A Low-Complexity Algorithm for Antenna Selection in Space-Time Block Coded Systems

Chiang-Yu Chen, Aydin Sezgin, John M. Cioffi, and Arogyaswami Paulraj
Department of Electrical Engineering
Stanford University
Stanford, CA 94305 USA
Email: {chiangyu.chen, sezgin, cioffi, apaulraj}@stanford.edu

Abstract—This paper presents a practical algorithm for antenna selection in multiple-input multiple-output wireless communication systems employing space-time block codes (STBC). It first shows that maximizing the channel Frobenius norm helps maximize the mutual information for both orthogonal STBC and quasi-orthogonal STBC. However, the computational complexity for finding the optimal antenna subset grows exponentially with the number of antennas. This paper identifies that the channel Frobenius norm maximization problem can be formulated as a quadratically constrained quadratic programming (QCQP) problem. Then, despite the fact that the problem is non-convex, a semidefinite relaxation of QCQP enables the problem to be solved approximately by semidefinite programming in polynomial time. Simulation results indicate that the loss of semidefinite relaxation is negligible. It is also shown that although the combination of STBC and antenna selection is not always beneficial, it is a robust transmission strategy in the high SNR regime when only imperfect channel information is available.

I. INTRODUCTION

Multi-antenna wireless communication systems have demonstrated significant performance enhancement in recent years. The improvements over traditional single-antenna systems originate from the intelligent use of the extra dimensions, and are widely characterized by spatial multiplexing and diversity gain [16], [19]. However, the deployment of multiple RF-chains, which includes analog-digital converters, low noise amplifiers, downconverters, etc., increases the cost and system complexity dramatically. To reduce the number of RF-chains while keeping the benefits of using multiple antennas, algorithms of antenna selection (AS) are proposed [6], [7], [10] to fed the most favorable subsets of transmit and/or receive antennas to RF chains.

The works on AS in space-time block coded (STBC) systems can be found in [7], [14], [5], [20]. Since the orthogonal STBC (OSTBC) decouples all the transmitted symbols, the SNR for each of them is proportional to the channel Frobenius norm. Therefore, maximizing the channel Frobenius norm minimizes the pairwise error probability (PEP) at the same time. Moreover, this selection criterion maximizes the mutual information. Nevertheless, the well-known Alamouti scheme for two transmit antennas [1] is the only existing full-rate complex OSTBC and all the other complex orthogonal designs suffer a rate loss [18], [9]. Consequently, quasi-orthogonal STBC (QSTBC) are proposed to provide better spectral efficiency [17], [8], [11]. Since QSTBC can not decouple all the transmitted symbols and suffers higher decoding complexity, the SNR is not completely proportional to the channel Frobenius norm. Several antenna selection criteria and their performance comparisons of QSTBC are shown in [14].

Despite many analytical and numerical results in the literature, this paper argues that using STBC in systems with transmit AS capability, however, is not always beneficial. This is because the STBC averages channel gains of all the selected transmit antennas. When the estimated channel is exact and the channel fades slowly, to excite the single best transmit antenna only always gives a larger channel gain than the averaged gain of the best few transmit antennas. Hence, the more transmit antennas are used together with STBC, the worse the system performance. However, STBC with AS makes the MIMO system robust when there exists channel estimation error, or the channel changes so fast that it becomes different at the time the signal is transmitted. In these situations, the diversity gain provided by STBC can protect the signals from being corrupted by the unwanted channel change or by imperfect channel estimation. Therefore, with different channel condition and SNR, the best STBC schemes can vary.

This paper also shows that the channel Frobenius norm maximization is a promising antenna subset selection criterion for both OSTBC and QSTBC-based systems towards maximizing the mutual information. Conventionally, the joint transmit and receive AS is done by an exhaustive search of all the possible combinations, which produces computational complexity growing exponentially with the number of antennas. To reduce the complexity, a quadratically constrained quadratic programming (QCQP) problem is formulated to solve the Frobenius norm maximization problem. The problem is non-convex and NP-hard. Fortunately, a semidefinite relaxation converts the QCQP to a semidefinite programming (SDP), which is convex and can be sub-optimally solved in polynomial time. Simulation results indicate that on average the proposed algorithm achieves a percentage of the maximum Frobenius norm close to 100%.

The organization of this paper is as follows: Section II introduces the system model for STBC-based systems and derives the selection criterion. Section III explains how the AS problem can be formulated as a QCQP. To solve the QCQP with polynomial complexity, the SDP relaxation technique is then described in Section IV. Section V provides the numerical results. Finally, conclusions are drawn in Section VI.

II. SYSTEM MODEL

This paper focuses on a MIMO system with a total of $M_T$ transmit antennas and $M_R$ receive antennas. The process of
the AS is divided into the following steps:

- The receiver obtains an estimation of a $M_T$-by-$M_R$ channel matrix $F$ and chooses $N_T$ transmit antennas and $N_R$ receive antennas. The channel matrix of the selected antennas is denoted as an $N_T$-by-$N_R$ matrix $M(F)$, a minor of $F$ with elements corresponding to the selected antennas.
- The receiver feedbacks the indices of the selected transmit antennas. The transmitter sends the space-time coded signal through those antennas.
- Assume the channel changes from $M(F)$ to $H$ and the receiver tracks it perfectly. The receiver then decodes the signal based on the new channel estimate $H$.

This paper assumes each path experience i.i.d. Rayleigh fading so that the entries of $F$ are modeled by independent zero-mean circularly symmetric complex Gaussian (ZMCSCG) random variables with unit variance. Also, the change of channel is modeled by a Gauss-Markov process:

$$H = \sqrt{1 - \sigma_e^2}M(F) + \sigma_e E,$$

(1)

where the entries of $E$ are also independent ZMCSCG random variables with unit variance. $\sigma_e \in [0, 1]$ represents the uncertainty of the channel. The larger $\sigma_e$ means the channel changes faster or the initial estimation of $F$ in the first step is less precise. The coefficient in front of $M(F)$ keeps the variance of the channel unchanged.

The STBC with block length $T$ maps its input symbols $x_1, x_2, \ldots, x_N$ to a $T$-by-$N_T$ transmission matrix $S$. The relationship between the transmitted and received signals can be expressed as:

$$Y = \sqrt{\frac{E_s}{N_T}}S(x_1, \ldots, x_N)H + V,$$

(2)

where $Y, V$ are $T$-by-$N_R$ matrices denoting received signals and additive white Gaussian noise with variance 1.

The Alamouti scheme is a well-known technique exploiting full transmit diversity for systems with two transmit antennas [1]. It is also proved to be the only existing full-rate (i.e. $T = N_T$) complex OSTBC [18]. The transmission matrix can be written as:

$$S(x_1, x_2) = \begin{pmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{pmatrix}.$$  

(3)

Because $x_1$ and $x_2$ arrive at the receiver in orthogonal directions, a simple linear receiver can decouple the transmitted symbols and achieve the maximum-likelihood (ML) decoding. As a result, the receive SNR $\gamma$ for each channel realization can be represented as

$$\gamma = \frac{E_s}{2}\|H\|^2_F.$$  

(4)

The mutual information achieved by Alamouti scheme $I_O$ becomes:

$$I_O = \log_2 \left( 1 + \frac{E_s}{2}\|H\|^2_F \right).$$  

(5)

Unfortunately, the advantages of Alamouti scheme cannot be extended to systems with more than two transmit antennas since the maximum code rate of OSTBC decreases as $N_T$ increases [9]. To compensate this rate loss, QSTBCs are proposed to achieve full-rate with only partial orthogonality [17], [11], [8]. An example of QSTBC for a 4-transmit antenna system is the TBH scheme in [17]:

$$S(x_1, x_2, x_3, x_4) = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\
 x_3 & x_4 & x_1 & x_2 \\ -x_4^* & x_3^* & -x_1^* & x_2^* \end{pmatrix}.$$  

(6)

It can be easily verified that each column is orthogonal with another two columns but coupled with the remaining one. Therefore, the four input symbols are decoupled into two symbol pairs $(x_1, x_3)$ and $(x_2, x_4)$, where ML decoding is accomplished by the search of all possible symbol pairs. For systems with more than 4 antennas, the corresponding full-rate QSTBC can be constructed as in [13]. [13] also shows that the mutual information achieved by QSTBC is:

$$I_Q = \frac{2}{N_T} \log_2 \left( 1 + \frac{E_s}{2} \sum_{j=1}^{N_T/2} \|h_j\|^2_F A_{N_T}^{kH} h_j \right),$$

(7)

where $h_j$ denotes the $j$-th column of $H$, and $(\cdot)^H$ is the Hermitian operation of vectors and matrices. The matrices $A_{N_T}^k$ for all $1 \leq k \leq N_T/2$ are idempotent, i.e. they are of rank two and both the non-zero eigenvalues are 1. Also, $\sum_{k=1}^{N_T/2} A_{N_T}^k = I_{N_T}$. The formulation of $A_{N_T}^k$ can be found in [13]. By A.M. $\geq$ G.M., an upper bound of $I_Q$ can be derived:

$$I_Q \leq \log_2 \left( 1 + \frac{E_s}{N_T} \|H\|^2_F \right).$$  

(8)

This upper bound is reasonably tight [11], and can be treated as an extension of (5).

It can be seen from (5) and (8) that the mutual information for both OSTBC and QSTBC increases as the channel Frobenius norm increases. In order to maximize the mutual information, the AS algorithm should select the antennas associated with the minor of $F$ having the largest Frobenius norm. Therefore, the following sections use the maximization of the channel Frobenius norm as the antenna selection criterion.

Although the amount of feedback required in the system combining STBC with AS is small, it does not imply that the scheme of interest is always beneficial. For example, when $\sigma_e = 0$ so that $H = M(F)$, compare STBC with a scheme that uses only a single antenna to transmit. Let $m$ be the row of $H$ with the largest norm. Then, $\|m\|^2_F \geq \|H\|^2_F/N_T$. Therefore, the mutual information achieved by this single antenna scheme becomes $\log_2(1 + E_s\|m\|^2_F)$, which is greater than the mutual information achieved by STBC in (5) or (8).

Since the selection of the single antenna already provides a full diversity gain $M_T-M_R$, STBC with AS cannot exploit extra diversity. Moreover, it suffers a lower coding gain than the single antenna transmission. This is because the STBC averages the channel gain of the best transmit antenna with that of other worse (but still better than those not selected) transmit antennas. On the other hand, the single antenna transmission...
simply picks the best one to transmit. This fact was not considered in the literature so far [5], [7]. Of course, by using multiple transmit antennas efficiently (e.g. eigenmode transmission), the mutual information achieved by AS can be greater than the single antenna transmission. However, since STBC allocates the energy of all the selected antenna equally, with the same $M_T$, the larger the $N_T$, the less the coding gain.

However, when $\sigma_r > 0$, selecting the best minor of $F$ does not guarantee that the channel $H$ remains good as the transmitted signal is being sent. In that case, choosing the best transmit antenna can not achieve the full diversity order. Thus, a better diversity can be obtained by averaging the channel gains of multiple transmit antennas. Consequently, even when the transmit antenna with the largest Frobenius norm changes, there is still high probability for the other antennas remaining in a good channel condition. The STBC with larger $N_T$ may still suffer from a loss of coding gain in the low SNR regime. But eventually it outperforms systems with fewer transmit antennas in the high SNR regime.

III. PROBLEM FORMULATION

Since maximizing the channel Frobenius norm is a suitable approach for both OSTBC and QSTBC, an exhaustive search of all possible $(M_T \times N_T) (N_R \times M_R)$ combinations of transmit and receive antennas can be done to solve the AS problem. However, the number of combinations grows exponentially with the number of antennas and the computational complexity of exhaustive search becomes intractable when either $M_T$ or $M_R$ or both is large. To solve the problem with reasonable complexity, the maximization of Frobenius norm is first formulated as:

$$\begin{align*}
\text{maximize} & \quad \sum_{i=1}^{M_T} \sum_{j=1}^{M_R} |f_{ij}|^2 t_i r_j \\
\text{subject to} & \quad \sum_{i=1}^{M_T} t_i = N_T, \quad \sum_{j=1}^{M_R} r_j = N_R \\
& \quad t_i, r_j \in \{0, 1\}, \quad \forall i, j,
\end{align*}$$

where $f_{ij}$ denotes the $(i,j)$-th entry, the path gain from the $i$-th transmit antenna to the $j$-th receive antenna, of $F$. \{t_1 \cdots t_{M_T}\}$ represents the indicators of the transmit AS and \{r_1 \cdots r_{M_R}\} denotes the indicators of receive AS. That is, the sets $I_T = \{i | t_i = 1, \forall i\}$ and $I_R = \{j | r_j = 1, \forall j\}$ correspond to the selected subsets of transmit and receive antennas, respectively. Thus, the sum of $t_i$ ($r_j$) should be equal to $N_T$ ($N_R$). Whenever the constraints are satisfied, the objective function becomes simply the channel Frobenius norm for selected antennas.

Let $v = [t_1 \cdots t_{M_T} r_1 \cdots r_{M_R}]^T$ be the vector cascading all of the optimization variables and $G$ be a $M_T$-by-$M_R$ matrix whose $(i,j)$-th element is $|f_{ij}|^2$. The problem can be rewritten as:

$$\begin{align*}
\text{maximize} & \quad v^T Q v \\
\text{subject to} & \quad A v = b \quad (10) \\
& \quad v_i \in \{0, 1\}, \quad \forall i \in \{1, \cdots, M_T + M_R\},
\end{align*}$$

where

$$\begin{align*}
Q & = \frac{1}{2} \begin{pmatrix} 0_{M_T \times M_T} & G \\ G^T & 0_{M_R \times M_R} \end{pmatrix}, \\
A & = \begin{pmatrix} 1_{1 \times M_T} & 0_{1 \times M_R} \\ 0_{1 \times M_T} & 1_{1 \times M_R} \end{pmatrix}, \quad \text{and} \\
b & = [N_T \ N_R]^T.
\end{align*}$$

Here $0_{m \times n}$ denotes an $m$-by-$n$ all-zero matrix and $1_{m \times n}$ is the all-one matrix. (10) has the form of $(0, 1)$-quadratic programming (QP), which is a special type of optimization problem. In general, $(0, 1)$-QP is NP-hard and cannot be solved optimally in polynomial time. Moreover, (10) can also be seen as a quadratically constrained quadratic programming (QCQP) by replacing the original constraints with their equivalents: $v_i(v_i - 1) = 0$, and $\|A v - b\|^2 = 0$. Then,

$$\begin{align*}
\text{maximize} & \quad v^T Q v \\
\text{subject to} & \quad v^T A^T A v - 2b^T A v + b^T b = 0 \\
& \quad \text{diag}(v^T v) - v = 0, \quad (11)
\end{align*}$$

where $\text{diag}(\cdot)$ is a column vector with elements equal to the diagonal elements of the input matrix. However, since $Q$ is indefinite, this problem is a non-convex QCQP.

IV. SEMIDEFINITE RELAXATION OF QCQP

Fortunately, nonconvex QCQP can be solved approximately by using semidefinite relaxations. The relaxation can be done by either primal side or the dual side ([12], [3]). Because the primal relaxation gives more intuitions to the solution vector, this paper adopts the primal side approach. First, a change of variable $x = [1 \ (2v^T - 1_{1 \times (M_T + M_R)})]^T$, a $(M_T + M_R + 1)$-dimensional vector, converts this problem from a $(0, 1)$-QP to a $(-1, 1)$-QP:

$$\begin{align*}
\text{maximize} & \quad x^T Q_c x \\
\text{subject to} & \quad x^T B x = 0 \\
& \quad \text{diag}(xx^T) = 1, \quad (12)
\end{align*}$$

where

$$\begin{align*}
Q_c & = \begin{pmatrix} 0 & I_{1 \times (M_T + M_R)} \\ I_{(M_T + M_R) \times 1} & Q \end{pmatrix}, \\
B & = \begin{pmatrix} b_c^T b_c & -b_c^T A \\ -A^T b_c & A^T A \end{pmatrix}, \quad \text{and} \\
b_c & = [2N_T - M_T \ 2N_R - M_R]^T.
\end{align*}$$

Since $x^T Q_c x = \text{Tr}(x^T Q_c x) = \text{Tr}(Q_c xx^T)$, (12) can be rewritten as:

$$\begin{align*}
\text{maximize} & \quad \text{Tr}(Q_c X) \\
\text{subject to} & \quad X = xx^T, \\
& \quad \text{Tr}(BX) = 0 \\
& \quad \text{diag}(X) = 1. \quad (13)
\end{align*}$$

The problem then is ready to be relaxed into a convex problem by replacing the non-convex constraint $X = xx^T$ with a convex constraint $X \succeq 0$, where the operation $A \succeq B$
means $A - B$ is a semidefinite matrix. While the original constraint was of rank 1, $X$ can be of any rank with this relaxation. The relaxed problem is a semidefinite programming (SDP) problem:

$$\begin{align*}
\text{maximize} & \quad \text{Tr}(QcX) \\
\text{subject to} & \quad \text{Tr}(BX) = 0 \\
& \quad \text{diag}(X) = 1 \\
& \quad X \succeq 0.
\end{align*}$$

(14)

Since problem in (14) is convex, it can be solved in polynomial time by convex optimization algorithms [4]. The solution to (14) gives an upper bound of the objective function and a relaxed solution $X^*$. If $X^*$ has rank 1, it can be directly decomposed to $X^* = x^*x^{*T}$, and the solution to the relaxed SDP problem is the same as that to (13). Moreover, in this case, $X^*$ consists of only $-1$ and $1$. Thus, the optimal $v$ can be obtained from the last $(MT + MR)$ entries of the columns of $X^*$ with $1$ as its first entry.

Nevertheless, unless $X^*$ is of rank 1, it does not give a direct hint on finding a feasible point of $x$ and $v$. Therefore, a rounding algorithm is needed to achieve a good rank-1 approximation. Typically, a randomized method that generates random independent samples and keeps the best feasible one (e.g., the feasible point corresponding to the greatest objective among all the random samples) is adopted. This paper considers an heuristic approach: let $u^*$ be the eigenvector associated with the largest eigenvalue ($\lambda_{\text{max}}$) of $X^*$ and $u^*_i$ be its $i$-th entry [2]. Without loss of generality, $u^*_1$ is assumed positive. Then, $\lambda_{\text{max}}u^*u^{*T}$ can be used directly as the rank 1 approximation of $X^*$. In order to assign the selected antennas, $\{u^*_2, \ldots, u^*_{MT+1}\}$ are sorted for the transmit AS while $\{u^*_{MT+2}, \ldots, u^*_{MT+MR+1}\}$ are sorted for the receive AS. The selected transmit (receive) antennas become those corresponding to the largest $NT$ ($NR$) $u^*_i$’s. Since this simple rounding algorithm is polynomial-time solvable as well, the polynomial-time solvability of the whole problem is kept. As we will see in the following section, the performance of the proposed algorithm is very close to that of the exhaustive search. Thus, it is not necessary to improve the rounding algorithm by sacrificing the complexity.

V. SIMULATION RESULTS

Fig. 1 shows the ergodic mutual information achieved by systems using a single transmit antenna ($NT = 1$), Alamouti scheme ($NT = 2$), and QSTBC ($NT = 4$) with different AS algorithms. $MT = 8$, $MR = 4$, and $NR = 2$. Three AS algorithms: exhaustive search of maximum mutual information in (5) or (7), maximization of Frobenius norm via the SDP relaxation of QCQP, and no AS (random selection), are simulated for Alamouti scheme and QSTBC. For single antenna transmission, the maximum achievable mutual information and the mutual information without AS are shown. Perfect channel estimation with $\sigma_e = 0$ are assumed. It can be observed that except for the no AS case, the mutual information increases as the number of selected antenna $NT$ decreases for all SNR.

Also, AS for the single antenna transmission provide about 5.6 dB gain while the gain for QSTBC is only 3dB. The proposed algorithm solves the problem with very little loss of mutual information.

Table I presents more details of how the low-complexity algorithms performs. Three different simulation settings, as indicated in the first three rows, are examined. The fourth and fifth row show how the proposed algorithm compares with the solutions obtained by solving (9) with an exhaustive search. The last row shows the percentage of optimal mutual information in 5 or 7 achieved by the proposed algorithm. It can be observed that although the probability of selecting the optimal antenna subsets in (9) is not very high, the average objective value $\|H\|^2_F$ for the proposed algorithm is still very close to the optimal. That implies that when the proposed algorithm does not solve the problem optimally, most of the time it still finds very good subsets of antennas with a channel Frobenius norm and mutual information very close to the optimum.

Fig. 2 shows the 10% outage mutual information achieved by maximizing the channel Frobenius norm under a channel estimation error. The simulation parameters are the same as in Fig. 1. Three value of $\sigma_e$: 0, 1, and 0.8 are compared, where $\sigma_e = 1$ is essentially the same as the random selection (or no AS) in Fig. 1 because the estimated channel is not reliable at
all. When $\sigma_e = 0.8$, using OSTBC and QSTBC achieves the almost same amount of outage information, which has also 1dB gain over the single transmit antenna system. It can be expected that for even high $\sigma_e$, both the Alamouti scheme and the QSTBC will eventually outperform the system with $N_T = 1$.

Fig. 3 shows the BER performance of the systems employing a single transmit antenna, Alamouti scheme, and QSTBC. The simulation settings are all the same as those in Fig. 2 except $\sigma_e = 0.5$. QPSK constellation and the rotated constellation are simulated to achieve full diversity for QSTBC [15]. When $\sigma_e = 0$, all the three schemes present the same diversity gain, while using the best transmit antenna provides 0.6dB and 1.8dB coding gain over the Alamouti scheme and the QSTBC, respectively. However, as $\sigma_e$ increases to 0.5, different scheme shows distinct diversity order. Using the best transmit antenna performs best only when SNR is less than 4dB. The Alamouti scheme outperforms the other two when SNR is between 4dB and 9dB. If SNR is higher than 9dB, the QSTBC should be adopted in order to achieve the best BER. Eventually, when $\sigma_e$ reaches 1, QSTBC becomes the best scheme regardless of the SNR. Since the change of slope is smaller for larger $N_T$, using STBC is a robust and advantageous transmission strategy, especially in the high SNR regime or when the uncertainty of the channel is not negligible.

VI. CONCLUSION

Maximizing the channel Frobenius norm is a proper selection criterion for AS for systems using STBC. This paper proposes a low-complexity algorithm for solving the problem in polynomial time by first formulating a non-convex QCQP. Then a SDP relaxation enables the problem to be solved with very slight performance in polynomial time. The situations where STBC improves the performance of the AS system are also shown. The analysis on the diversity order of the coding schemes in different channel conditions is subject of future investigation.

REFERENCES


