dijkstra’s algorithm for valued fuzzy graphs, where valuations of the links are L-fuzzy numbers. Another algorithm has been proposed by Okada and Soper [19] for the valued fuzzy graphs with L–R fuzzy numbers. The proposed algorithm defines a relation order between L–R fuzzy numbers. It is based on the multiple labeling method to obtain all nondominated paths. The multiple labeling can be considered as a generalization of Dijkstra’s algorithm [19].

A formulation of the fuzzy shortest path problem, not doing reference to the concept z-cut or parametric orders, has been proposed by Klein [12]. Klein’s algorithm is based on multi-criteria dynamic programming, and can find a path or paths for a level of membership set by a decision maker. This algorithm, however, assumes that the valued fuzzy graphs are acyclic graphs. To apply the Klein’s algorithm to other graphs, Klein proposed a transformation for these graphs according to the following remark owed to Lawler [17]: each graph that has no cycles of negative weight can easily be converted to a directed acyclic graph. Nevertheless, the transformation procedure is NP-Hard. Hence, for computational aspects, the Klein’s algorithm is restricted to acyclic graphs.

The present work proposes a structure of dioid (path algebra) to solve the fuzzy shortest path problem in a fuzzy graph [2,4]. This structure agrees to solve the problem of the K-best fuzzy shortest path. This first result generalizes the Klein’s work. This paper starts the extension of Gondran path algebra [10,15] given for the crisp case to the valued fuzzy graphs.

### 4.1. Valued fuzzy graph

Let $\Omega$ be a finite set which is assumed equal to $\{1, \ldots, n\}$. The structure $(\Omega, \sigma, \mu, \phi)$ will be called a valued fuzzy graph on $\Omega$ where:

- $\sigma : \Omega \to [0, 1]$ and stands for the membership level of each node;
- $\mu : \Omega \to [0, 1]$ and stands for the membership level of each link;
- $\phi : \Omega^2 \to $ a totally ordered set. In this paper $\mathbb{R} = [0, 1]^{\Theta}$, where $\Theta \subset \mathbb{N}$ the set of natural numbers.

### 4.2. Graphs and path algebra

Closed semiring or path algebra played a sophisticated role in solving general path problems. According to Kuich and Salomaa [15], a closed semiring is a set $S$ with $(e, e) \in S$ and $\oplus, \otimes : S \times S \to S$ where $(S, \oplus, e)$ is a commutative monoid, $(S, \otimes, e)$ is a monoid, $\otimes$ is distributives over $\oplus$, $e$ is a null-element w.r.t. $\otimes$, infinite sums exist, commutativity holds for infinite sums, and $\otimes$ is distributives over infinite sums. Moreover, the law $\ast$ denotes the closure operation which is defined by $\forall S \in S : s' = \sum_{i \geq 1} s$. 

Given a graph $G = (V, E)$ and a labeling of edge $\beta : E \to S$, Kleene’s or generalized Gauss–Seidel’s algorithm [10] in solving general path problem can be realized. Then, it is clear that all pairs shortest path problem can be solved by choosing the semiring $(S, \oplus = \min, \otimes = +, e = \infty, e = 1)$. For efficiency aspects, we can use Dijkstra’s algorithm for a class of semirings. Anyhow, in the spirit of this paper it would be more appropriate to create an own operator for semiring applications (see Fig. 7). The main idea developed below for the fuzzy path-finding on fuzzy spatial networks is based on semiring theory using a generalized Gauss–Seidel’s algorithm (Ford’s algorithm is an instance of Gauss–Seidel’s algorithm in the classical case). Numerous mathematical algorithms to find the shortest path from a given point to another over a network are proposed. In recent works, we can find a study of several shortest path algorithms that run fastest on real road network. Three
important algorithms are used: (i) the graph growth algorithm implemented with two queues, 
(ii) the Dijkstra’s algorithm implemented with approximate buckets, and (iii) the Dijkstra’s algo-
rithm implemented with double buckets. Performance of a particular shortest path algorithm partly 
depends on how basic operations in labeling method are implemented. Two aspects are particu-
larly important to the performance of a shortest path algorithm: strategies used to select next tem-
porarily labeled node to be scanned and data structure used to maintain labeled nodes set.

4.3. Path algebra for the K-best fuzzy shortest path

Let \( \Omega_k \) be the set of all subset of \( \mathbb{N} \) the set of natural numbers, with cardinality \( k \).

Let \( \lfloor \cdot \rfloor_k \) denote the selection operator that returns only the \( k \) first ordered elements of the con-
sidered set. Let \( A \oplus B \) denote the addition operator defined on fuzzy sets using the extension 

principle \([12,22]\).

Let \( A \cup B \) denote the union operator defined on fuzzy sets.

The path algebra \((\Omega_k, \oplus, \otimes, e, e)\) proposed for the \( K \)-best fuzzy shortest path problem is de-
veloped below \([2,4]\):

Let \( A \in \Omega_k \), and \( A \in \Omega_k \), then we have:

- the fuzzy set \( A \oplus B \) of \( \Omega_k \) is obtained as \( A \oplus B = [A \cup B]_k \),
- the fuzzy set \( A \otimes B \) of \( \Omega_k \) is obtained \( A \otimes B = [A \oplus B]_k \),
- \( e = \emptyset \) and \( e = \) the fuzzy set \( \{1/0\} \).
**Proposition 1.** The structure \((\Omega_k, \oplus, \otimes, e, e)\) is a path algebra.

Let \(G(\Omega, \sigma, \mu, \phi)\) a valued fuzzy graph. Assume that the structure \((N, \otimes, \epsilon, e)\) is a path algebra, then, in the case of a graph without \(p\)-absorbing cycles the generalized Gauss–Seidel's algorithm [10] for shortest path problem is given below. Here the labels \(\pi(i)\) give the length of the shortest path or paths from vertex 1 to vertex \(i\).

**Proposition 2.** The generalized Gauss–Seidel's algorithm always converges in \((\Omega_k, \oplus, \otimes, e, e)\) path algebra for the \(K\)-best fuzzy shortest path of a valued fuzzy graph without \(p\)-absorbing cycles.

**Generalized Gauss–Seidel’s algorithm**

The function \(\Gamma\) gives the set of successors of each node.

\[
\begin{align*}
(a) & \quad \pi(1) = e, \pi(i) = \phi(1, i) \text{ for } i \geq 2 \\
(b) & \quad \text{at step } k, \\
\pi(1) & \leftarrow \sum_{j \in \Gamma^{-1}(1)} \pi(j) \otimes \phi(j, 1) + e \\
\end{align*}
\]

\(\pi(1)\) for \(i = 2 \ldots n\) do

\[\pi(i) \leftarrow \sum_{j \in \Gamma^{-1}(i)} \pi(j) \otimes \phi(j, i)\]

(\(\beta\) Repeat (\(\beta\) until stabilization of \(\pi(i)\).)

The generic manipulations around graphs are very often used by operators to monitor the urban transportation network. These tasks concern the routing and the assessment of the itinerary risk in the case of the HazMat application. It is in this point of view that a Graphs Operators Library (GOL) has been developed. New operators based on the paths algebra and making reference to the fuzzy set theory is proposed in this Library (Figs. 7 and 8).

### 4.4. Fuzzy graphs and object oriented modelling

Our main practical objective is to develop a Fuzzy Graph Object Components. The software component could be integrated in many applications. To this end, we have used mainly the object-oriented-programming methodologies. We use the C++ programming language for the implementation of concepts. Frequently, basic Abstract Data Type are implemented in the Standard
Template Library (STL). STL is assumed to become a part of the C++ standard library and therefore it is an ideal basis when writing portable programs. Unfortunately, STL has no support for graphs. Abstract Data Type. We decided to implement a fuzzy graph library based on STL. The built library contains classes needed to work with fuzzy graphs, nodes and edges and some basic fuzzy path-finding algorithms: Klein’s algorithm, Dijkstra’s algorithm for parametric and relation orders, and Generalized Gauss–Seidel’s algorithm with the corresponding path algebra for the K-best shortest path problem.

5. Conclusion

In this paper two fundamental aspects are considered. The first is bound to the telegeomatic transportation systems, and around this concept an application have been achieved, which is relative to the routing and HazMat monitoring. In this paper we have proposed a new algebraic structure for the K-best fuzzy shortest path-finding in the valued fuzzy graphs. The proposed path algebra structure permits to solve the problem of the K-best fuzzy shortest path. The main result introduced in this paper concerns the extension of the path-finding based on path algebra paradigm to the valued fuzzy graphs. Future investigations would involve analysis of the obtained paths and a development of procedures to help decision maker to choose paths in the context of HazMat routing. The general algorithm with the proposed path algebra helps us to solve the K-best fuzzy shortest path problem. Further orientations would be considered to improve computing time aspect. The transposition of results given in this paper to solve some operational problems is foreseen (scheduling problem with duration expressed with fuzzy variables, fuzzy routing problem in the area of telecommunication and transportation integrating the fuzzy modelling).

References


