A SIGNAL-ADAPTIVE DISCRETE EVOLUTIONARY TRANSFORM: GENERALIZED TIME-BANDWIDTH PRODUCT OPTIMAL STFT

Sultan Aldirmaz¹, Lutfiye Durak-Ata², Aydin Akan³, and Luis F. Chaparro⁴

¹,²Dept. of Electronics and Commun. Eng., Yildiz Technical Univ., Yildiz, Besiktas, 34349, Istanbul Turkey.
Phone: +90 212 383 24 98, +90 212 383 24 86 ¹aldirmaz@yildiz.edu.tr, ²lutfiye@ieee.org
³Dept. of Electrical and Electronics Eng., Istanbul Univ., Avciilar, 34320, Istanbul, Turkey. akan@istanbul.edu.tr
⁴Dept. of Electrical and Computer Eng., University of Pittsburgh, Pittsburgh, Pennsylvania 15261, USA, chaparro@ee.pitt.edu

ABSTRACT

The discrete evolutionary transform is applied to signals in a blind–way without any signal parameters. For this reason, discrete evolutionary transform is not optimal and needs an improvement by using some information about the signal. In this paper, we propose an improvement for the discrete evolutionary transform. We redefine the generalized time-bandwidth product optimal short–time Fourier transform as a special case of the discrete evolutionary transform. The optimized kernel function of the short–time Fourier transform is determined according to the instantaneous frequency of LFM-type signals. Even in case of quadratic FM signals, the resulting localization of the time-frequency representations improves remarkably. The performance of this optimal discrete evolutionary transform is presented on signals whose chirp rates vary by time.

1. INTRODUCTION

In nature, most of the signals vary both in time and frequency simultaneously and their characterization in both domains is an important issue for various applications such as processing speech, acoustic and biomedical signals, modeling and estimating the impulse responses of wireless communication channels, and analyzing sonar, radar and seismic signals. To acquire more information about the signal, we need to reveal its joint time-frequency behavior besides individual time-domain or frequency-domain structures separately. Thus, signal processing applications become much more powerful and accurate.

Time-frequency analysis tools such as short–time Fourier transform (STFT), spectrogram, continuous wavelet transform, Wigner Distribution (WD) and its derivatives have been used for this purpose for a long time. WD generates sharp and well-localized time-frequency representations of single–component signals, however it becomes inefficient for multi-component signals as it introduces cross-terms on the time-frequency plane [1]. Being a linear distribution, STFT is a reliable choice for multi-component signals. Moreover, to avoid the resolution problem of wavelet transform at high frequencies, STFT becomes an attractive alternative providing uniform resolution for all frequency bands on the time-frequency plane.

$STFT_x(t, f) = \int x(t')h(t' - t)e^{-j2\pi f t'} dt'$

where $h(t)$ is a low-pass unit-energy window function. Up until now, STFT improved by various techniques, i.e., in [2], an instantaneous frequency (IF) estimation technique is added to STFT to choose the window function appropriately. In [3, 4], fractional Fourier transform (FrFT) is used to obtain high resolution STFT images. Moreover, Durak et. al. introduce the generalized time-bandwidth product (GTBP)–optimal STFT by using FrFT and time-band-width product (TBP) of the signal in fractional Fourier domains [5, 6]. This way, it has been possible to adapt the STFT of signals to both IF and fractional-domain TBP of the signal simultaneously. In the simulations they presented the improvement imposed by the GTBP–optimal STFT on linear and quadratic FM signals. The GTBP definition provides a rotation-invariant measure of signal support on the time-frequency plane. Then, the optimal STFT kernel that provides the most compact representation is obtained by considering the GTBP of signal components. The proposed time-frequency analysis is shown to be equivalent to an ordinary STFT analysis conducted in a scaled fractional Fourier transform domain [5, 6].

On the other hand, the discrete evolutionary transform (DET) is introduced for non-stationary signal–analysis in [15]. Two different DET models have been defined as sinusoidal basis-DET and chirp basis-DET, previously. The appropriate DET can be selected depending on the application. If the signal is narrow-band, sinusoidal bases are more suitable to represent in time-frequency plane. On the other hand, chirp basis-DET is more capable when the signal contains wide-band components. Time and/or frequency dependence is provided to selecting the window type. For example, Malvar-based windows are both time and frequency dependent orthogonal bases, on the contrary Gabor-based windows are time-dependent. In the DET analysis, Malvar-based and Gabor-based windows are used in [7]. The DET is used in various applications such as a jammer excision algorithm [8], estimation of multipath fading and frequency selective channels [9].

The goal of this paper is to prove the GTBP–optimal STFT as a special case of DET. The remainder of this paper is organized as follows. In Section 2, preliminary information is given on discrete evolutionary transform, time-frequency localization, GTBP–optimal STFT and instantaneous frequency (IF) estimation procedure. Section 3 shows the link between the optimal STFT and DET. Simulation results are given in Section 4. Finally, conclusions are drawn in Section 5.
2. PRELIMINARIES

2.1 Discrete Evolutionary Transform

The DET is a time-frequency method that provides a representation of non-stationary signals as well as their spectra. The DET can be defined by using sinusoidal or chirp basis. When sinusoidal basis are used, the evolutionary kernel of the DET becomes

\[ X(n, f_k) = \sum_{l=0}^{N-1} x(l)W_k(n, l)e^{-j2\pi f_k l}, \quad 0 \leq k \leq K - 1 \] (2)

where \( W_k(n, l) \) represents the time–frequency dependent window function. The inverse DET is stated as

\[ x(n) = \sum_{k=0}^{K-1} X(n, f_k)e^{j2\pi f_k n}, \quad 0 \leq n \leq N - 1. \] (3)

If chirp bases are used, the kernel is changed to

\[ X_p(n, f_k) = \sum_{m} x_p(m)W_k(m, n)e^{-j(2\pi f_k m + \phi_p(m))} \] (4)

where \( x_p(n) \) and \( \phi_p(n) \) represent each of the individual signal components and phases, respectively. In case of chirp basis, the inverse transform is defined by

\[ x(n) = \sum_{p=0}^{P-1} \sum_{k=0}^{K-1} X_p(n, f_k)e^{j(2\pi f_k n + \phi_p(n))}. \] (5)

When Gabor–based DET is calculated, both the analysis function \( \gamma(.) \) and its dual pair synthesis function \( h(.) \) are employed. Figure 1 shows the block diagram that illustrates the calculation of the evolutionary kernel of sinusoidal–DET for \( x(t) \) when Gabor-bases are used. Gabor–coefficients \( a(m, k) \) are obtained as

\[ a(m, k) = \sum_{n=0}^{N-1} x(n)\gamma^*(n - mL)e^{-j2\pi n f_k}. \] (6)

Then, the evolutionary kernel becomes

\[ X^G(n, f_k) = \sum_{m=0}^{M-1} a(m, k)h(n - mL). \] (7)

The time-varying DET window is defined as

\[ w(l, n) = \sum_{m=0}^{M-1} \gamma^*(n - mL)h(n - mL). \] (8)

The main difference between the ordinary STFT and the DET is that the analysis window varies by time in DET. The time-varying window has been expressed as a function of a set of orthogonal functions in [15]. The evolutionary spectrum is defined as the magnitude square of the DET kernel as

\[ S_E(n, f_k) = |X(n, f_k)|^2. \] (9)

In Malvar-based DET, the analysis–window length is chosen with respect to the analyzed signal by using a cost function optimization. However, Gabor–based DET does not depend on the signal. Thus, it can be said that the DET provides a signal representation and its corresponding spectrum without using any of the characteristics of the signal, and thus it can be improved. Moreover, when the signal consists of multi–components, DET separates each of the components of the signal by using a mask as an offline–procedure. Then, each of the components is analyzed individually.

2.2 Time–Frequency Localization of Signals

Localization of a signal on time–frequency domain gives information about the signal support. According to the well–known uncertainty principle, there is a lower bound on the spread of a signal’s energy in both time and frequency domains together. This concentration may be measured by the time-bandwidth product (TBP), which is defined as the product of time-width \( T_x \) and bandwidth \( B_x \), and it is bounded by [10, p.50]

\[ T_xB_x \geq \frac{1}{4\pi} \] (10)

where

\[ T_x = \left[ \int |t - \eta_x| |x(t)|^2 dt \right]^\frac{1}{2} \] (11)

\[ B_x = \left[ \int |f - \eta_f| |X(f)|^2 df \right]^\frac{1}{2} \] (12)

and \( \eta_x, \eta_f \) and \( ||.|| \) are the time and frequency mean values and the norm operator, respectively. \( X(f) \) is the Fourier transform of \( x(t) \). The Gaussian function is the best localized function in both time and frequency domain having the lowest TBP equal to \( 1/(4\pi) \).

2.3 GTBP–Optimal STFT Definition

GTBP–optimal STFT is introduced as a signal dependent representation by Durak et al. [5, 6]
\[ D_x(t,f) = e^{-j\pi \psi} \int x(\tau) g_{GTBP}(\tau-t) e^{-j2\pi f\tau} d\tau \]  
(13)

where \( \psi = (t^2 - f^2) \sin \phi_0 \cos \phi_0 + 2tf \sin^2 \phi_0 \) and the optimal kernel is

\[ g_{GTBP}(\tau) = K e^{-j\pi \gamma} e^{-2\pi^2 \frac{\gamma \cos^2 \phi_0}{\gamma^2 + \sin^2 \phi_0}} \]  
(14)

with \( K = \sqrt{1 + \cot \phi_0} \) and \( \gamma = B_{x0}/T_{x0} \). \( B_{x0} \) and \( T_{x0} \) denote the bandwidth and time–width of \( a_0 \)-order fractional domain signal, respectively. Since the phase \( \psi \) can be ignored in \( D_x(t,f) \), it is easy to see that the desired representation in Eq. 13 has the form of STFT with kernel \( g_{GTBP}(\tau) \). Except the fractional order \( a_0 \) determination, the computational complexity of Eq. 13 is the same as the computational complexity of the ordinary STFT.

The discretized version of the optimal STFT is defined as

\[ D_x(m,k) = e^{-j\pi \psi} \sum_{n=0}^{N-1} x(n) g_{GTBP}(n-m) e^{-j\frac{2\pi}{N} kn} \]  
(15)

where \( N \) is the number of frequency bins.

Furthermore when the second term of Eq. (14) is included in the exponential term in Eq. (13), the optimal STFT can be recognized as a generalized discrete Fourier transform (GDFT), which is recently introduced by Akansu [14].

In [5], TBP is chosen as a suboptimal measure of support and TBP–optimal STFT kernel is obtained by using the following optimization scheme.

\[ \min_{T_B, B_\gamma} \left( T_B^2 + T_\gamma^2 \right)^{1/2} \cdot \left( B_{\gamma}^2 + B_\gamma^2 \right)^{1/2} \]  
(16)

It is shown that the TBP–optimal solution \( g_{TB}(t) \) must be the Gaussian kernel

\[ g_{TB}(t) = e^{-\pi x^2 B_\gamma/T_B} \]  
(17)

By using GTBP–optimal STFT technique an appropriate analysis window is found for the signal \( x(t) \) that varies in time, so that time-frequency distribution with the maximum concentration is obtained. The desired time-frequency representation of \( x(t) \) can equivalently be obtained as the counter-clockwise rotation of the optimal STFT for \( x_{a0}(t) \) by an angle of \( \phi_0 \) where

\[ \text{STFT}_{x_{a0}}(t,f) = \left( \int x(\tau) \left( \int h(t'-\tau) e^{j2\pi f'\tau} B_{-a_0}(t', \tau) d\tau \right) d\tau \right)^{1/2} \]  
(18)

with optimal Gaussian kernel \( h(t) = e^{-\pi y^2 \tau^2} \). The desired representation of \( x(t) \) is expressed as

\[ \text{D}_{x}(t,f) = R_{\phi_0} \{ \text{STFT}_{x_{a0}}(t,f) \} \]  
(19)

In Eq. 22, the expression in the brackets can be recognized as the \( -a_0^{th} \)-order FrFT of \( h(t' - \tau) e^{j2\pi t'\tau} \), which is simply the time and frequency shifted form of the kernel \( h(t) \). In [5] it is shown that Eq. 22 is equivalent to Eq. 13.

2.4 Estimation of the Instantaneous Frequency

Instantaneous frequency (IF) of a signal \( x(t) = A(t) e^{j\phi(t)} \) can be defined as \( f_I(t) = \frac{d}{dt} \phi(t) \). IF gives the information about the frequency variation of a signal by time. There are a lot of IF estimation techniques in the literature [2, 11–13]. STFT has been employed to estimate IF of the signal in [2]. In [11], two different IF estimation algorithms are proposed by using optimization scheme, which makes use of the maximum fractional time–bandwidth ratio, and a minimum essential bandwidth, which is expressed as the minimum sum of the bandwidths of the separate signal components. Genetic algorithms are employed to determine the IF of the signal components. Image processing techniques are used in [12] for multi–components LFM signals. Except these, one way of determining the IF of an LFM signal component is to search for the peaks of the FrFT magnitudes computed at various fractional orders. This method makes use of the relationship between the Radon–Wigner transform of a signal and its corresponding FrFT [13]. Figure 3 shows the FrFT order estimation by searching the maximum peak value among all FrFTs of the signal. The analyzed signal includes AWGN noise of 5dB SNR. Thus, this IF procedure is robust against the noise.

3. REPRESENTATION OF THE GTB–OPTIMAL STFT AS A DET

DET uses time and time-frequency dependent windows to analyze the signal \( x(t) \). Eq. 4, which belongs to DET, looks like the STFT equation. In the optimal STFT, the optimal

![Figure 3: FrFT order estimation with respect to the IF value of the signal.](image-url)
window is determined according to time—width and band—width of \( x(t) \) on the appropriate fractional Fourier domain. Eq. 13 denotes the STFT calculation by using the optimal window for the signal. The optimum window, which contains the appropriate chirp component of the analyzed signal, is given in Eq. 15. This component is considered as a time—varying window. Under these circumstances, it can be said that the optimal STFT is a special case of the DET. Moreover, this technique improves the DET taking into account of signal—specific information such as \( T_r \), \( B_c \) and \( IF \).

Thanks to the RWT–FRFT relations, IF values are estimated robustly for each of the components when a multi—component signal is analyzed. As future work, we will present high resolution optimal STFT images for multi—component signals. It will introduce substantial improvement for DET, when multi—component signals are considered. Since, DET uses a mask window to separate each of the components of the analyzed signal by an offline procedure.

4. SIMULATIONS

Time—frequency domain localization by using TBP—optimal STFT and GTBP—optimal STFT of a synthetic LFM signal and a real bat echolocation signal are computed in simulations. Figure 3 shows a synthetic LFM signal. Its IF value is estimated as 0.5 by using representation IF estimation algorithm. Its time variation and its WD are given in Figure 3 (a) and (b). When the signal is mono-component, WD provides a sharp time—frequency representation. Figure 3 (c) and (d) represent the STFT and G—optimal STFT images. The performance of optimal STFT is very high and almost equivalent to WD’s.

The bat echolocation signal includes four non-linear chirp components. First, we obtained STFT of the signal by using a Gaussian window \( h(t) = e^{−πt^2} \). Then, time—width and band—width of the signal are calculated in the fractional domain which gives the minimum TBP. By using these values to construct the optimal window, TBP—STFT is obtained. It is shown in Figure 2.4(c). Last, \( g_{opt}(t) \), which is given in Eq. 14, is used to calculate the optimal STFT. It can be easily seen that, optimal STFT images have higher concentrations than ordinary—STFT’s.

5. CONCLUSIONS

In this paper, we link up between DET and the GTBP—STFT. The optimal STFT enriches the DET by taking into account of the three parameters related to signals. These are time—width and band—width terms in a fractional domain and the IF parameter. The signal—adaptive DET results are presented by an LFM signal and bat echolocation signals.

6. APPENDIX A

The \( \alpha \)-th order FrFT of \( x(t) \) is defined as

\[
x_\alpha(t) = F^\alpha \{ x(t) \} = \int B_\alpha(t,t')x(t')dt
\]

where \( 0 < |\alpha| < 2 \) and the transformation kernel \( B_\alpha(t,t') \) is

\[
B_\alpha(t,t') = A_\phi e^{-j\pi(1/2\cot(\phi)−2\alpha/\sin(\phi)+\alpha/2\cot(\phi))}
\]

with

\[
A_\phi = e^{-\pi\alpha\sin(\phi)/4+j(\phi)^2/2/|\sin(\phi)|^{1/2}}
\]

where the transform angle (\( \phi \)) and the FrFT order is related by \( \phi = \alpha \pi/2 \).
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Figure 5: Real Bat echolocation signal (a) in time, (b) its STFT (c) TBP–opt.STFT and (d) GTBP–opt.STFT.