An application of genetic algorithm in a marketing oriented inventory model with interval valued inventory costs and three-component demand rate dependent on displayed stock level

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Abstract

The objective of this research is to investigate an inventory model for a single item with imprecise inventory costs, by considering the impact of marketing strategies such as pricing and advertising on three component demand rate. This rate is dependent on selling price, frequency of advertisement and displayed stock level (DSL) in a show room/shop. Here, the impreciseness of inventory costs like carrying cost, purchase cost, ordering cost and advertisement cost has been represented by interval valued numbers. Analyzing the relative size of the storage capacity of the show room/shop and the stock level dependency parameters of demand, different scenarios with sub scenarios of each have been mentioned. Then, for each sub scenario, the model has been formulated as a constrained optimization problem with interval objective. To solve these problems, an advanced genetic algorithm (GA) for mixed integer non-linear programming has been developed with interval valued fitness function. In this developed GA, the order relations of interval valued numbers have been used with respect to pessimistic decision maker’s point of view. This approach has been used in the ranked based selection process for selecting better chromosomes/individuals for the next generation and also for finding the best chromosomes/individuals in each generation. Finally, the model has been illustrated with some numerical examples and the performance of the GA has been tested by computing appropriate statistical measures and the computing time.

Keywords: Inventory; Genetic algorithm; Interval numbers; Order relations

1. Introduction

In the present business scenario, a product becomes widely popular in the society, when it is aggressively promoted through the mass media and/or by the attractive display of items in the show-room at the market...
place. Normally customers are motivated to buy more units of an item due to glamorous display of that item in large numbers with the help of modern light and electronic arrangements. Observing this phenomenon, many marketing researchers/practitioners have been attracted to investigate its modeling aspects. Over the last few years, the concept of the existence of a functional relationship between the demand rate of a product and its displayed stock level (DSL) has consistently gained importance and popularity, as reflected in the various models developed in this area by marketing researchers and practitioners. Considering only the displayed stock level dependent demand, Paul et al. [12], Giri et al. [5], Sarkar et al. [14], Pal et al. [11], etc. developed different types of inventory models. From the practical point of view, demand rate is dependent on DSL only within a range and beyond this range it is constant. Again periodic advertisements (in popular media like newspaper, magazine, TV, etc., and through sales representatives) of an item and variations in its selling price also affect its demand.

Only a few OR researchers have studied the effects of price variations and/or the advertisement on the demand of an item. Among them, one may refer to the recent works of Abad [1], Bhunia and Maiti [2], Goyal and Gunasekaran [7], Pal et al. [11] and others. In all the aforesaid works, to the best of our knowledge, the inventory costs like the carrying cost, ordering cost, purchase cost and advertising cost have been assumed to be known at a fixed level. However, in real life situations, these costs should be imprecise numbers instead of fixed real numbers because inventory costs might fluctuate due to different reasons. For example, inventory carrying cost may be different in rainy season compared to summer or winter seasons (costs of taking proper action to prevent deteriorations of items in different seasons and also the labour charges in different seasons are different). Ordering cost, being dependent on the transportation facilities may also vary from season to season. Changes in the price of fuels, mailing charges, and telephonic charges may also make the ordering cost fluctuating. Unit purchase cost is highly dependent on the costs of raw materials and labour charges, which may fluctuate over time. Advertisement cost is also prone to fluctuation. To solve the problem with such imprecise numbers, stochastic, fuzzy and fuzzy-stochastic approaches may be used. In stochastic approach, the parameters are assumed to be random variables with known probability distribution. In fuzzy approach, the parameters, constraints and goals are considered as fuzzy sets with known membership functions. On the other hand, in fuzzy-stochastic approach, some parameters are viewed as fuzzy sets and others, as random variables. However, it is not always easy for a decision maker to specify the appropriate membership function or probability distribution. For these reasons, we have represented the imprecise numbers by interval numbers. These types of imprecise problems can be solved by a powerful computerized heuristic search and optimization method, viz., genetic algorithm (GA), which is based on the mechanics of natural selection (dependent on the evolution principle “Survival of the fittest”) and natural genetics. It is executed interactively on the set of real/binary coded solution called population. In each iteration (which is called generation), three basic genetic operations i.e., selection/reproduction, crossover and mutation are performed. Prof. J.H. Holland, University of Michigan, envisaged the fundamental concept of this algorithm in the mid sixties and published his seminal work. Thereafter, a number of researchers have contributed to the development of this field. Most of the initial research works have been published in several conference proceedings. However, at present, there exist several text books on GAs by Goldberg [6], Michalawicz [10], Sakawa [13], Gen and Cheng [4] and others.

Till now, only a few researchers have modeled different types of real life inventory problems and solved them with the help of GA. In this connection, the works of Mondal and Maiti [16], Razaei and Davoodi [17,18], Khourja et al. [19], Mahapatra et al. [20], Pal et al. [21], Sarkar and Newton [22] are worth mentioning.

In this paper, our objective function would be interval valued, as some of the parameters are interval valued. To solve this type of problem by GA method, order relations of interval numbers are essential for selection/reproduction operation as well as for finding the best chromosome in each generation. To the best of our knowledge, very few researchers have defined the order relations of interval valued numbers. Among them, one may refer to the works of Ishibuchi and Tanaka [8], Chanes and Kuchta [3]. However, their definitions are not complete. In the year 2000, Sengupta and Pal [15] proposed two different approaches (deterministic and fuzzy) to compare any two interval numbers with respect to the decision makers point of view. However, in some cases, their approaches fail to find out the order relations between two interval numbers. Very recently, Majumdar and Bhunia [9] proposed a new concept for order relations with respect to optimistic and pessimistic decision maker’s point of view in solving assignment problem with interval objective.
In this paper, we have developed an inventory model considering the impact of marketing strategies such as pricing and advertising on three component demand rate. This rate is dependent on selling price, frequency of advertisement and displayed stock level (DSL) in a show room/shop. Moreover, the inventory costs like carrying cost, ordering cost, purchase cost and advertisement cost have been considered to be interval valued. Shortages, if any are allowed and backlogged partially and the storage capacity of the showroom/shop is assumed to be limited (finite). Analyzing the relative size of the storage capacity of the show room/shop and stock level dependency parameters of demand, different scenarios with sub scenarios of each have been mentioned. For each sub scenario, we have formulated mixed integer constrained optimization problem with interval objective with the help of interval arithmetic. To solve these problems, we have developed an advanced genetic algorithm with ranking selection, whole arithmetical crossover and mutation for mixed integer non-linear programming considering the order relations of interval numbers with respect to pessimistic decision maker’s point of view. In the case of mutation, to maintain the diversity, we have considered uniform and non-uniform mutation for integer and non-integer variables respectively. In the whole arithmetic crossover, we have taken the convex combinations of genes corresponding to non-integer variables and intermediate value of gene corresponding to the variable of two randomly selected chromosomes. Finally, the model has been illustrated with some numerical examples and the performance of the developed GA has been tested by computing appropriate statistical measures and the computing time.

2. Finite interval arithmetic

An interval valued number is a closed interval defined by

\[ A = [a_L, a_R] = \{ x : a_L \leq x \leq a_R, \ x \in R \}, \]

where \( a_L \) and \( a_R \) are the left and right limits respectively and \( R \), the set of all real numbers. It is also defined by its centre and radius as

\[ A = (a_C, a_W) = \{ x : a_C - a_W \leq x \leq a_C + a_W, \ x \in R \}, \]

where the centre and radius are denoted by \((a_L + a_R)/2\) and \( a_W = (a_R - a_L)/2 \).

Particularly, each real number can be regarded as an interval, such as, for all \( x \in R \), \( x \) can be written as an interval \([x, x]\), which has zero width.

Now, we shall give the concise definitions of first four arithmetical operations of intervals.

**Definition 1.** Let \(* \in (+, -, \cdot, /)\) be a binary operation on the set of real numbers. If \( A \) and \( B \) are two closed intervals, then

\[ A * B = \{ a * b : a \in A \text{ and } b \in B \} \]

defines a binary operation on the set of closed intervals. In the case of division, it is assumed that \( 0 \notin B \).

For two interval numbers \( A = [a_L, a_R] \) and \( B = [b_L, b_R] \), the operations on interval used in this paper may be explicitly calculated from **Definition 1** as

\[ A + B = [a_L + b_L, a_R + b_R], \]
\[ A - B = [a_L - b_R, a_R - b_L], \]
\[ \lambda A = \begin{cases} \lfloor \lambda a_L, \lambda a_R \rfloor & \text{if } \lambda \geq 0, \\ \lceil \lambda a_R, \lambda a_L \rfloor & \text{if } \lambda < 0, \end{cases} \]

where \( \lambda \) is a real number.

Now, we shall discuss the order relations of interval numbers for maximization problems with respect to pessimistic decision makers’ point of view. Let the imprecise numbers from two alternative be represented by two closed intervals \( A = [a_L, a_R] \) and \( B = [b_L, b_R] \), respectively. It is also assumed that the value of each alternative lies in the corresponding interval. These two intervals \( A \) and \( B \) may be of the following three types:
Type-I: Both the intervals are disjoint.
Type-II: Intervals are partially overlapping.
Type-III: One interval is contained in the other.

According to Majumdar and Bhunia [9], the pessimistic decision making of the order relations of interval numbers for minimization problems is as follows:

**Definition 2.** For minimization problem, let us define the order relation $\leq_p$ between $A = [a_L, a_R] = (a_C, a_W)$ and $B = [b_L, b_R] = (b_C, b_W)$ as

\[
A \leq_p B \iff a_C \leq b_C,
\]

\[
A <_p B \iff A \leq_p B \land A \neq B.
\]

**Definition 3.** Using **Definition 2**, we can easily define the order relation $\geq_p$ between two intervals $A$ and $B$ for maximization problem as

\[
A \geq_p B \iff a_C \geq b_C,
\]

\[
A >_p B \iff A \geq_p B \land A \neq B.
\]

3. **Assumptions and notations**

The following assumptions and notations are used in developing the proposed model.

3.1. **Assumptions**

(i) The demand rate is dependent on the frequency of advertisement, selling price and the instantaneous displayed stock level.

(ii) The size of replenishment is finite, but the replenishment rate is infinite.

(iii) The inventory-planning horizon is infinite and the inventory system involves only one item and one stocking point.

(iv) A single order will be placed at the beginning of each cycle and the entire lot is delivered in one batch.

(v) The item is neither repaired nor replaced.

(vi) Lead-time is assumed to be negligible.

(vii) There is no quantity discount.

(viii) Shortages, if any, are allowed and partially backlogged.

(ix) The storage capacity of the showroom/shop is limited

(x) The whole stocks are displayed in the showroom/shop.

(xi) The inventory costs (like, holding/carrying cost, purchase cost, ordering cost, advertisement cost, etc.) lie in the known intervals.

**Notations**

$T$ cycle length

$A$ frequency of advertisement

$W$ storage capacity of the showroom/shop

$S(\leq W)$ initial on-hand inventory level at the beginning of the cycle after fulfilling the backlogged quantity, if any

$Q$ order quantity

$q(t)$ instantaneous displayed stock level at time $t$

$R_p$ Stock level at the reorder point

$\delta (0 \leq \delta < 1)$ backlogging parameter

$C_1 \in [C_{1L}, C_{1R}]$ Carrying cost per unit item per unit time

$C_2$ shortage cost per unit item per unit time
$C_3 \in [C_{3L}, C_{3R}]$ unit purchase cost  
$C_4 \in [C_{4L}, C_{4R}]$ ordering cost per order  
$G \in \{G_L, G_R\}$ advertisement cost per advertisement  
$p (> C_{3R})$ unit selling price  
$D(A,p,q)$ demand rate  
$S_0, S_1$ displayed stock level dependency parameters  
$a, b, c, m$ demand parameters  
$C_{\text{HOL}}, C_{\text{SHO}}$ total carrying cost and shortage cost respectively over the entire cycle  
$Z \in [Z_L, Z_R]$ net profit of Scenario 2.3  
$\Pi$ profit per unit time or profit function for Scenario 2.3  
$p\_\text{size}$ population size  
$p\_\text{cros}$ crossover rate or probability of crossover  
$p\_\text{mute}$ mutation rate or probability of mutation  
$m\_\text{gen}$ maximum number of generation

4. The demand rate function

Under realistic situations, to some extent the demand rate is dependent on the displayed stock, but this stock dependency nature of demand exists up to a certain level of displayed stock. Again, the demand rate is not fully dependent on the displayed stock level, but it also depends on some marketing parameters (e.g., selling price, frequency of advertisements). Thus the demand rate, $D(A,p,q)$ is deterministic and is a known function of marketing parameters (such as selling price $p$ per unit and the number $A$ of time the product/item is advertised) and the current stock level in the showroom/shop within the range $S_0$ to $S_1$. Outside this range, the demand rate becomes constant with respect to the displayed stock level. During stock-out period, the demand rate differs from the stock-in period by a given fraction $\delta$. Thus the demand rate takes the following form:

$$
D(A,p,q) = \begin{cases} 
 f(A,p,S_1) & \text{for } q > S_1 \\
 f(A,p,q) & \text{for } S_0 < q \leq S_1 \\
 f(A,p,S_0) & \text{for } 0 \leq q \leq S_0 \\
 \delta f(A,p,S_0) & \text{for } q < 0,
\end{cases}
$$

where $0 < \delta \leq 1$ and $f(A,p,q)$ is a function of $A$, $p$ and $q$.

The stock level $q$ and the selling price $p$ in the demand function $f(A,p,q)$ may appear in different forms like power form, exponential form, linear form, quadratic form etc. Here we have considered the linear form of $p$ and $q$ in $f(A,p,q)$. Hence $f(A,p,q)$ is proportional to $(a - bp + cq)$ for fixed $A$. Again, the demand of an item is likely to increase with the increase of the frequency of advertisement, but is not directly proportional to the frequency of advertisement. So, we have taken $f(A,p,q)$ proportional to $A^m$ for fixed $p$ and $q$. Thus, in this paper, we have considered the function $f(A,p,q)$ in the following form:

$$
f(A,p,q) = A^m(a - bp + cq) \quad \text{where } m > 0, \ a, b, c > 0.
$$

5. Mathematical model description

Initially, an enterprise purchases an amount of $S(<W)$ units of the item. This amount will be depleted to meet up the customer's demand. This demand is dependent on the relative size of $S_0$, $S_1$ and $W$. To analyze this situation, the following three scenarios may arise:

Scenario 1: $S_0 \leq W \leq S_1$.  
Scenario 2: $W > S_1$.  
Scenario 3: $W \leq S_0$.

In the above inequalities, the equal sign indicates the smooth transition from one scenario to another.
In the third scenario, the demand rate is always constant. It never becomes DSL dependent. This contradicts the proposed demand pattern. So, we shall reject the scenario \( W \leq S_0 \). For this purpose, we can assume that the capacity \( W \) of the show room/shop is greater than or equal to \( S_0 \).

Again analyzing Scenario 1 and 2, there may arise some sub scenarios for each scenario as follows:

Scenario 1.1: \( S_0 < R_p < S \leq W < S_1 \).
Scenario 1.2: \( 0 \leq R_p \leq S_0 < S < W < S_1 \).
Scenario 1.3: \( S_0 \leq S < W < S_1 \) and \( R_p < 0 \).
Scenario 2.1: \( S > S_1 \) and \( R_p > 0 \).
Scenario 2.2: \( S > S_1 \) and \( 0 < R_p < S_0 \).
Scenario 2.3: \( S > S_1 \) and \( R_p < 0 \).

In Scenario 1.1, the demand rate never becomes constant; the inventory is replenished before reaching \( S_0 \). In the situation of Scenario 1.2, the DSL-dependent demand rate is observed where the stock-level drops from \( S_1 \) to \( S_0 \). Beyond the stock level \( S_0 \), there is a constant demand rate up to the level \( R_p \) at reorder point. In Scenario 1.3, the same demand rate is observed in the stock-in position whereas in stock-out position, the decreased demand rate is realized. In Scenario 2.1, the demand rate is constant between the stock-level \( S \) and \( S_1 \). Then it becomes DSL-dependent up to the reorder point. In Scenario 2.2, initially the demand rate is constant, then it becomes DSL-dependent and next it is constant. In the Scenario 2.3, all possible forms of demand rate are considered and the stock-out position is reported.

Now we shall study Scenario 2.3 in details. In this scenario, the order quantity is

\[
Q = S + |R_p|
\]

and the total time period (cycle length) \( T \) is given by

\[
T = (S - S_1)/f(A, p, S_1) + \int_{S_0}^{S_1} \frac{dq}{f(A, p, q)} + \frac{S_0}{f(A, p, S_0)} - \frac{R_p}{\delta f(A, p, S_0)}
\]

\[
= A^{-m} \left[ \frac{S - S_1}{(a - bp + cS_1)} + \log \left( \frac{a - bp + cS_1}{a - bp + cS_0} \right) + \frac{S_0}{(a - bp + cS_0)} - \frac{R_p}{\delta(a - bp + cS_0)} \right].
\]

Now the total inventory carrying cost is given by

\[
C_{\text{HOL}} = C_1 \left[ \int_{S_1}^{S} \frac{q dq}{f(A, p, S_1)} + \int_{S_0}^{S_1} \frac{q dq}{f(A, p, q)} + \int_{0}^{S_0} \frac{q dq}{f(A, p, S_0)} \right].
\]

and the total shortage cost \( C_{\text{SHO}} \) is given by

\[
C_{\text{SHO}} = C_2 \int_{R_p}^{0} \frac{(q dq)}{f(A, p, S_0)}.
\]

During the cycle, the total cost (TC) of this sub scenario is given by

\[
\text{TC} = \langle \text{ordering cost} \rangle + \langle \text{purchase cost} \rangle + \langle \text{inventory carrying cost} \rangle + \langle \text{inventory shortage cost} \rangle + \langle \text{advertisement cost} \rangle.
\]

i.e.,

\[
\text{TC} = C_4 + C_3(S + |R_p|) + C_{\text{HOL}} + C_{\text{SHO}} + C_{\text{ADU}}.
\]

So, the net profit for this sub scenario, i.e., the difference between the sales revenue and TC is given by

\[
Z = (p - C_3)(S + |R_p|) - C_4 - C_1 A^{-m} \left[ \frac{S^2 - S_1^2}{2(a - bp + cS_1)} + \frac{S_1 - S_0}{c} + \frac{(a - bp)}{c^2} \right]
\]

\[
\times \log \left( \frac{a - bp + cS_0}{a - bp + cS_1} + \frac{S_0}{2(a - bp - cS_0)} \right) - \frac{C_2 A^{-m}(R_p)^2}{2\delta(a - bp + cS_0) - AG}.
\]
Hence, for the fixed value of selling price \( p \), the profit function \( \pi(S, R_p, A) \) (i.e., the profit per unit time for the cycle) of Scenario 2.3 is given by

\[
\pi(S, R_p, A) = Z/T
\]

where \( S, R_p \) are continuous variables and \( A \), discrete variable. For the interval valued

\[
C_1, C_3, C_4 \quad \pi \in \left[ \frac{Z_L}{T}, \frac{Z_R}{T} \right],
\]

where

\[
Z_L = (p - C_{3R})(S + |R_p|) - C_{4R} - C_{1RA}^{-m} \left[ \frac{S^2 - S_1^2}{2(a - bp + cS_1)} + \frac{S_1 - S_0}{c} + \frac{(a - bp)}{c^2} \log \left( \frac{a - bp + cS_0}{a - bp + cS_1} \right) + \frac{S_0^2}{2(a - bp + cS_0)} \right] - \frac{C_{2A}^{-m}(R_p)^2}{2\delta(a - bp + cS_0)} - AG_R
\]

and

\[
Z_R = (p - C_{3L})(S + |R_p|) - C_{4L} - C_{1LA}^{-m} \left[ \frac{S^2 - S_1^2}{2(a - bp + cS_1)} + \frac{S_1 - S_0}{c} + \frac{(a - bp)}{c^2} \log \left( \frac{a - bp + cS_0}{a - bp + cS_1} \right) + \frac{S_0^2}{2(a - bp + cS_0)} \right] - \frac{C_{2A}^{-m}(R_p)^2}{2\delta(a - bp + cS_0)} - AG_L.
\]

It is to be noted that the expressions of \( Z_L \) and \( Z_R \) in (8) and (9) are obtained by using interval arithmetic. Hence, our problem is

Maximize \( \pi(S, R_p, A) \)

subject to \( S_1 < S \leq W, R_p < 0 \) and \( A \) is an integer.

This is a non-linear maximization problem with interval objective.

6. Implementation of GA

Now, we have to solve the mixed integer maximization problem (10) with interval objective. Generally, deterministic inventory problems are solved by different gradient methods. However, being mixed integer maximization problem, it is rather difficult to solve the problem (10) by gradient method. Hence we shall develop an advanced genetic algorithm for solving the mixed integer maximization problem (10) with interval objective.

The different steps of this algorithm have been described as follows:

6.1. Algorithm-1

Step-1: Initialize the parameters of Genetic Algorithm, bounds of variables and different parameters of the proposed inventory system.

Step-2: \( t = 0 \) [\( t \) represents the number of current generation].

Step-3: Initialize \( P(t) \) [\( P(t) \) represents the population at \( t \)-th generation].

Step-4: Evaluate \( P(t) \).

Step-5: Find the best result from \( P(t) \).

Step-6: \( t = t + 1 \).

Step-7: If \( t > \text{maximum generation number} \) go to Step-14.

Step-8: Select \( P(t) \) from \( P(t - 1) \) by ranking selection process(with the help of Definition 3, discussed under Section 2).
Step-9: Alter $P(t)$ by crossover and mutation process.
Step-10: Evaluate $P(t)$.
Step-11: Find the best result from $P(t)$.
Step-12: Compare the best results of $P(t)$ and $P(t - 1)$ and accept the better one.
Step-13: Go to Step-6.
Step-14: Print the result.
Step-15: Stop.

For implementing the above GA in solving the maximization problem developed in this model, the following basic components are considered.

- Parameters of GA.
- Chromosome representation and initialization of population.
- Evaluation function.
- Selection process.
- Genetic operators (crossover and mutation).

### 6.2. Parameters of genetic algorithm

Genetic Algorithm (GA) is dependent on some parameters like population size ($p\_size$), maximum number of generations ($m\_gen$), probability of crossover ($p\_cros$) and probability of mutation ($p\_mute$). Though there is no hard and fast rule for selecting the population size of GA, storing of the data for large population size may invite some difficulties at the time of computation with the help of computer. However, if the population size is too small, then good crossovers cannot be implemented. Again, according to the genetics, it is obvious that probability of crossover is always greater than that of mutation.

### 6.3. Chromosome representation and initialisation

For the proper application of GA, the designing of an appropriate chromosome representation of solutions of the problem is an important task. In many situations, in solving the optimization problem with larger decision variables, the classical binary coding is not well adopted. In this case, a chromosome is coded in the form of matrix of real numbers and every component of that chromosome represents a decision variable of the problem. As our proposed problem is non-linear containing one discrete and two continuous variables, a real number representation is used here. A real row matrix $V_j = [V_{j1}, V_{j2}, V_{j3}]$ is used as a chromosome where $V_{j1}, V_{j2}$ and $V_{j3}$ represent $S$, $R$ and $A$ respectively, to represent the solution of the problem.

After the selection of chromosome representation, the next step is to initialize the chromosomes that will take part in the artificial genetic operations like natural genetics. This procedure produces population size number of chromosomes in which every component for each chromosome is randomly generated within the bounds of the corresponding decision variable. There are different processes for selecting a random number, of which the uniform distribution is used for first two components of each chromosome. For third component, a random value can be selected from the discrete set of values within the bounds.

### 6.4. Evaluation of fitness function

After getting a population of potential solutions, we need to check how good they are. For this purpose, we have to calculate the fitness value for each chromosome. In this work, the value of objective function (interval valued) corresponding to the chromosome $V_j$ is taken as the fitness value of $V_j$.

### 6.5. Selection

The selection operator plays a crucial role in GA. Usually, it is the first operator applied to the population. The primary objective of this operator is to emphasize on the above average solutions and eliminate below
average solutions from the population for the next generation under the well-known evolutionary principle ‘Survival of the fittest’. Here, the fitness value of each chromosome/individual is an interval valued number. So, the rank based selection procedure for finding the rank of chromosomes is an important task. This task has been performed from the point of view of pessimistic decision making. In this decision making, we have used ranking selection following the definition (Definition 3) for comparing interval numbers. The probability of the \( r \)th chromosome being selected in this method is defined by

\[
\text{prob}(r\text{th individual}) = p_c (1 - p_c)^{r-1},
\]

where \( p_c \) is the probability of selecting the best individual and \( r \) is the rank of the individual.

6.6. Crossover

After the selection process, the resulting chromosomes (those which have survived) undergo genetic operations – crossover and mutation. Crossover is an operation that really empowers the GA. It operates on two parent solutions at a time and generates offspring by recombining the features of both the parent solutions. For this operation, expected \( p\text{ _cros} \cdot p\text{ _size} \) (\( \ast \) denotes the product) number of chromosomes will take part. Hence in order to determine the parents for crossover operation, select \( p\text{ _cros} \cdot p\text{ _size} \) number of chromosomes. After selection of chromosomes, the crossover operation is applied. Here crossover operation is done in the following manner:

**Step-1:** Find the integral value of \( p\text{ _cros} \cdot p\text{ _size} \) and store it in \( N \).

**Step-2:** Select the chromosomes \( V_k \) and \( V_i \) randomly among the population for crossover.

**Step-3:** Generate a random real number \( \lambda \) in \([0,1]\).

**Step-4:** The first two components \( V'_{kj} \) and \( V'_{ij} (j = 1, 2) \) of two offspring will be created by

\[
V'_{kj} = \lambda V_{kj} + (1 - \lambda) V_{ij},
\]

\[
V'_{ij} = \lambda V_{ij} + (1 - \lambda) V_{kj}.
\]

**Step-5:** The third component of \( V'_{k3} \) and \( V'_{i3} \) will be produced by either \( V'_{k3} = V_{k3} - g \) and \( V'_{i3} = V_{i3} + g \) if \( V_{k3} > V_{i3} \) or \( V'_{k3} = V_{k3} + g \) and \( V'_{i3} = V_{i3} - g \), where \( g \) is a random integer number between 0 and \( |V_{k3} - V_{i3}| \).

**Step-6:** Repeat Step-2 to Step-5 for \( N/2 \) times.

6.7. Mutation

Mutation introduces random variations into the population. It is applied to a single chromosome only. It is usually performed with low probability; otherwise it would defeat the order building being generated through selection and crossover. Mutation attempts to bump the population gently into a slightly better course. This means that mutation changes single or all the genes of a randomly selected chromosome slightly. Here, we use non-uniform mutation whose action is dependent on the age of the population. If the element (gene) \( V_{ik} \) of chromosome \( V_i \) is selected for mutation and domain of \( V_{ik} \) is \([l_{ik}, u_{ik}]\), then the reduced value of \( V_{ik} \) is given by

\[
V'_{ik} = \begin{cases} 
V_{ik} + \Delta(t, u_{ik} - V_{ik}), & \text{if a random digit is 0,} \\
V_{ik} - \Delta(t, V_{ik} - l_{ik}), & \text{if a random digit is 1,}
\end{cases}
\]

where \( k \in \{1, 2, 3\} \) and \( \Delta(t,y) \) returns a value in the range \([0,y]\).

In our study, we have taken

\[
\Delta(t,y) = yr \left(1 - \frac{t}{m \cdot \text{gen}}\right)^b,
\]

for non-integer variables = a random integer between\([0,y]\) for an integer variable.

where \( r \) is a random number in \([0,1]\), \( t \) represents the current generation and \( b \) (which is called the non-uniform mutation parameter) is constant.
7. Solution procedure

Comparing the values of \( S_0, S_1 \) and \( W \), it can be easily observed whether the given system falls under Scenario 1 \( (S_0 \leq W \leq S_1) \) or Scenario 2 \( (W \geq S_1) \). If it falls under Scenario 1, then the optimal solution of the given system is \( S_{\text{opt}} = S_1, R_{p(\text{opt})} = R^*_{p_1}, A_{\text{opt}} = A^*_1, T_{\text{opt}} = T'_1 \), and \( (NP)_{\text{opt}} = (NP_1)_{\text{opt}} \). Otherwise, the optimal solution will be \( S_{\text{opt}} = S_2, R_{p(\text{opt})} = R^*_{p_2}, A_{\text{opt}} = A^*_2, T_{\text{opt}} = T'_2 \), and \( (NP)_{\text{opt}} = (NP_2)_{\text{opt}} \), where \( (NP)_{\text{opt}} \) denotes the profit per unit time in the inventory system.

We suggest the following algorithm for finding the optimal or nearer to optimal solution of the proposed inventory system.

7.1. Algorithm-2

Step 1: Read all the system parameters.

Step 2: If \( S_0 \leq W \leq S_1 \) (Scenario 1) then go to Step 3, else go to Step 7.

Step 3: Ignoring the constraint \( S \leq W \), solve \( i \) th sub scenario of Scenario 1 (for \( i = 1, 2, 3 \)) by genetic algorithm, that is, find the values \( S^*, R^*_{p}, A^* \) and \( \pi^*_{i} \). (* Denotes the best-found value).

Step 4: If \( S^* \geq W \), find the boundary profit with \( S = W \) by optimizing \( \pi^*_1 \cdot (W, R_{p}, A) \) for \( R_{p} \) (best found value of \( R_{p} \)), and \( A^* \) (best found value of \( A \)). Set \( S^* = W, R^*_{p} = R^*_{p}, A^* = A^*, \pi^*_1 = \pi^*_1(S^*, R^*_{p}, A^*) \).

Step 5: Find the best found solution of Scenario 1, that is, find \( (NP_1)_{\text{opt}} = \max \pi^*_1 \) and the corresponding values of \( S, R_{p}, A \). These are denoted by \( S_1, R^*_{p}, A^*_1 \).

Step 6: Set \( S_{\text{opt}} = S_1, R_{p(\text{opt})} = R^*_{p}, A_{\text{opt}} = A^*_1 \) and \( (NP)_{\text{opt}} = (NP_1)_{\text{opt}} \). Go to Step 11.

Step 7: Solve \( i \) th sub scenario of Scenario 2 (for \( i = 1, 2, 3 \)), that is, find the values of \( S^*, R^*_{p}, A^* \), and \( \pi^*_{i} \).

Step 8: If \( S^* \geq W \), solve \( \pi^*_2 \). (\( S, R_{p}, T, A \) for \( R_{p} \) (best found value of \( R_{p} \)) and \( A^* \) (best found value of \( A \)) for finding the boundary profit. Set \( S^* = W, R^*_{p} = R^*_{p}, A^* = A^*, \pi^*_2 = \pi^*_2(S^*, R^*_{p}, T^*, A^*) \).

Step 9: Find the optimal solution of Scenario 2, that is, find \( (NP_2)_{\text{opt}} = \max \pi^*_2 \) and the corresponding values of \( S, R_{p}, A \). These are denoted by \( S_2, R^*_{p}, A^*_2 \).

Step 10: Set \( S_{\text{opt}} = S_2, R_{p(\text{opt})} = R^*_{p}, A_{\text{opt}} = A^*_2 \) and \( (NP)_{\text{opt}} = (NP_2)_{\text{opt}} \).

Step 11: Find \( T_{\text{opt}} \) and \( Q_{\text{opt}} \) from the sub scenario which gives \( (NP)_{\text{opt}} \).

Step 12: Print \( S_{\text{opt}}, R_{p(\text{opt})}, T_{\text{opt}}, A_{\text{opt}} \), and \( (NP)_{\text{opt}} \).

Step 13: Stop.

8. Numerical examples

To illustrate the model discussed earlier, the following four numerical examples have been considered. The values of the model parameters considered in these numerical examples are not selected from any real life case.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Example 1</th>
<th>Example 2</th>
<th>Example 3</th>
<th>Example 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W )</td>
<td>1500</td>
<td>1500</td>
<td>1500</td>
<td>1500</td>
</tr>
</tbody>
</table>
| \( C_1 \)  | [1.2, 1.5]| [1.2, 1.5]| –         | [1.2, 1.5]|}
| \( C_2 \)  | 15        | 15        | 15        | 15        |
| \( C_3 \)  | [25, 28]  | –         | [25, 28]  | [25, 28]  |
| \( C_4 \)  | [85, 90]  | [85, 90]  | [85, 90]  | –         |
| \( p \)    | 40        | 40        | 40        | 40        |
| \( G \)    | [205, 210]| [205, 210]| [205, 210]| [205, 210]|}
| \( m \)    | 0.3       | 0.3       | 0.3       | 0.3       |
| \( a \)    | 200       | 200       | 200       | 200       |
| \( b \)    | 0.6       | 0.6       | 0.6       | 0.6       |
| \( c \)    | 0.2       | 0.2       | 0.2       | 0.2       |
| \( \delta \)| 0.8      | 0.8       | 0.8       | 0.8       |
| \( S_0 \)  | 50        | 50        | 50        | 50        |
| \( S_1 \)  | 150       | 150       | 150       | 150       |
study, but these values considered here are all feasible. The different values of the different parameters used in each of the four examples are displayed in Table 1. Example 1, for different values of shortage cost $C_2$, Example 2, for different values of purchase cost $C_3$ (interval valued), Example 3, for different values of carrying cost $C_1$ (interval valued), and Example 4, for different values of ordering cost $C_4$ (interval valued) have been solved to find out the best found values of $S_{\text{opt}}$, $R_{\text{opt}}$, $A_{\text{opt}}$, $T_{\text{opt}}$ along with the best found profit per unit time of the system. In each case, 20 independent runs have been performed by proposed GA of which the best interval value of the profit function has been taken according to the Definition 3 of interval order relations. Also, to test the performance of our developed GA for solving the problems for each example, mean, coefficient of variation of the center values of interval profits and the average time of 20 independent runs have been calculated. In this computation, the following values of GA parameters are used.

Population size ($p_{\text{size}}$) = 100, Probability of crossover ($p_{\text{cros}}$) = 0.9, Probability of mutation ($p_{\text{mute}}$) = 0.1, Maximum generation ($m_{\text{gen}}$) = 500.

All the results are displayed in Tables 2–5. From these tables it is observed that the coefficient of variations and the average computational times for different costs in different examples are very small.

Table 2
Results of numerical example 1, for different values of $C_2$

<table>
<thead>
<tr>
<th>$C_2$</th>
<th>$S_{\text{opt}}$</th>
<th>$R_{\text{opt}}$</th>
<th>$A_{\text{opt}}$</th>
<th>$T_{\text{opt}}$</th>
<th>Best found net profit, $\pi_{\text{opt}}$</th>
<th>Remarks (result from)</th>
<th>Mean</th>
<th>Coefficient of variation</th>
<th>Average time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>05</td>
<td>1257.64</td>
<td>−278.91</td>
<td>23</td>
<td>3.135</td>
<td>[2638.94,4449.69]</td>
<td>Scenario 2.3</td>
<td>3544.018</td>
<td>0.021895</td>
<td>0.2482</td>
</tr>
<tr>
<td>07</td>
<td>1167.87</td>
<td>−210.72</td>
<td>22</td>
<td>2.87</td>
<td>[2634.58,4412.82]</td>
<td>Scenario 2.3</td>
<td>3523.672</td>
<td>0.005575</td>
<td>0.2443</td>
</tr>
<tr>
<td>10</td>
<td>881.17</td>
<td>48.23</td>
<td>13</td>
<td>1.868</td>
<td>[2661.53,4389.87]</td>
<td>Scenario 2.2</td>
<td>3524.742</td>
<td>0.010186</td>
<td>0.2495</td>
</tr>
<tr>
<td>15</td>
<td>881.17</td>
<td>48.23</td>
<td>13</td>
<td>1.868</td>
<td>[2661.53,4389.87]</td>
<td>Scenario 2.2</td>
<td>3524.742</td>
<td>0.010186</td>
<td>0.2495</td>
</tr>
</tbody>
</table>

Table 3
Results of numerical example 2, for different values of $C_3$

<table>
<thead>
<tr>
<th>$C_3$</th>
<th>$S_{\text{opt}}$</th>
<th>$R_{\text{opt}}$</th>
<th>$A_{\text{opt}}$</th>
<th>$T_{\text{opt}}$</th>
<th>Best found net profit, $\pi_{\text{opt}}$</th>
<th>Remarks (result from)</th>
<th>Mean</th>
<th>Coefficient of variation</th>
<th>Average time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[20,22]</td>
<td>1399.24</td>
<td>−112.96</td>
<td>32</td>
<td>2.688</td>
<td>[5533.05,7114.60]</td>
<td>Scenario 2.3</td>
<td>6338.225</td>
<td>0.006563</td>
<td>0.2662</td>
</tr>
<tr>
<td>[28,30]</td>
<td>434.85</td>
<td>48.52</td>
<td>05</td>
<td>1.17</td>
<td>[1979.86,2897.31]</td>
<td>Scenario 2.2</td>
<td>2438.498</td>
<td>0.000409</td>
<td>0.2605</td>
</tr>
<tr>
<td>[30,35]</td>
<td>220.01</td>
<td>49.25</td>
<td>02</td>
<td>0.7</td>
<td>[433.915,2123.68]</td>
<td>Scenario 2.2</td>
<td>1277.764</td>
<td>0.0879553</td>
<td>0.2730</td>
</tr>
<tr>
<td>[32,34]</td>
<td>230.47</td>
<td>49.44</td>
<td>02</td>
<td>0.73</td>
<td>[755.10,1489.71]</td>
<td>Scenario 2.2</td>
<td>1122.389</td>
<td>0.0062698</td>
<td>0.2705</td>
</tr>
<tr>
<td>[34,35]</td>
<td>259.20</td>
<td>49.42</td>
<td>02</td>
<td>0.848</td>
<td>[448.32,867.45]</td>
<td>Scenario 2.2</td>
<td>657.852</td>
<td>0.0058902</td>
<td>0.2674</td>
</tr>
</tbody>
</table>

Table 4
Results of numerical example 3, for different values of $C_1$

<table>
<thead>
<tr>
<th>$C_1$</th>
<th>$S_{\text{opt}}$</th>
<th>$R_{\text{opt}}$</th>
<th>$A_{\text{opt}}$</th>
<th>$T_{\text{opt}}$</th>
<th>Best found net profit, $\pi_{\text{opt}}$</th>
<th>Remarks (result from)</th>
<th>Mean</th>
<th>Coefficient of variation</th>
<th>Average time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1,2]</td>
<td>691.05</td>
<td>44.05</td>
<td>10</td>
<td>1.588</td>
<td>[2306.36,4414.20]</td>
<td>Scenario 2.2</td>
<td>3360.104</td>
<td>0.004074</td>
<td>0.2653</td>
</tr>
<tr>
<td>[0.6,0.8]</td>
<td>1446.05</td>
<td>49.04</td>
<td>20</td>
<td>2.77</td>
<td>[3458.81,5376.62]</td>
<td>Scenario 2.2</td>
<td>4417.387</td>
<td>0.002826</td>
<td>0.2621</td>
</tr>
<tr>
<td>[2.9,3.1]</td>
<td>439.00</td>
<td>−89.97</td>
<td>08</td>
<td>1.49</td>
<td>[1826.48,2989.59]</td>
<td>Scenario 2.3</td>
<td>2407.995</td>
<td>0.002815</td>
<td>0.2605</td>
</tr>
<tr>
<td>[3.4,3.6]</td>
<td>400.04</td>
<td>−90.17</td>
<td>07</td>
<td>1.45</td>
<td>[1695.446,2800.07]</td>
<td>Scenario 2.3</td>
<td>2247.716</td>
<td>0.013556</td>
<td>0.2671</td>
</tr>
</tbody>
</table>

Table 5
Results of numerical example 4, for different values of $C_4$

<table>
<thead>
<tr>
<th>$C_4$</th>
<th>$S_{\text{opt}}$</th>
<th>$R_{\text{opt}}$</th>
<th>$A_{\text{opt}}$</th>
<th>$T_{\text{opt}}$</th>
<th>Best found net profit, $\pi_{\text{opt}}$</th>
<th>Remarks (result from)</th>
<th>Mean</th>
<th>Coefficient of variation</th>
<th>Average time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[40,140]</td>
<td>776.12</td>
<td>46.45</td>
<td>11</td>
<td>1.74</td>
<td>[2657.326,4344.05]</td>
<td>Scenario 2.2</td>
<td>3500.526</td>
<td>0.006527</td>
<td>0.2647</td>
</tr>
<tr>
<td>[40,50]</td>
<td>948.02</td>
<td>48.66</td>
<td>15</td>
<td>1.95</td>
<td>[2628.42,4444.635]</td>
<td>Scenario 2.2</td>
<td>3536.409</td>
<td>0.004498</td>
<td>0.2638</td>
</tr>
<tr>
<td>[150,200]</td>
<td>747.22</td>
<td>47.26</td>
<td>10</td>
<td>1.716</td>
<td>[2637.63,4251.32]</td>
<td>Scenario 2.2</td>
<td>3444.385</td>
<td>0.008372</td>
<td>0.2652</td>
</tr>
</tbody>
</table>
9. Conclusion

In this paper, we have developed an inventory model with the inventory costs (excluding shortage cost) as interval valued numbers. Here the modified ideas about the order relations have been used with respect to the point of view of pessimistic decision making for the selection of better chromosomes for the next generation as well as for finding the best chromosomes in each iteration. Here only one inventory cost i.e. shortage cost is not considered as interval valued, since this cost is under the control of manufacturer/retailer/wholesaler. The model has been developed considering more realistic demand rate (dependent mainly on displayed inventory level in the show-room/shop and partially on the frequency of advertisement and the selling price of the item) and different scenarios, depending on the level of displayed stock, storage capacity and the order quantity of the system.

The impact of proposed demand rate and the optimal profit is reported. The results indicate that the effects of proposed demand on the system behaviour are significant and hence should not be ignored in developing the model.

Another feature of our model is the incorporation of a new type of selling situation that can occur in cases where the customers are induced to purchase goods because of the aggressive marketing of the product. This effect continues within a certain range of displayed inventory in the show room. Beyond the upper level, only a limited number (may be considered as constant) of customers arrive to purchase the goods due to different factors – such as goodwill of the shop, good quantity, genuine price-level of the item, locality of the shop, discount facilities, prompt services, etc. In case of unavailability of goods (when shortages occur), only a fraction of customers switch over to a different shop for purchasing goods due to their urgency and other reasons.

For future research, this model can further be generalized to include the case of deteriorating items, finite replenishment of the stock as well as the interval valued demand.

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References