Certain characterizations of LA-semigroups by soft sets

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Abstract. In this paper, a new approach to LA-semigroup theory is proposed by obtaining significant characterizations of regular, intra-regular, completely regular, weakly regular and quasi-regular LA-semigroups via soft intersection left (right, two-sided) ideals, (generalized) bi-ideals, interior ideals, quasi-ideals of LA-semigroups.

Keywords: Soft set, Soft intersection left (right, two-sided, interior, quasi) ideal, Soft intersection (generalized)-bi ideal, Regular LA-semigroup

1. Introduction

Molodtsov [1] in 1999 introduced the fundamental concept of soft set which provides a naturel framework for generalizing several basic notions of algebra such as groups [2,3], semirings [4], rings [5], BCK/BCI-algebras [6,7,8], BL-algebras [9], near-rings [10,28] and soft substructures and union soft substructures [12,13], and some other structures such as [14,15,16,17].

Many related concepts with soft sets, especially soft set operations, have also undergone tremendous studies. Maji et al. [18] presented some definitions on soft sets and based on the analysis of several operations on soft sets Ali et al. [19] introduced several operations of soft sets and Sezgin and Atan [20] and Ali et al. [21] studied on soft set operations as well.

The theory of soft set and its applications has been growing rapidly nowadays. These applications are used in various fields such as computer science and soft decision making as in the following studies [22,23] and some other fields such as [24,25,26,27,28].

In this paper, we make a new approach to the LA-semigroup theory via soft set theory, with the concept of soft intersection LA-semigroups and soft intersection LA-ideals. The paper reads as follows: First, we remind some basic definitions about soft sets, LA-semigroups, soft intersection product and soft characteristic function, soft intersection LA-semigroup, soft intersection left (right, two-sided) ideal, bi-ideal, interior ideal, quasi-ideal, generalized bi-ideal and soft semiprime ideals in LA-semigroups. Moreover, we show that all of these ideals coincide in an intra-regular LA-semigroup with left identity. In the following sections, regular, intra-regular, completely regular, weakly regular and quasi-regular LA-semigroups are characterized by the properties of these soft intersection ideals.

2. Preliminaries

The idea of generalization of a commutative semigroup (which we call the left almost semigroup) was introduced by M.A. Kazim and M. Naseeruddin in 1972 [32]. They introduced braces on the left of the tenary commutative law \(abc = cba\) to get a new pseudo associative law, that is

\[(ab)c = (cb)a\] for all \(a, b, c\).

It is since then called the left invertive law. A groupoid satisfying the left invertive law is called a left almost semigroup and is abbreviated by LA-semigroup. P. Holgate call it simple invertive groupoid [29]. It is also known as Abel-Grassmann’s groupoid [30]. It is a mid structure between a groupoid and a commutative semigroup, having many applications in the theory of flocks [31]. In an LA-semigroup \(S\), medial law [32] holds:

\[(ab)(cd) = (ac)(bd)\] for all \(a, b, c, d \in S\).

There can be a unique left identity in an LA-semigroup [33]. In an LA-semigroup with left identity, paramedial law holds:
\[(ab)(cd) = (dc)(ba)\] for all \(a, b, c, d \in S\).

Moreover, in an LA-semigroup with left identity, the following law holds:

\[a(bc) = b(ac)\] for all \(a, b, c \in S\).

It is interesting to see that an LA-semigroup \(S\) with left identity becomes medial and commutative, however an LA-semigroup with a right identity becomes a commutative semigroup with identity [33].

We denote by \([a^2], R[a^2], J[a^2]\), the principal left ideal, right ideal, two-sided ideal of an LA-semigroup \(S\) generated by \(a^2 \in S\). Note that the principal left ideal, right ideal, two-sided ideals of an LA-semigroup \(S\) generated by \(a^2 \in S\) are equal, that is, \([a^2] = R[a^2] = J[a^2] = a^2 \Sigma = = sa^2 : s \in S\). For further information about LA-semigroups and fuzzy LA-semigroups, we invite the reader to [34,35,36,37].

A semilattice is a structure \(S = (S, \cdot)\), where “\(\cdot\)” is an infix binary operation, called the semilattice operation, such that “\(\cdot\)” is associative, commutative and idempotent. From now on, \(U\) refers to an initial universe, \(E\) is a set of parameters, \(P(U)\) is the power set of \(U\) and \(A, B, C \subseteq E\).

**Definition 2.1.** ([23,1]) A soft set \(A\) over \(U\) is a set defined by

\[f_A : E \rightarrow P(U)\] such that \(f_A(x) = \emptyset\) if \(x \notin A\).

Here \(f_A\) is also called an approximate function. A soft set over \(U\) can be represented by the set of ordered pairs

\[f_A = \{(x, f_A(x)) : x \in E, f_A(x) \in P(U)\} \subseteq E \times P(U)\]

It is clear to see that a soft set is a parametrized family of subsets of the set \(U\). Note that throughout this paper, the set of all soft sets over \(U\) will be denoted by \(S(U)\).

**Definition 2.2.** [23] Let \(f_A, f_B \in S(U)\). Then, \(f_A\) is called a soft subset of \(f_B\) and denoted by \(f_A \subseteq f_B\), if \(f_A(x) \subseteq f_B(x)\) for all \(x \in E\).

**Definition 2.3.** [23] Let \(f_A, f_B \in S(U)\). Then, union of \(f_A\) and \(f_B\), denoted by \(f_A \cup f_B\), is defined as \(f_A \cup f_B = f_A \cup f_B\), where \(f_A \cup f_B(x) = f_A(x) \cup f_B(x)\) for all \(x \in E\).

**Definition 2.4.** [23] Let \(f_A, f_B \in S(U)\). Then, intersection of \(f_A\) and \(f_B\), denoted by \(f_A \cap f_B\), is defined as \(f_A \cap f_B = f_A \cap f_B\), where \(f_A \cap f_B(x) = f_A(x) \cap f_B(x)\) for all \(x \in E\).

**Definition 2.5.** [23] Let \(f_A, f_B \in S(U)\). Then, \(\Lambda\)-product of \(f_A\) and \(f_B\), denoted by \(f_A \Lambda f_B\), is defined as \(f_A \Lambda f_B = f_A \Lambda f_B\), where \(f_A \Lambda f_B(x, y) = f_A(x) \Lambda f_B(y)\) for all \((x, y) \in E \times E\).

**Definition 2.6.** [38] Let \(S\) be an LA-semigroup and \(f_s\) and \(g_s\) be soft sets over the common universe \(U\). Then, soft intersection product \(f_s \cap g_s\) is defined by

\[(f_s \cap g_s)(x) = \begin{cases} \bigcup_{y \in y} \{f_s(y) \cap g_s(z)\} & \text{if } \exists y, z \in S \\ \emptyset & \text{otherwise} \end{cases} \]

for all \(x \in S\).

**Theorem 2.7.** [38] Let \(f_s, g_s, h_s \subseteq S(U)\). Then,

i) \((f_s \cap g_s) \cap h_s = f_s \cap (g_s \cap h_s)\).

ii) \((f_s \cup g_s) \cup h_s = (f_s \cup g_s) \cap (f_s \cup h_s)\).

iii) \((f_s \cap g_s) \cap h_s = (f_s \cap g_s) \cap (f_s \cap h_s)\).

iv) \((f_s \cap g_s) \cup h_s = (f_s \cap g_s) \cup (f_s \cap h_s)\).

v) \((f_s \cap g_s) \cap h_s = (f_s \cap g_s) \cap (f_s \cap h_s)\).

**Proposition 2.8.** [38] Let \(S\) be an LA-semigroup, then the medial law holds in \(S(S)\).

**Theorem 2.9.** [38] Let \(S\) be an LA-semigroup with left identity and \(f_s, g_s, h_s\) be any elements of \(S(S)\). Then, the following properties hold in \(S(S)\):

i) \((f_s \cap g_s) \cap h_s = g_s \cap (f_s \cap h_s)\).

ii) \((f_s \cap g_s) \cap (h_s \cap k_s) = (k_s \cap h_s) \cap (g_s \cap f_s)\).

**Definition 2.10.** [38] Let \(X\) be a subset of \(S\). We denote by \(S_X\) the soft characteristic function of \(X\) and define as

\[S_X(x) = \begin{cases} U & \text{if } x \in X \\ \emptyset & \text{if } x \notin X \end{cases} \]

**Theorem 2.11.** [38] Let \(X \subseteq Y\) be nonempty subsets of an LA-semigroup \(S\). Then, the following properties hold:

i) \(S_X \subseteq S_Y\).

ii) \(S_X \cup S_Y = S_X \cup S_Y\).

iii) \(S_X \cap S_Y = S_X \cap S_Y\).

**Definition 2.12.** [38] Let \(S\) be an LA-semigroup and \(f_s\) be a soft set over \(U\). Then, \(f_s\) is called a soft intersection LA-semigroup of \(S\), if

\[f_s(xy) \subseteq f_s(x) \cap f_s(y)\]

for all \(x, y \in S\).

**Definition 2.13.** [38] A soft set over \(U\) is called a soft intersection left (right) ideal of \(S\) over \(U\) if

\[f_s(ab) \supseteq f_s(b) \cap f_s(ab) \supseteq f_s(a)\]
for all \( a, b \in S \). A soft set over \( U \) is called a soft intersection two-sided ideal (soft intersection ideal) of \( S \) if it is both soft intersection left and soft intersection right ideal of \( S \) over \( U \).

**Definition 2.14.** [38] A soft intersection LA-semigroup \( f_S \) over \( U \) is called a soft intersection bi-ideal of \( S \) over \( U \) if

\[
f_S(xyz) \supseteq f_S(x) \cap f_S(z)
\]

for all \( x, y, z \in S \).

**Definition 2.15.** [38] A soft set over \( U \) is called a soft intersection interior ideal of \( S \) over \( U \) if

\[
f_S(xyz) \supseteq f_S(x) \cap f_S(z)
\]

for all \( x, y, z \in S \).

**Definition 2.16.** [38] A soft set over \( U \) is called a soft intersection generalized bi-ideal of \( S \) over \( U \) if

\[
f_S(xyz) \supseteq f_S(x) \cap f_S(z)
\]

for all \( x, y, z \in S \).

For the sake of brevity, soft intersection LA-semigroup, soft intersection right (left, two-sided, interior, quasi, generalized bi-) ideal are abbreviated by \( SI - LA \)-semigroup, \( SI \)-right (left, two-sided, interior, quasi, generalized bi-) ideal, respectively.

It is easy to see that if \( f_S(x) = U \) for all \( x \in S \), then \( f_S \) is an \( SI - LA \)-semigroup (right ideal, left ideal, ideal, bi-ideal, interior ideal, generalized bi-ideal) of \( S \) over \( U \). We denote such a kind of \( SI - LA \)-semigroup (right ideal, left ideal, ideal, bi-ideal, interior ideal, generalized bi-ideal) by \( \widetilde{S} [38] \).

**Definition 2.17.** [38] A soft set over \( U \) is called a soft intersection quasi-ideal of \( S \) over \( U \) if

\[
(f_S \circ \widetilde{S}) \cap (\widetilde{S} \circ f_S) \subseteq f_S.
\]

**Lemma 2.18.** [38] Let \( f_S \) be any \( SI - LA \)-semigroup over \( U \). Then, we have the followings:

i) \( \widetilde{S} \circ \widetilde{S} \subseteq \widetilde{S} \).

ii) \( f_S \circ \widetilde{S} \subseteq \widetilde{S} \) and \( \widetilde{S} \circ f_S \subseteq \widetilde{S} \).

iii) \( f_S \circ \widetilde{S} = \widetilde{S} \) and \( f_S \circ \widetilde{S} = f_S \).

**Theorem 2.19.** [38] Let \( X \) be a nonempty subset of an LA-semigroup \( S \). Then, \( X \) is sub LA-semigroup (left, right, two-sided ideal, bi-ideal, interior ideal, quasi-ideal, generalized bi-ideal) of \( S \) if and only if \( S_X \) is an \( SI - LA \)-semigroup (left, right, two-sided ideal, bi-ideal, interior ideal, quasi-ideal, generalized bi-ideal) of \( S \).

**Proposition 2.20.** [38] Let \( f_S \) be a soft set over \( U \). Then,

i) \( f_S \) is an \( SI - LA \)-semigroup over \( U \) if and only if \( f_S \circ f_S \subseteq f_S \).

ii) \( f_S \) is an \( SI \)-left (right) ideal of \( S \) over \( U \) if and only if \( \widetilde{S} \circ f_S \subseteq f_S \) and \( f_S \circ \widetilde{S} \subseteq f_S \).

iii) \( f_S \) is an \( SI \)-bi-ideal of \( S \) over \( U \) if and only if \( \widetilde{S} \circ f_S \subseteq f_S \) and \( f_S \circ \widetilde{S} \subseteq f_S \).

iv) \( f_S \) is an \( SI \)-interior ideal of \( S \) over \( U \) if and only if \( f_S \circ f_S \subseteq f_S \) and \( f_S \circ \widetilde{S} \subseteq f_S \).

v) \( f_S \) is an \( SI \)-generalized bi-ideal of \( S \) over \( U \) if and only if \( f_S \circ f_S \subseteq f_S \).

**Theorem 2.21.** [38] Every \( SI \)-left (right, two sided) ideal of a semigroup \( S \) over \( U \) is an \( SI \)-bi-ideal of \( S \) over \( U \).

**Proposition 2.22.** [38] For an LA-semigroup \( S \), the following conditions are equivalent:

1) Every \( SI \)-ideal of an LA-semigroup \( S \) over \( U \) is an \( SI \)-ideal of \( S \) over \( U \).

2) Every \( SI \)-quasi-ideal of \( S \) is an \( SI \)-quasi-ideal of \( S \).

3) Every one-sided \( SI \)-ideal of \( S \) is an \( SI \)-quasi-ideal of \( S \).

4) Every \( SI \)-quasi-ideal of \( S \) is an \( SI \)-bi-ideal of \( S \).

### 3. Regular LA-semigroups

In this section, we characterize a regular LA-semigroup in terms of \( SI \)-ideals.

An LA-semigroup \( S \) is called regular if for every element \( a \) of \( S \) there exists an element \( x \) in \( S \) such that

\[
a = (ax)a.
\]

**Example 3.1.** [37] Consider the set \( S = \{1, 2, 3, 4, 5, 6, 7\} \) defined by the following table:

<table>
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<th>( \cdot )</th>
<th>1</th>
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<th>4</th>
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<td>6</td>
<td>1</td>
<td>3</td>
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</tbody>
</table>

\( S \) is an LA-semigroup also \( (1 \cdot 4 \cdot 6) \neq 1 \cdot (4 \cdot 6) \), so \( S \) is non-associative and \( S \) is regular, since \( 1 = (1 \cdot 4) \cdot 1 = (2 \cdot 7) = 2 \cdot 3 = (3 \cdot 3) \cdot 3 = (4 \cdot 6) \cdot 4, 5 = (5 \cdot 2) \cdot 5, 6 = (6 \cdot 2) \cdot 6 \) and \( 7 = (7 \cdot 1) \cdot 7 \).

Note that in a regular LA-semigroup, \( S^2 = S \).
Theorem 3.2. Let $S$ be an LA-semigroup. If $S$ is regular, then $f_S \circ g_S = f_S \tilde{\circ} g_S$ for every $S$-right ideal $f_S$ of $S$ and $S$-left ideal $g_S$ of $S$ over $U$.

Proof. Let $S$ be a regular LA-semigroup and $f_S$ be an $S$-right ideal of $S$ and $g_S$ be an $S$-left ideal of $S$ over $U$. Then,

$$f_S \circ g_S \subseteq f_S \circ \tilde{g}_S \subseteq f_S \circ \tilde{g}_S,$$

thus, $f_S \circ g_S \subseteq f_S \tilde{\circ} g_S$ for every $S$-right ideal $f_S$ of $S$ and $S$-left ideal $g_S$ of $S$ over $U$. Therefore, it suffices to show that $f_S \tilde{\circ} g_S \subseteq f_S \circ g_S$. Let $s$ be any element of $S$. Then, since $S$ is regular, there exists an element $x$ in $S$ such that $s = (sx)$. Thus, we have

$$(f_S \circ g_S)(s) = \bigcup_{a=ab} (f_S(a) \cap g_S(b))$$

$$\supseteq f_S(sx) \cap g_S(s)$$

$$\supseteq f_S(s) \cap g_S(s)$$

$$= (f_S \tilde{g}_S)(s)$$

Thus, $f_S \circ g_S = f_S \tilde{\circ} g_S$.

Corollary 3.3. Let $S$ be an LA-semigroup. If $S$ is regular, then $f_S \circ g_S = f_S \tilde{\circ} g_S$ for every $S$-right ideal $f_S$ and $S$-left ideal $g_S$ of $S$ over $U$.

Corollary 3.4. Let $S$ be an LA-semigroup. If $S$ is regular, then $f_S \circ g_S = g_S \circ f_S$ for every $S$-right ideal $f_S$ and $S$-left ideal $g_S$ of $S$ over $U$.

Proposition 3.5. Every $S$-right ideal of a regular LA-semigroup is idempotent.

Proof. Let $h_S$ be an $S$-right ideal of $S$. Then,

$$h_S \circ h_S \subseteq h_S \circ \tilde{h}_S \subseteq h_S \circ h_S.$$

Now, we show that $h_S \circ h_S \subseteq h_S \circ h_S$. Since $S$ is regular, there exists an element $x \in S$ such that $a = (ax)a$ for all $a \in S$. So, we have;

$$(h_S \circ h_S)(a) = \bigcup_{a=(ax)a} (h_S(ax) \cap h_S(a))$$

$$\supseteq h_S(a) \cap h_S(a)$$

$$= h_S(a)$$

Hence, $h_S \subseteq h_S \circ h_S$ and so $(h_S)^2 = h_S \circ h_S = h_S$.

Corollary 3.6. Every $S$-ideal of a regular LA-semigroup is idempotent.

Corollary 3.7. The set of all $S$-ideals of a regular LA-semigroup $S$ forms a semilattice under the soft int-product.

Proposition 3.8. Let $S$ be a regular LA-semigroup. Then every $S$-right ideal of $S$ is an $S$-left ideal of $S$.

Proof. Let $f_S$ be an $S$-right ideal of $S$. Since $S$ is regular, for any $x \in S$, there exist $n \in S$ such that $x = (xn)x$. Thus,

$$f_S(xy) = f_S((xn)x)y = f_S((yx)(xn)) \supseteq f_S(yx) \supseteq f_S(y)$$

Thus, $f_S$ is an $S$-left ideal of $S$.

Proposition 3.9. Let the set of all $S$-ideals of $S$ be a regular LA-semigroup of $S$ under the soft int-product. Then, every $S$-ideal of $S$ has the form $f_S = (f_S \circ \tilde{S}) \circ f_S$.

Proof. Let $f_S$ be an $S$-ideal of $S$. Then, by assumption, there exists an $S$-ideal $g_S$ of $S$ such that

$$f_S = (f_S \circ g_S) \circ f_S.$$ 

Thus, we have

$$f_S = (f_S \circ g_S) \circ f_S.$$

since

$$(f_S \circ \tilde{S}) \circ f_S \subseteq (f_S \circ \tilde{S}) \circ g_S \subseteq f_S \tilde{\circ} g_S$$

and

$$(f_S \circ \tilde{S}) \circ f_S \subseteq (f_S \circ \tilde{S}) \circ f_S \subseteq f_S \circ \tilde{S}.$$ 

Hence, $f_S = (f_S \circ \tilde{S}) \circ f_S$.

Proposition 3.10. If $f_S$ is an $S$-ideal of $S$, where $S$ is a regular LA-semigroup, then $f_S$ is an $S$-ideal of $S$ over $U$.

Proof. Let $a$ be any elements of $S$. Then, since $S$ is regular, there exist elements $x$ in $S$ such that

$$a = (ax)a.$$

Then, since $f_S$ is an $S$-ideal of $S$, we have

$$f_S(ab) = f_S((ax)a)b \supseteq f_S(a).$$

This means that $f_S$ is an $S$-ideal of $S$.

Theorem 3.11. Let $S$ be an LA-semigroup. If $S$ is regular, then $f_S = (f_S \circ \tilde{S}) \circ f_S$ for every $S$-ideal (generalized bi-ideal) $f_S$ of $S$ over $U$.

Proof. Let $f_S$ be any $S$-ideal $f_S$ of $S$ over $U$ and $s$ be any element of $S$. Then, since $S$ is regular, there exists an element $x \in S$ such that $s = (sx)s$. Thus, we
Let \( S \) be a semigroup in terms of \( \tilde{S} \). Then, \( \tilde{S} \) is an \( S \)-quasi-ideal of \( S \), since for each \( a \in S \), there exist \( x, y \in S \) such that \( a = (xa^2)y \). So, by using left invertive law, \( f_S(ab) = f_S((xa^2)y)b = f_S((by)(xa^2)) \supseteq f_S(by) \supseteq f_S(b) \).

Thus, \( f_S \) is an \( S \)-left ideal of \( S \). Conversely, assume that \( f_S \) is an \( S \)-left ideal of \( S \). Then, by using left invertive law, \( f_S(ab) = f_S(((xa^2)y)b) = f_S((by)(xa^2)) \supseteq f_S(xa^2) \supseteq f_S(a^2) \supseteq f_S(a) \).

Thus, \( f_S \) is an \( S \)-right ideal of \( S \).

**Proposition 4.3.** Every \( S \)-two-sided ideal of an intra-regular LA-semigroup \( S \) with left identity is idempotent.

**Proof.** Assume that \( f_S \) is an \( S \)-two-sided ideal of \( S \), then
\[
f_S \circ f_S \subseteq f_S \circ \tilde{S} \subseteq f.
\]

Since \( S \) is intra-regular, so for each \( a \in S \), there exist \( x, y \in S \) such that \( a = (xa^2)y \). So, \( a = (xa^2)y = (x(aa))y = (a(xa))y = (y(xa))a \).

Thus, we have\[
(f_S \circ f_S)(a) = \bigcup_{a=(y(xa))} \{f_S(y(xa)) \cap f_S(a)\}
\]
\[
\supseteq f_S(y(xa)) \cap f_S(a)
\]
\[
\supseteq f_S(a) \cap f_S(a)
\]
\[
= f_S(a)
\]

Hence, \( f_S \circ f_S = f_S \).

**Corollary 4.4.** Every \( S \)-left ideal of an intra-regular LA-semigroup \( S \) with left identity is idempotent.

**Proposition 4.5.** Let \( S \) be an intra-regular LA-semigroup with left identity. If \( S \) is intra-regular, then \( f_S = (\tilde{S} \circ f_S)^2 \) for any \( S \)-left ideal \( f_S \) of \( S \).

**Proof.** Let \( f_S \) be any \( S \)-left ideal of an intra-regular LA-semigroup \( S \) with left identity \( S \). Then, \( \tilde{S} \circ f_S \subseteq f \) and since \( \tilde{S} \circ f_S \) is an \( S \)-left ideal of \( S \), it is idempotent. Thus, \( (\tilde{S} \circ f_S)^2 = \tilde{S} \circ f_S \circ f_S \). Moreover, \( f_S = f_S \circ f_S \subseteq f_S \circ f_S = (\tilde{S} \circ f_S)^2 \), which implies that \( f_S = (\tilde{S} \circ f_S)^2 \).

**Theorem 4.6.** For an LA-semigroup \( S \) with left identity, the following conditions are equivalent:

1) \( f_S \) is an \( S \)-ideal of \( S \).
2) \( f_S \) is an \( S \)-bi-ideal of \( S \).

**Proof.** (1) implies (2) follows from Theorem 2.21. Let \( f_S \) be an \( S \)-bi-ideal of \( S \). Since \( S \) is intra-regular, so for each \( a, b \in S \), there exist \( x, y \) and \( u, v \in S \) such that \( a = (xa^2)y \) and \( b = (ub^2)v \). So,
There exist elements 1) in a LA-semigroup $S$.

**Corollary 4.7.** An $S_1$-right ideal of an LA-semigroup $S$ with left identity is an $S_1$-bi-ideal of $S$.

**Proposition 4.8.** For a soft set $f_S$ of an intra-regular LA-semigroup $S$ with left identity, the following conditions are equivalent:

1) $f_S$ is an $S_1$-ideal of $S$.
2) $f_S$ is an $S_1$-interior ideal of $S$.

**Proof.** (1) implies (2) is clear. Assume that (2) holds. Let $a$ and $b$ be any elements of $S$. Then, since $S$ is intra-regular, there exist elements $x, y$ and $v$ in $S$ such that $a = (x^2a)y$ and $b = (ub^2)v$. Since $f_S$ is an $S_1$-interior ideal of $S$, we have

$$f_S(ab) = f_S(((x^2a)y)b) = f_S((by)(x^2a)) = f_S((by)(x(xa))) = f_S((by)(a(xa))) = f_S((ba)(y(xa))) \supseteq f_S(a)$$

and

$$f_S(ab) = f_S(a((ub^2)v)) = f_S((ub^2)(av)) = f_S((v(a)(v(a)v))b) = f_S((v(v(a)v))b) = f_S((v(v(a)v))b) = f_S((v(v(a)v))b) \supseteq f_S(b)$$

Hence, $f_S$ is an $S_1$-ideal of $S$.

**Proposition 4.9.** For a soft set $f_S$ of an intra-regular LA-semigroup $S$ with left identity, the following conditions are equivalent:

1) $f_S$ is an $S_1$-bi-ideal of $S$.
2) $f_S$ is an $S_1$-generalized bi-ideal of $S$.

**Proof.** (1) implies (2) is clear. Assume that (2) holds. Let $a$ be any element of $S$. Then, since $S$ is intra-regular, there exist elements $x, y$ in $S$ such that $a = (x^2a)y$. Thus, we have

$$f_S(ab) = f_S(((x^2a)y)b) = f_S(((x^2a)(ey)b) = f_S(((ye)(a^2)x)b) = f_S(((ye)(x)(aa))b) = f_S(((ye)(x)(aa))b) \supseteq f_S(a) \cap f_S(b).$$

Hence, $f_S$ is an $S_1$-bi-ideal of $S$.

**Proposition 4.10.** For a soft set $f_S$ of an intra-regular LA-semigroup $S$ with left identity, the following conditions are equivalent:

1) $f_S$ is an $S_1$-ideal of $S$.
2) $f_S$ is an $S_1$-quasi-ideal of $S$.

**Proof.** (1) implies (2) is clear. Assume that (2) holds. Let $a$ be any element of $S$. Then, since $S$ is intra-regular, there exist elements $x, y$ such that $a = (x^2a)y$. Thus, we have

$$a = (x^2a)y = (x^2a)(ey) = (xe)(a^2y) = a^2((xe)y) = (aa)((xe)y) = (ya)((xe)a) = (y(x)e)(a) = (y(e)(x)a).$$

Also,

$$\bar{\bar{s}} \circ f_S = (\bar{\bar{s}} \circ \bar{\bar{s}}) \circ f_S = (f_S \circ \bar{\bar{s}}) \circ \bar{\bar{s}}.$$ Therefore,

$$(\bar{\bar{s}} \circ f_S)(a) = ( (f_S \circ \bar{\bar{s}})(a) = \bigcup_{a=\bar{\bar{a}}(y(x)e)} \{(f_S \circ \bar{\bar{s}})(a) \cap \bar{\bar{s}}((y(x)e)a) \}\supseteq (f_S \circ \bar{\bar{s}})(a)$$

Therefore,

$$f_S \circ \bar{\bar{s}}(f_S \circ \bar{\bar{s}})(f_S \circ \bar{\bar{s}}) \supseteq f_S.$$ Hence, $f_S$ is an $S_1$-right ideal of $S$. And by Proposition 4.2, $f_S$ is an $S_1$-left ideal of $S$. Hence, $f_S$ is an $S_1$-ideal of $S$.

**Theorem 4.11.** For a soft set $f_S$ of an intra-regular LA-semigroup $S$ with left identity, the following conditions are equivalent:

1) $f_S$ is an $S_1$-ideal of $S$.
2) $f_S$ is an $S_1$-left ideal of $S$.
3) $f_S$ is an $S_1$-right ideal of $S$.
4) $f_S$ is an $S_1$-bi-ideal of $S$.
5) $f_S$ is an $S_1$-generalized bi-ideal of $S$.
6) $f_S$ is an $S_1$-interior ideal of $S$.
7) $f_S$ is an $S_1$-quasi-ideal of $S$.

**Definition 4.12.** A soft set $f_S$ over $U$ is called soft semiprime if for all $a \in S$,

$$f_S(a) \supseteq f_S(a^2).$$

**Theorem 4.13.** For a nonempty $A$ of $S$, the following conditions are equivalent:
1) $A$ is semiprime.
2) The soft characteristic function $S_A$ of $A$ is soft semiprime$^*$. 

Proof. First assume that (1) holds. Let $a$ be any element of $S$. We need to show that $S_A(a) \supseteq S_A(a^2)$ for all $a \in S$. If $a^2 \in A$, then since $A$ is semiprime, $a \in A$. Thus,

$$S_A(a) = U = S_A(a^2)$$

If $a^2 \notin A$, then

$$S_A(a) \supseteq \emptyset = S_A(a^2)$$

In any case, $S_A(a) \supseteq S_A(a^2)$ for all $a \in S$. Thus, $S_A$ is soft semiprime$^*$. Hence (1) implies (2).

Conversely assume that (2) holds. Let $a^2 \in A$. Since $S_A$ is soft semiprime$^*$, we have

$$S_A(a) \supseteq S_A(a^2) = U$$

implying that $S_A(a) = U$ and that $a \in A$. Hence, $A$ is semiprime. Thus, (2) implies (1).

Theorem 4.14. For any $SI$-$LA$-semigroup $f_S$, the following conditions are equivalent:

1) $f_S$ is soft semiprime$^*$. 
2) $f_S(a) = f_S(a^2)$ for all $a \in S$.

Proof. (2) implies (1) is clear. Assume that (1) holds. Let $a$ be any element of $S$. Since $f_S$ is an $SI$-$LA$-semigroup, we have;

$$f_S(a) \supseteq f_S(a^2) = f_S(aa) \supseteq f_S(a) \cap f_S(a) = f_S(a)$$

So, $f_S(a^2) = f_S(a)$ and (1) implies (2). This completes the proof.

Proposition 4.15. In an intra-regular $LA$-semigroup, every $SI$-$interior$ ideal is soft semiprime$^*$. 

Proof. Let $f_S$ be any $SI$-$interior$ ideal of $S$ and $a$ be any element of $S$. Since $S$ is intra-regular, there exist elements $x, y$ in $S$ such that $a = (xa^2)y$. Thus, we have

$$f_S(ab) = f_S((xa^2)y) \supseteq f_S(a^2).$$

Hence, $f_S$ is soft semiprime$^*$. 

Theorem 4.16. For an $LA$-semigroup $S$, the following conditions are equivalent:

1) $S$ is intra-regular. 
2) Every $SI$-ideal of $S$ is soft semiprime$^*$. 
3) $f_S(a) = f_S(a^2)$ for all $SI$-ideal of $S$ and for all $a \in S$. 

Proof. First assume that (1) holds. Let $f_S$ be any $SI$-ideal of $S$ and $a$ any element of $S$. Since $S$ is intra-regular, there exist elements $x$ and $y$ in $S$ such that $a = (xa^2)y$. Thus,

$$f_S(a) = f_S((xa^2)y) \supseteq f_S(xa^2) = f_S(xaa) \supseteq f_S(aa) \supseteq f_S(a)$$

so, we have $f_S(a) = f_S(a^2)$. Hence, (1) implies (3).

Conversely, assume that (3) holds. It is known that $J[a^2]$ is an ideal of $S$. Thus, the soft characteristic function $S_{J[a^2]}$ is an $SI$-ideal of $S$. Since $a^2 \in J[a^2]$, we have;

$$S_{J[a^2]}(a) = S_{J[a^2]}(a^2) = U$$

Thus, $a \in J[a^2] = (S\cap a)S$. Here, one can easily show that $S$ is intra-regular. Hence (3) implies (1).

It is obvious that (3) implies (2). Now, assume that (2) holds. Let $f_S$ be an $SI$-ideal of $S$. Since $f_S$ is a soft semiprime$^*$ ideal of $S$,

$$f_S(a) \supseteq f_S(a^2) = f_S(aa) \supseteq f_S(a)$$

Thus, $f_S(a) = f_S(a^2)$. Hence (2) implies (3). This completes the proof.

Theorem 4.17. For an $LA$-semigroup $S$ with left identity, the following conditions are equivalent:

1) $S$ is intra-regular. 
2) $L \cap R \subseteq LR$ for every left ideal $L$ and every right ideal $R$ and $R$ is semiprime. 
3) $f_S \circ g_S \subseteq f_S \circ g_S$ for every $SI$-left ideal $f_S$ and every $SI$-right ideal $g_S$ of $S$ and $SI$-right ideal $g_S$ is soft semiprime$^*$. 

Proof. First assume that (1) holds. Let $f_S$ and $g_S$ be any $SI$-left ideal and $SI$-right ideal of $S$, respectively and $a$ any element of $S$. Since $S$ is intra-regular, there exist elements $x$ and $y$ in $S$ such that $a = (xa^2)y$. Thus,

$$a = (xa^2)y = (a(xa))y = (y(xa))a = (y(xa))(ea) = y((xe)a) = (xa)((ye)a) = (xa)((ae)y).$$

Thus, we have

$$(f_S \circ g_S)(a) = \bigcup_{a=(xa)(ae)y} \{f_S(xa) \cap f_S((ae)y)\}$$

$$\supseteq f_S(a) \cap g_S(a)$$

$$= (f_S \circ g_S)(a)$$

Thus, $f_S \circ g_S \supseteq f_S \circ g_S$ and

$$g_S(a) = g_S((xa^2)y) = g_S((xa^2)(ey)) = g_S((ye)(a^2x)) = g_S(a^2((ye)x)) \supseteq g_S(a^2).$$
Hence, $g_S$ is soft semiprime*.

Now assume that (3) holds. Let $L$ and $R$ be any left ideal and right ideal of $S$, respectively. Since $S_L$ and $S_R$ are $SI$-left and $SI$-right ideal of $S$, respectively. Let $a \in L \cap R$. Then, $a \in L$ and $a \in R$. So, by assumption, we have

$$U = S_{L \cap R}(a) = (S_L \cap S_R)(a) \supseteq (S_L \circ S_R)(a) = (S_{LR})(a).$$

Thus, $L \cap R \subseteq LR$. By assumption, the soft characteristic function $S_R$ is soft semiprime*, and so $R$ is semiprime by Theorem 4.13.

Now assume that (2) holds. Let $a \in S$. Then, obviously $a \in S_a$, where $S_a$ is a left ideal of $S$ and $a^2 = a^2S$ and $a^2S$ is a right ideal of $S$. By assumption, the right ideal $a^2S$ is semiprime, which implies that $a \in a^2S$. Thus, by using medial law, we have

$$a \in S_a \cap a^2S \subseteq (S_a)(a^2S) = (S_a)(aS) \subseteq (S_a)(SS) = (S_a^2)S.$$

Therefore, there exist $x, y \in S$ such that $a = (xa^2)y$. Hence, $S$ is intra-regular.

**Theorem 4.18.** For an LA-semigroup $S$ with left identity, the following conditions are equivalent:

1) $S$ is intra-regular.
2) $R \cap L = RL$ for every left ideal $L$ and every right ideal $R$ of $S$ and $S$ is semiprime.

**Theorem 4.19.** For an LA-semigroup $S$ with left identity, the following conditions are equivalent:

1) $S$ is intra-regular.
2) $f_S \circ g_S = f_S \circ g_S$ for every $SI$-left ideal $g_S$ and every $SI$-right ideal $f_S$ of $S$ and $f_S$ is soft semiprime*.

**Proof.** Assume that $S$ is intra-regular. Let $f_S$ and $g_S$ be any $SI$-right and $SI$-left ideal of $S$, respectively. Then, we have

$$f_S \circ g_S \subseteq f_S \circ g_S.$$

Since $S$ is intra-regular, for each $a \in S$, there exist $x, y \in S$ such that $a = (xa^2)y$. Thus,

$$a = (xa^2)y = (x(aa))(y) = (a(xa))a = ((ey)(xa))a = ((ax)(ye))a.$$

Hence,

$$(f_S \circ g_S)(a) = \bigcup \{f_S((ax)(ye)) \cap g_S(a)\} \supseteq f_S((ax)(ye)) \cap g_S(a) \supseteq f_S(a) \cap g_S(a) = (f_S \circ g_S)(a).$$

Thus, $f_S \circ g_S = f_S \circ g_S$. Moreover,

$$f_S(a) = f_S((xa^2)y) = f_S((xa^2)(ey)) = f_S((ye)(a^2x)) = f_S(a^2((ye)x)) \supseteq f_S(a^2).$$

Hence, $f_S$ is soft semiprime*.

Now assume that (2) holds. Let $L$ and $R$ be any left ideal and right ideal of $S$, respectively. Then, $S_L$ and $S_R$ are $SI$-left and $SI$-right ideal of $S$, respectively. Let $a \in R \cap L$. Then, $a \in R$ and $a \in L$. So, by assumption, we have

$$U = S_{R \cap L}(a) = (S_R \cap S_L)(a) = (S_R \circ S_L)(a) = (S_{RL})(a).$$

Thus, $R \cap L \subseteq RL$ and clearly $RL \subseteq R \cap L$. Hence, $S$ is intra-regular by Theorem 4.18. Moreover, by assumption, the soft characteristic function $S_R$ is soft semiprime*, and so $R$ is semiprime by Theorem 4.13.

**Corollary 4.20.** Let $S$ be an intra-regular LA-semigroup $S$ with left identity. Then, $f_S \circ g_S = f_S \circ g_S$ for every $SI$-ideals $f_S$ and $g_S$ of $S$.

**Proposition 4.21.** [38] Let $f_S; g_S$ be $SI$-left (right, two-sided) ideals of $S$, where $S$ is an LA-semigroup with left identity. Then, $f_S \circ g_S$ is an $SI$-left (right, two-sided) ideal of $S$ over $U$.

**Theorem 4.22.** The set of all $SI$-ideals of an intra-regular LA-semigroup $S$ with left identity forms a semilattice structure with identity $S$.

**Proof.** Let $I_S$ be the set of all $SI$-ideals of an LA-semigroup $S$ and $f_S g_S, h_S \in I_S$. It is obvious that $I_S$ is closed by Proposition 4.21. Moreover, we have $f_S = (f_S)^2$ by Proposition 4.3 and by Corollary 4.20, $f_S \circ g_S = f_S \circ g_S$, where $f_S$ and $g_S$ are $SI$-ideals. Obviously, $f_S \circ g_S = g_S \circ f_S$. Moreover, by using left invertive law,

$$(f_S \circ g_S) \circ h_S = (h_S \circ g_S) \circ h_S = f_S \circ (g_S \circ h_S).$$

Also, by using left invertive law and Proposition 4.21, $f_S \circ (f_S \circ g_S) = (\tilde{g_S} \circ f_S) \circ f_S = f_S \circ f_S = f_S$.

**Theorem 4.23.** For an LA-semigroup $S$ with left identity, the following conditions are equivalent:

1) $S$ is intra-regular.
2) $f_S \circ g_S \subseteq f_S \circ g_S$ for every $SI$-right ideal $f_S$ and every $SI$-left ideal $g_S$ of $S$ and $SI$-right ideal $f_S$ is soft semiprime*.
3) $f_S \circ g_S \subseteq f_S \circ g_S$ for every $SI$-right ideal $f_S$ and every $SI$-bi-ideal (or quasi-ideal) $g_S$ of $S$ and $SI$-right ideal $f_S$ is soft semiprime*.

**Proof.** (1) implies (3) and (3) implies (2) is obvious. (2) implies (1) follows from Theorem 4.19.

**Theorem 4.24.** For an LA-semigroup $S$ with left identity, the following conditions are equivalent:
Thus, \( f \) and \( g \) be any \( S \)-right and \( S \)-bi-ideal of \( S \), respectively. Then, since \( S \) is intra-regular, for each \( a, b \in S \), there exist \( x, y \in S \) such that \( a = (xa^2)y \). Thus,

\[
\begin{align*}
\alpha &= (xa^2)y = (a(xa))y = (y(xa))a = \\
&= (y((xa^2)y))a = ((y((xa^2)y))a = \\
&= ((a(xa))(y(xy)y))a = ((y((xy)(xa))a)a = \\
&= ((ax)((xy)y))a.
\end{align*}
\]

Thus, we have

\[
(fg \circ g)(a) = \bigcup_{\alpha = ((ax)((xy)y))a} \{f, g\}(\alpha) \cap g(\alpha) \cap f(\alpha) \supseteq \bigcup_{\alpha = ((ax)((xy)y))a} f(\alpha) \cap g(\alpha) \cap f(\alpha).
\]

Thus, \( f \) is soft semiprime*. (3) implies (2) is obvious.

Now let (2) hold. Assume that \( f \) and \( g \) be any \( S \)-right and \( S \)-left ideal of \( S \), respectively. Then, by assumption, we have

\[
gf \subseteq f \subseteq g \subseteq f \subseteq g \subseteq f \subseteq g \subseteq f \subseteq g \subseteq f.
\]

Hence, by Theorem 4.17, \( S \) is intra-regular.

### 5. Completely regular LA-semigroups

In this section, we characterize completely regular LA-semigroups in terms of \( S \)-ideals. An element \( a \) of an LA-semigroup \( S \) is called left regular if there exists \( x \in S \) such that

\[
a = axa^2 = (aa)x.
\]

and \( S \) is called left regular if all elements of \( S \) are left regular. An element \( a \) of an LA-semigroup \( S \) is called right regular if there exists \( x \in S \) such that

\[
a = a^2x = (aa)x.
\]

and \( S \) is called right regular if all elements of \( S \) are right regular. An element \( a \) of an LA-semigroup \( S \) is called completely regular if \( a \) is regular and left and right regular and \( S \) is called completely regular if all elements of \( S \) are completely regular.

**Theorem 5.1.** For an LA-semigroup \( S \), the following conditions are equivalent:

1. \( S \) is left regular.
2. For every \( S \)-left ideal \( I \) of \( S \), \( f(s) = f(s) \) for all \( s \in I \).

**Proof.** First assume that (1) holds. Let \( f \) be any \( S \)-left ideal of \( S \) and \( a \) be any element of \( S \). Since \( S \) is left regular, there exists an element \( x \in S \) such that \( a = x(aa) \). Thus, we have

\[
f(s) = f(x(aa)) \supseteq f(aa) \supseteq f(s)
\]

implying that \( f(s) = f(s) \). Hence (1) implies (2).

Conversely, assume that (2) holds. Let \( a \) be any element of \( S \). Since \( f \) is a left ideal of \( S \), the soft characteristic function \( Sf = f(a^2) \) is an \( S \)-left ideal of \( S \).

Since \( a^2 \in L[a^2] \), we have

\[
Sf = f(a^2) = U
\]

implying that \( a \in L[a^2] = S(aa) \). This obviously means that \( S \) is left regular. So (2) implies (1). This completes the proof.

**Theorem 5.2.** For an LA-semigroup \( S \), the following conditions are equivalent:

1. \( S \) is right regular.
2. For every \( S \)-right ideal \( I \) of \( S \), \( f(s) = f(s) \) for all \( s \in I \).

### 6. Weakly Regular LA-semigroups

In this section, we characterize a weakly regular LA-semigroup in terms of \( S \)-ideals. An element \( a \) of an LA-semigroup \( S \) is called weakly regular if there exist \( x, y \in S \) such that \( a = (ax)(ay) \) and \( S \) is called weakly regular if all elements of \( S \) are weakly regular.

**Theorem 6.1.** Let \( S \) be an LA-semigroup. If \( S \) is weakly regular, then \( f \) is a \( S \)-right ideal of \( S \). For every \( S \)-right ideal \( f \) of \( S \) and for every \( S \)-right ideal \( g \) of \( S \).

**Proof.** Let \( f \) be an \( S \)-right ideal of \( S \), \( g \) be an \( S \)-left ideal of \( S \) and \( x \in S \). Then, since \( S \) is weakly regular, \( x = (xs)(xt) \) for some \( s, t \in S \). Hence,

\[
(fg)(s) = \bigcup_{x = (xs)(xt)} (fg)(s)(xt)
\]

\[
\supseteq f(s)(x) \cap g(s)(x)
\]

Since \( f \) is a \( S \)-right ideal of \( S \) and \( g \) is a \( S \)-left ideal of \( S \), \( f \cap g = f \).
7. Quasi-regular LA-semigroups

An element \( a \) of an LA-semigroup \( S \) is called left (right) quasi-regular if there exist \( x, y \in S \) such that \( a = (xa)(ya) \) and \( S \) is called left (right) quasi-regular if all elements of \( S \) are left (right) quasi-regular.

**Theorem 7.1.** An LA-semigroup \( S \) is left (right) quasi-regular if and only if every \( SI \)-left (right) ideal is idempotent.

**Proof.** Assume that \( f_S \) is an \( SI \)-left ideal. Then, there exist \( x, y \in S \) such that \( a = (xa)(ya) \). So, we have:

\[
(f_S \circ f_S)(a) = \bigcup_{a = (xa)(ya)} (f_S(xa) \cap f_S(ya)) \\
\geq f_S(xa) \cap f_S(ya) \\
\geq f_S(a) \cap f_S(a) \\
= f_S(a)
\]

and so, \( f_S \circ f_S \supseteq f_S \). Thus, \( f_S \circ f_S = f_S \) and \( f_S \) is idempotent.

Conversely, assume that every \( SI \)-left ideal of \( S \) is idempotent. Let \( a \in S \). Then, since \( L[a] \) is a principal left ideal of \( S \), the soft characteristic function \( S_L[a] \) is an \( SI \)-left ideal of \( S \). Thus, by assumption

\[
S_{L[a]}L[a](a) = (S_L[a] \circ S_{L[a]})(a) = S_L[a](a) = U
\]

and so,

\[
a \in L[a]L[a] = (Sa)(Sa)
\]

Hence, \( S \) is left quasi-regular. The case when \( S \) is right quasi-regular can be similarly proved.

**Proposition 7.2.** If \( f_S \) is an \( SI \)-right ideal of an left (right) quasi-regular LA-semigroup \( S \), then \( f_S \) is an \( SI \)-ideal of \( S \).

**Proof.** Let \( f_S \) be an \( SI \)-right ideal of an left (right) quasi-regular LA-semigroup \( S \). Then, since \( S \) itself is an \( SI \)-right ideal of \( S \), and by assumption \( S \) is idempotent, we have

\[
\tilde{S} \circ f_S = (S \circ \tilde{S}) \circ f_S = (f_S \circ \tilde{S}) \circ \tilde{S} \subseteq f_S \circ \tilde{S} \subseteq f_S \circ f_S = f_S
\]

**Theorem 7.3.** Let \( f_S \) be an LA-semigroup \( S \). If \( f_S = (f_S \circ \tilde{S})^2 \cap (\tilde{S} \circ f_S)^2 \) for every \( SI \)-ideal \( f_S \) of \( S \), then \( S \) is quasi-regular.

**Proof.** Let \( f_S \) be any \( SI \)-right ideal of \( S \). Thus, we have

\[
f_S = (f_S \circ \tilde{S})^2 \cap (\tilde{S} \circ f_S)^2 \subseteq f_S \circ f_S \subseteq f_S \circ f_S = f_S
\]

and so \( f_S = (f_S)^2 \). It follows that \( S \) is right quasi-regular by Theorem 7.1. One can similarly show that \( S \) is left quasi-regular. This completes proof.

8. Conclusion

In this paper, we first remind the concepts of soft intersection left (right, two-sided) ideals, (generalized) bi-ideals, interior ideals, quasi-ideals of LA-semigroups defined in [38]. We then show that all of these ideals coincide in an intra-regular LA-semigroup with left identity. Moreover, we characterize regular, intra-regular, completely regular, weakly regular and quasi-regular LA-semigroups by the properties of these soft intersection ideals. We are of the opinion that some results in this paper have already constituted a foundation for further investigation related with the further development of LA-semigroups. In the following study of soft intersection LA-semigroups, applying soft intersection LA-semigroups to some applied fields, such as decision making, data analysis and forecasting and so on are worth to be considered.

References