3D Vascular Decomposition and Classification for Computer-aided Detection

Ashirwad Chowriappa*, Sarthak Salunke, Maxim Mokin, Peter Kan, Peter D. Scott

Abstract—In this work we propose a Weighted Approximate Convex Decomposition (WACD) and Classification methodology for computer-aided detection (CADe) and analysis. We start by addressing the problem of vascular decomposition as a cluster optimization problem and introduce a methodology for compact geometric decomposition. Classification of decomposed vessel sections is performed using the most relevant eigenvalues obtained through feature selection. The method was validated using pre-segmented sections of vasculature archived for 98 aneurysms in 112 patients. We test first for vascular decomposition and next for classification. The proposed method produced promising results with an estimated 81.5% of the vessel sections correctly decomposed. Recursive Feature Elimination was performed to find the most compact and informative set of features. We showed that the selected sub-set of eigenvalues produces minimum error and improved classifier precision. The method was also validated on a longitudinal study of four cases having internal cerebral aneurysms. Volumetric and surface area comparisons were made between expert segmented sections and WACD Classified sections containing aneurysms. Results suggest that the approach is able to classify and detect changes in aneurysm volumes and surface areas close to that segmented by an expert.

Index Items—Vascular decomposition, segmentation, classification, aneurysm, spectral.

I. INTRODUCTION

SUBARACHNOID hemorrhage (SAH) caused by the rupture of an aneurysm is one of the common causes of stroke. Each year, approximately 795,000 Americans experience a new or recurrent stroke [1]. To perform pre or post stroke assessment some form of analysis of the vasculature has to be carried out [2], [3]. Such as, comparative assessment among and between patient vasculatures. In the case of population-based comparisons, and in diagnosis of neuro-vascular diseases [4], [5]. In this paper we focus on shape decomposition and characterization as a fundamental approach to understand the nature of complex vascular structures in computer-aided detection (CADe) and analysis. The Weighted Approximate Convex Decomposition (WACD) methodology for vessel decomposition was first proposed in [6]. We extend this prior work and address the problem of vascular classification. Our approach is twofold. First we decompose the cerebral vasculature into sub-structures. Next, the decomposed structures are classified to identify aneurysms. To our knowledge this is the first time that this approach has been used to address the problem of 3D vascular decomposition and classification.

A. Previous work on decomposition and segmentation

The literature shows several 2D/3D vascular decomposition and segmentation methods [6]-[11]. For extended reviews on vessel segmentation algorithms we refer the reader to the following surveys [12]-[15]. Methods adapted from generic segmentation algorithms generally use a model based approach whereas others use the notion of centerline for vessel tracking. Model based techniques naturally capture the physics and geometry of vascular structures (more generally, tube-like structures) that vary spatially. A geometric deformable model for segmenting tubular structures is presented in [16]. The main advantage of this technique is its ability to segment twisted, convoluted, and occluded structures without the need for user interaction. Christian et al. [17] presents an approach for simultaneously separating and segmenting multiple interwoven tubular tree structures. Also in [18] the authors discuss a deformable model for detecting bifurcations and providing structural analysis.

B. Our vascular decomposition approach

In this work we propose a model based approach that decomposes the vasculature into sub-components. Our approach uses convex decomposition, a topic that has been investigated in [19], [20]. However, exact convex decomposition is not well suited for complex shapes such as vascular structures (figure 1 b). An exact decomposition of a complex shape such as a vascular tree can lead to an unacceptably large number of components which may not have meaningful relationships in a global sense. To overcome this problem we propose a Weighted Approximate Convex Decomposition methodology (WACD) that is well suited for vessel decomposition illustrated in figure 1 (c). It can be seen that an approximate decomposition (c) produces a clearer and more parsimonious representation of the vessel structures than an exact decomposition (b).

In the next step of our approach the decomposed vessel sections are classified based on the information coded by the Laplacian spectrum, represented by relevant eigenvalues.

C. Related work

Classification requires an efficient shape representation, and depends on the shape descriptors [21]. The literature shows several schemes for vascular classification in computer-aided
diagnosis and detection [22], [23]. In [24] the authors propose a method for vascular geometry classification to detect cerebral aneurysms using a color indicator. They use curvatures as the base of their vascular coloring scheme. An approach that uses texture analysis is performed in [25] for vascular classification. Lauric et al. [26] introduce a write number as a novel aneurysm surface descriptor. Their approach was shown to be effective for both detection and classification.

D. Our WACD classification approach

Vessel classification is addressed by analysis in the spectral domain. We obtain a 3D shape representation that is invariant under natural deformations, and at the same time, contains enough information to perform shape characterization. Although classification can be directly performed without the need for vascular decomposition. When dealing with complex and highly tortuous structures seen in the neuro-vasculature, such classification would require high dimensional features and an exponentially large training data set. The WACD methodology is used to simplify vascular structures into parsimonious descriptions with useful physiological significance.

Spectral approaches in general use the $k$ smallest eigenvalues and embed the manifold onto a $k$ dimensional subspace spanned by their associated eigenvectors [27]. These eigenvalues can be used as simple shape descriptor that works quite well on vessel structures. The literature shows spectral graph theory used extensively to provide adequate characterization of geometric shapes [28] and also for shape matching [27]-[29]. In [30] the authors propose a deformation invariant shape representation. They use the eigenvalues and eigenvectors of the geodesic distance matrix after application of some kernel. One useful property of spectral methods is their ability to compute bend invariant signatures for manifold structures [28].

The remainder of the paper is organized as follows. In section II A we introduce surface mesh generation for clinical data. In section II B we compute the dual graph and define curvature parameters that drive the decomposition cost function. Vessel shape representation and feature selection is discussed in section II C. Finally we present our results in section III and conclude in section V.

![Fig. 1](image1.png)

Fig. 1. (a) Reconstructed surface mesh of a tortuous vessel tree, (b) exact convex decomposition of vessel tree section having 286 clusters, (c) decomposition using a weighted approximate convex decomposition strategy having 9 clusters for the vessel tree. Clusters are shown as different colored convex hulls of varying sizes.

II. METHODOLOGY

A. Data preprocessing and mesh generation

The clinical data set consisting of contrast enhanced CTA data (silk thickness: 1.25 mm, stored using DICOM standards) was used for mesh generation. Volumes of interest (VOI) around identified vascular structures are specified using maximum intensity projections (MIP). More specifically the following vessel sections are reconstructed; Basilar arteries (BA), Anterior Communicant (ACoA), Posterior Communicant (PCoA), Middle Cerebral (MCA), Internal Carotid (ICA) and Superior Cerebelar (SCA). Prior to segmentation and reconstruction an anisotropic diffusion filter is applied [31]. This filter reduces the noise while preserving small vascular structures enabling better segmentation which is important in the region of the brain. Next, we remove the skull bones, the sinuses and the skin, having similar intensity as the vessels, using double thresholding $(d_1, d_2, d_3, d_4, d_5)$. A 3D model of the vessel is obtained as an iso-surface of intensity zero, resulting from the level set evolution, implemented utilizing ITK [32]. For a detailed description we refer the reader to [33]. In order for small vessels to be included iso-surface $I_{iso} = 118$ value is used. Values for the level set threshold $t_i = 36$, variance $\sigma_i^2 = 0.3$ and number of iteration $i = 120$ in ITK were chosen. The parameter settings that were used in double thresholding were $dt_1 = 183$ (bone), $dt_2 = 318$ (bone boundary), $dt_3 = 714$ (mean bone boundary) and $dt_4 = \max(I)$ (maximum intensity of the data set).

B. Vascular decomposition

In this section we introduce our WACD methodology. From the computed vessel surface representation, convex decomposition is employed to partition the mesh into a minimal set of convex sub-surfaces, $S = \{s_1, s_2, ... , s_n \}$. The 2-manifold surface $S$ is defined in $R^3$ as $V = \{v_1, v_2, ... , v_j \}$, $T = \{t_1, t_2, ... , t_i \}$ and $E = \{e_i, e_2, ... , e_j \}$ (where $V$ is the vertex set, $T$ the set of triangles and $E$ is the edge set). We define the dual graph $D'$ associated with the surface mesh $S$. Edge weights for $D'$ are computed from shape indices to favor certain features over others. In mesh decimation vertices of $D'$ are iteratively clustered by applying a decimation operator that minimizes a weighted cost function.

1) Computing shape index and edge weights

The curvature of a point on the surface can be defined by its maximum and minimum curvatures ($k_1, k_2$). Using these curvature measures we determine whether the given point lies on a concave, convex, ridge or saddle region (figure 2). Saddle regions are characterized by being concave on one plane and convex from another (eg. horse saddle) and most frequently correspond to surfaces at vessel bifurcations. Ridges on the other hand can be found on aneurysm heads and ostiums (neck).

For each node in the graph $D'$ we define a set of rings around the node as follows: the $i^{th}$ ring around node $v_j$ is defined as the set of vertices $v_i \in V$ for which there exists a shortest path from $v_j$ to $v_i$ containing $e$ edges. The $\mathcal{L}$-ring neighborhood of node $v_j$ is defined as the set of rings $i < \mathcal{L}$ about node $v_j$. Koenderink et al. presented the shape index in [34]. To capture the shape of the $\mathcal{L}$ -ring neighborhood (in our implementation a 3-ring neighborhood) we use the shape
index introduced by [35], where the shape index $SI$ derived from the principal curvatures is given as

$$SI = \frac{1}{2} - \frac{1}{\pi} \arctan \frac{k_{\text{min}} + k_{\text{max}}}{k_{\text{min}} - k_{\text{max}}}$$

The shape index varies in the interval $[0, 1]$ and provides a continuous gradation between shapes, such as concave shapes, hyperboloid shapes and convex shapes (table 1).

This decimation process is governed by a cost function $C(u,v)$ that is weighted on $\lambda$ and minimizes for concavity $Con$ and the aspect ratio $\rho$. The cost associated with each $hcol(u,v)$ is given by

$$C(u,v) = \alpha g(u,v) + \lambda \frac{Con(S(u,v))}{N}$$

where, $Con(S(u,v))$ is the concavity [19] of surface $S(u,v)$, formed by the $L$-ring neighborhoods of $(u,v)$, and $N$ is the normalization factor which is set as the diagonal of the bounding box of $S(u,v)$. The parameters $\alpha$ and $\lambda$ control contributions of the aspect ratio $\rho$ and concavity $Con(S(u,v))$. Setting $\alpha \geq 2$ favors the generation of compact disks in which case the cost is unity. The aspect ratio $AR$ of the surface $S(u,v)$ is defined by

$$\rho(u,v) = \frac{\gamma(S(u,v))^2}{2\pi^* \sigma(S(u,v))}$$

where, $\gamma$ is the perimeter and $\sigma$ is the area of the surface $S(u,v)$. After each edge collapse operation $\lambda$ is locally recomputed for surface $S(u,v)$ and $D'$ is updated by $(1-\delta)\lambda$. Where, $\delta$ is an influence parameter used to minimize the influence of newly formed surface features caused by the decimation process in successive iterations of $hcol$.

3) Mesh partitioning

With each iteration of the $hcol$ operator, the lowest mesh decimation cost $C$ is applied, and the new partition $\varphi(n) = \{ \varphi_1^n, \varphi_2^n, \varphi_3^n, \ldots, \varphi_M^n \}$ is computed as follows

$$\forall k, \varphi_k^n = p_k^n \cup H(p_k^n)$$

where, $p_k^n$ represents $k$th cluster in $D$ after $n$ edge collapse operations. This procedure is iteratively performed until all edges of $D'$ are in clusters with concavity lower than a determined concavity resolution value $\varepsilon$. In figure 3 we illustrate the decomposition of a vascular branch section.

2) Mesh decimation

Following the assignment of edge weights to the dual graph $D'$, convex decomposition (mesh decimation) is initiated by iteratively applying half edge operations on neighboring vertices. We define a half edge collapse operator $hcol(u,v)$ such that when applied to two vertices $(u,v)$ connected by an edge in $D'$ merges $v$ with $u$ and all incident edges on $v$ are connected to $u$. To keep track of the unified (merged) vertices caused by mesh decimation we define $H(u)$ (initially $\emptyset$) to be a vector containing the history of vertex $u$. With each operation of $hcol(u,v)$ the vector $H(u)$ is updated as follows

$$H(u) \leftarrow H(u) \cup \{v\}$$

Table 1

<table>
<thead>
<tr>
<th>Shape type</th>
<th>Shape index (SI) range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Cup</td>
<td>[ 0.0 - 0.2 ]</td>
</tr>
<tr>
<td>2. Rut</td>
<td>[ 0.2 - 0.4 ]</td>
</tr>
<tr>
<td>3. Saddle</td>
<td>[ 0.4 - 0.6 ]</td>
</tr>
<tr>
<td>4. Ridge</td>
<td>[ 0.6 - 0.8 ]</td>
</tr>
<tr>
<td>5. Cap</td>
<td>[ 0.8 - 1.0 ]</td>
</tr>
</tbody>
</table>

Edge weights are then computed from the SI ranges as follows. Two neighboring vertices connected by an edge in the dual graph $D'$ are assigned an edge weight $\lambda = \{\lambda_1, \lambda_2, \lambda_3\}$ determined by the following criteria

$$\lambda = \begin{cases} 
\lambda_1, & 0 < SI \leq 0.5 \text{ (umbilic points)} \\
\lambda_2, & 0.5 < SI \leq 0.75 \text{ (hyperbolic points)} \\
\lambda_3, & 0.75 < SI \leq 1.0 
\end{cases}$$

One advantage of using shape indices as edge weights is that transition from one shape type to another is continuous; hence they can be used to describe subtle shape variations.

Fig. 2. The primitive shape types for corresponding surface characteristics of an aneurysm sac.

Fig. 3. Decomposition of vessel branch section for $\varepsilon$ values.

Table 2

<table>
<thead>
<tr>
<th>Branch section</th>
<th>Aneurysm section</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concavity resolution $\varepsilon$</td>
<td>WP</td>
</tr>
<tr>
<td>50-100</td>
<td>17</td>
</tr>
<tr>
<td>100-150</td>
<td>11</td>
</tr>
<tr>
<td>150-200</td>
<td>7</td>
</tr>
<tr>
<td>200-250</td>
<td>5</td>
</tr>
</tbody>
</table>
The graph Laplacian operator is derived from this manifold and represented by the affinity matrix \( A \). 
(c) The matrix \( A \) is eigen decomposed into eigenvectors \( V \) and eigenvalues \( \lambda_i \). (f) The vertices of the manifold with vertex coordinates \( y \) are projected onto the subspace spanned by the first \( k \) eigenvectors. (b) Shows the resulting manifold-2 structure lying in this \( k \) dimensional subspace. (below) Shows embedding of aneurysm using eigenvectors ranging from 200 to 15.

4) Parameter estimation and computation time

Estimation of the concavity resolution \( \varepsilon \) was determined by volumetric comparisons made on \( n = 65 \) preselected sections that were segmented by an expert (containing cerebral aneurysms and vessel bifurcations). In order to obtain topological consistence it is essential that the WACD algorithm not separate the volumes into more parts (figure 3). Vessel sections were decomposed using WACD and voxelized. In our experimental setup parameters \( \lambda_1 = -0.78 \), \( \lambda_2 = 0.04 \), \( \lambda_3 = 0.83 \) and \( \delta = 0.66 \) were used. Table 2 shows the decomposition obtained (for a single branch section and aneurysm section) for various concavity resolution ranges. We determined on optimal range of \( 150 < \varepsilon < 200 \) that match closest to expert segmented section both in volume and number of decomposed parts.

We measured the computation time of WACD on a PC running Windows 7, 64-bits with an i-7 processor clock speed 2.66 GHz and 6 GB of RAM. On average the estimated decomposition time for a triangulated mesh with 3000 vertices was 250 seconds (figure 5).

C. Vessel shape representation and feature selection

Classification requires an efficient shape representation. We obtain a 3D shape representation that is invariant under natural deformations, and at the same time, contains enough information to perform shape characterization.

1) Shape representation and characterization

The Laplacian spectrum is computed for decomposed sections of vasculature. The intuition behind spectral shape descriptors as features is that the Laplacian eigenvalues are meaningful descriptors of the input surface due to their intrinsic definition and invariance with respect to deformations [28] (figure 6). From the definition of the Laplacian of a graph \( L = D - W \), the normalized graph Laplacian \( L = I - D^{1/2}WD^{1/2} \) is considered (in our case \( L \) is geometric), where \( W \) is the weight matrix whose elements are \( W(i,j) \) if \( (i,j) \) is an edge in \( \mathcal{D} \) and zero otherwise and \( I \) is the identity matrix. \( D_\nu = \sum W_{\nu \nu} \) is an \( n \times n \) matrix of the row sums of \( W \). The eigen decomposition of \( L \) results in eigenvalues \( 0 = \lambda_0 \leq \lambda_1 \leq \ldots \leq \lambda_n \), with smallest eigenvalue corresponding to the constant eigenvector (which is the smoothest mesh function). The corresponding eigenvectors \( \phi_i \), \( i = 0 \ldots n-1 \), are orthogonal and span the eigenspace \( \mathbb{R}^{0 \times n} \). This Laplace spectrum provides a descriptive feature vector, which is used to characterize the input vessel shape. Geometric information intrinsic to the decomposed vessel section is captured using the weight matrix \( W \) [36, 37], where \( W \) is constructed using principal curvatures.

Form the weighted dual graph representation \( \mathcal{D}' \) each edge (connecting vertices \( (i,j) \) ) has an associated weight \( W_{ij} \). For edge \( e = (i,j) \) the weight matrix \( W_{ij} \) is constructed among all pairwise vertices given by

\[
W_{ij} = \begin{cases} 
\left( |K_i| + |K_j| \right) - |\{i, j\}| & \text{if } K_i < 0 \text{ or } K_j < 0 \\
\delta & \text{otherwise}
\end{cases}
\]  

where, \( K_i \) and \( K_j \) are the minimal principal curvature of
vertices $i$ and $j$. When $K_i \geq 0$ and $K_j \geq 0$ implies convexity at $e$ (table 1). To ensure vertex connectivity we set $\delta = 0.01$. The Laplacian eigenvalues become invariant to shape scales by normalizing the spectrum of the input shape. We define $\bar{e}$ to be the normalized average of the principal curvature directions for $K_i$ and $K_j$, $\bar{n}$ is the (normalized) direction of $e$, and $l$ is the length of $e$ normalized by the average length of all edges in the mesh. In figure 4 we show the projection of a vascular 2-manifold surface $S \subset \mathbb{R}^{n \times 3}$ onto the subspace spanned by the first k eigenvectors given by

$$S' = \hat{V}_{(1..k)} \hat{V}_T \text{S}$$

where, $\hat{V}_{(1..k)}$ is a matrix containing the first $k$ columns of $\hat{V}$ and $\text{S}$ is a vector of mesh coordinates in Euclidian space.

![Fig. 6. (a) Bifurcation section with deforming transformation (b) applied to the inlets. Plots of the first $k = 30$ eigenvalues for the deformed and normal vessel sections (c) show little or no change to the first eight eigenvalues and provide invariant shape representations.](image)

We use the ARPACK package [38] together with Super LU [39] for eigen decomposition. Neumann boundary condition [40] are applied due to the presence of small holes or missing triangles in the reconstruction of $S$. This results in the first eigenvalue $\lambda_0$ to be zero and its corresponding eigenvector $\hat{V}_0$ is the constant vector. We ignore it and start with $\lambda_1$. The eigenvalues can be considered as mesh frequencies, whose corresponding low frequency eigenvectors are used to capture global shape characteristics. One useful property of spectral methods is their ability to compute invariant signatures for manifold structures [28]. In figure 6 we demonstrate invariance to vessel deformation. Very little or no change is observed in the first $k = 8$ eigenvalues, that correspond to low frequency mesh characteristics. This invariance to deformation property also makes them robust to noise. Next, eigenvalues of vessel shapes that are more relevant for shape characterization are selected through Feature Selection and used for classification.

2) Validation and test data

Pre-segmented sections (containing cerebral aneurysms and vessel bifurcations) of vasculature was archived for 98 aneurysms in 112 patients was utilized for validation and testing. This data set consisted of $n = 732$ vascular sections that included the same vessel shape and isometrically deformed variations of the vessel shape class (figure 7). These sections were classified by a clinical radiological expert into regular, aneurysm and bifurcation sections yielding the data set (ground truth).

3) Feature selection

In general, Feature Selection (FS) addresses the problem of finding the most compact and informative set of features. Classification performance depends heavily on the dimension of this feature vector [41]. As the dimension of the feature vector increase, the number of samples required to train the system increase exponentially. By using the eigenvalues as vascular shape features, the dimension of the feature space can be reduced without compromising much from the content.

We used the Recursive Feature Elimination (RFE) method, which uses a Support Vector Machines (SVM) [42]-[45]. Recursive Feature Elimination is a wrapper method which performs backward feature elimination. The idea is to find the features which lead to the largest margin of class separation. This combinatorial problem is solved in a greedy fashion. The algorithm begins with the set of all features and successively eliminates the feature which induces the smallest change in the cost function. Finally, RFE computes a ranking of the selected features.

We used 65% of the data, $n = 476$ vascular sections for feature and model parameter selection. The Bootstrap Cross-validation methodology [46] was used to split this data set into training and validation sets. The Bootstrap methodology in general is random resampling with replacement. The training set was used to select relevant features and the validation set was used to determine the performance of the classifier on the selected features. Using RFE, the subset of eigenvalues that produce best performances with respect to classification accuracy (i.e., the lower is the classification error, the higher is the accuracy) are selected.

A SVM with a Gaussian kernel was used as the classifier; the kernel width and the soft-margin parameter were selected through 10 Bootstrap iterations using all 200 eigenvalues. We train a different classifier for each class of vessel sections (bifurcation, aneurysm and regular, shown in figure 7), where relevant eigenvalues are selected during RFE. These eigenvalues represent the shape features that are shared by the class members (table 2) and maximize the discrimination with respect to the non-member shapes. If the selected eigenvalues change at each Bootstrap iteration, this indicates that the selection based on eigenvalues strongly depends on the training set; such selections are less representative for the class.

![Fig. 7. Data set used for bifurcation, aneurysm and regular class](image)
of vessel shapes. The eigenvalues that are selected several times within all the Bootstrap iterations are selected.

Using RFE the number of eigenvalues selected for each vessel class ranged from 8 to 11 (table 2). This tells us that a very small fraction of eigenvalues (about the 6% of the input spectrum) characterize the vessel shape features shared by a given class. We observed that the classifier uses almost the same selected features for all iterations of Bootstrap Cross-Validation. This demonstrates the stability of the eigenvalues selected by the classifier.

Fig. 8. (a, d, g) Shows the reconstructed mesh from clinical data, and (b, e, h) the decomposition using WACD, where $t$ is the number of triangle and $k$ is the number of clusters. Classification of decomposed sections labeled as vessel class (VE) and aneurysm class (AN) in green are shown in (c, f, i).

III. RESULTS

The proposed vascular decomposition and classification methodology was tested on labelled sections of the neuro-vasculature: BA, AcoA, PcoA, MCA, ICA and SCA extracted from clinical data sets. We test first for vascular decomposition and next for classification. To demonstrate the clinical benefits of this approach for computer-assisted detection (CADe) we also test the method using longitudinal study data on four patients with a prior history of unruptured and untreated cerebral aneurysms.

<table>
<thead>
<tr>
<th>Class</th>
<th>CR</th>
<th>ES</th>
<th>CDS</th>
<th>EV</th>
<th>CE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>1-3</td>
<td>96</td>
<td>81</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>Aneurysm</td>
<td>1-3</td>
<td>68</td>
<td>53</td>
<td>11</td>
<td>21</td>
</tr>
<tr>
<td>Branch</td>
<td>3-4</td>
<td>92</td>
<td>76</td>
<td>11</td>
<td>19</td>
</tr>
</tbody>
</table>

A. Experimental study 1: WACD decomposition

Table 2 documents our results; using WACD approximately 81.5% of the structures were clustered into the same number of components identified as the ground truth. The results suggested that the proposed WACD method is capable of producing effective decomposition, a parsimonious description basis on which vessel classification can be performed. In figures 8 and 9 we illustrate the decomposition using WACD where close conformity with ground truth was obtained. In figure 8 (b, e) it can be seen that WACD contained the aneurysm sac within a single cluster, the same results can be seen around the Circle of Willis in figure 11 (b). For vessel branches WACD clustered bifurcations with minimal number of cluster while maintaining close structural representation of the branched section (figure 8 h). In figure 9 (d, f) WACD was used to cluster vessel sections fused with the skull. Although we did not intend to use it for separation of vessels fused to other structures, preliminary results seemed to favor its use.
B. Experimental study 2: Vessel classification

Performance of the classifier was tested on expert labeled data. Using the ground truth data set, 35% (n = 256) of vessel sections that remained after REF, was used to test the SVM classifier. Comparisons were made between classifier performance using the first k (10, 20, 40 and 80) eigenvalues and the selected set of eigenvalues obtained by REF. For the two class problem (i.e. aneurysm or not aneurysm, bifurcation or not bifurcation, regular or not regular) we obtained a classification error of 21% for aneurysm sections, 12% for regular vessel sections and 19% for bifurcation sections (table 2). However, in classifying vascular sections our primary concern lies with the classifier error in detection of cerebral aneurysms, given by the False Negative (FN) error (aneurysm sections that were classified as vessel sections) and the False Positive (FP) error (vessel sections that were classified as aneurysm sections) in classifying aneurysm sacs. We evaluate the classifier for precision PRE (positive predictive value) and Error Rate (ER) on classification of cerebral aneurysms. Table 3 summarizes the classification error rate and precision rate with respect to the optimal set of eigenvalues (using RFE) and the first k (10, 20, 40, 80) eigenvalues. Our results showed the selected set of eigenvalues using RFE produces minimum error and improved classifier precision.

Table 3

<table>
<thead>
<tr>
<th></th>
<th>FPR</th>
<th>FNR</th>
<th>ER</th>
<th>PRE</th>
</tr>
</thead>
<tbody>
<tr>
<td>RFE</td>
<td>0.08</td>
<td>0.11</td>
<td>0.18</td>
<td>0.91</td>
</tr>
<tr>
<td>First 10</td>
<td>0.11</td>
<td>0.18</td>
<td>0.28</td>
<td>0.86</td>
</tr>
<tr>
<td>First 20</td>
<td>0.11</td>
<td>0.23</td>
<td>0.34</td>
<td>0.84</td>
</tr>
<tr>
<td>First 40</td>
<td>0.13</td>
<td>0.27</td>
<td>0.39</td>
<td>0.82</td>
</tr>
<tr>
<td>First 80</td>
<td>0.18</td>
<td>0.31</td>
<td>0.49</td>
<td>0.74</td>
</tr>
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</table>

In figure 8 (a) the reconstruction of a large Anterior Communicating Artery (ACOM) aneurysm is shown. Using WACD (b) the aneurysm section is decomposed, and aneurysm sac classified (c) shown in green. Also, in (d) a medium sized ACOM aneurysm of nearly half the mesh density than that in (a) is decomposed using WACD and classified (e, f). In decomposing arterial branch structures a section of the anterior cerebral artery (ACA) is shown in (h). (i) However, it is misclassified for an aneurysms section (shown in green).

WACD-based classification of the Circle of Willis

It has been suggested that geometric characteristics of the cerebral vessels of the Circle of Willis may provide meaningful association with vascular disease formation and progression [47]. We test if the methodology is capable of effectively decomposing the Circle of Willis and detecting cerebral aneurysms. In particular, cerebral aneurysms (pathological dilation of the vessel) are frequently located at or near arterial bifurcations and after regions of high vascular curvature in arteries of the Circle of Willis [48]. In figure 11 we illustrate the decomposition and classification of the Circle of Willis using the WACD classification approach. The decomposed sections are labeled (c) and classified as aneurysms and vessel sections. Anterior communicating artery (ACoA) aneurysms are the most common aneurysms [48]. Figure 10 shows the classification and labeling of ACoA aneurysm sacs.

Fig. 10. Shows decomposed and labeled vessel section where the green sections are labeled as aneurysm class (AN), and yellow sections are labeled as vessel class (VE).

Fig. 11. (a) The Circle of Willis is reconstructed for a CTA dataset where t is the mesh triangles. (b) WACD is performed generating k sub-sections on the reconstructed vasculature. In (c) the labeled aneurysm section (AN) is highlighted in green and non-aneurysm sections are highlighted in yellow (VE).
Aneurysm data

Data was acquired on the above cases that had pretty much diffused vascular aneurysms. CTA datasets obtained were used for vascular reconstruction and analysis. Measurements were performed on the sections containing aneurysms; aneurysm maximum diameters ranged from 1 to 5 mm (mean, 2 mm ± 1 [standard deviation]), and aneurysm neck sizes ranged from 1 to 3 mm (mean, 1.4 mm ± 1.2). Volume of aneurysm in terms of voxels ranged from 116 to 873 and aneurysm surface area ranged from 10.63 to 35.31 mm² (table 4). None of the aneurysms had previously been treated with coils.

Vascular decomposition and classification

Segmentation of the neuro-vasculature and 3D surface reconstruction was performed (10 CTA data sets). Using WACD the reconstructed vasculature was decomposed resulting in sub-parts of vessel sections (figure 13). Next, decomposed vessel sections were classified as aneurysm and non-aneurysm (vessel) sections. Follow-up analysis (volumetric and surface area computation) was performed on the extracted (classified) aneurysm sacs from longitudinal data (preliminary, follow-up I, follow-up II). Expert segmented aneurysm sections are used as ground truth. In our study, 8 of the 10 aneurysms were correctly classified. However, by specifying a Volume of Interest (VOI) around aneurysm sections (in the experimental setup a 8 cm³ VOI was used) all aneurysms were correctly classified and extracted. Table 4 documents our results; volumetric and surface area comparisons were made between expert segmented aneurysm sections and WACD classified aneurysm sections. We obtained a mean error rate of 7.78% for volumetric comparison and 10.38% for surface area comparisons between expert segmented aneurysm sections. Figure 12 (a, b) illustrates Case 3: where MCA aneurysm on right hand side was followed over a period of 2 months, MIP indicated noticeable change in aneurysm size (table 4). Using WACD and communicating artery aneurysm on the right. A 5x5 mm incidental R Pcomm aneurysm, unchanged in size was followed over a period of two months. Case 3: 62 year old female was followed for incidental right-sided middle cerebral artery (MCA) aneurysm, overall appeared stable (shown in figure 12). Case 4: 54 year old female with post carotid sacrifice and clipping of right MCA aneurysm and clipping of the left Pcom aneurysm. Follow up studies shows a small A1 aneurysm that appeared stable in size.

Table 4

<table>
<thead>
<tr>
<th>Case</th>
<th>Preliminary Volume</th>
<th>Preliminary Surface area</th>
<th>Follow up I Volume</th>
<th>Follow up I Surface area</th>
<th>Follow up II Volume</th>
<th>Follow up II Surface area</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Vox</td>
<td>EV</td>
<td>% err</td>
<td>SA</td>
<td>EA</td>
<td>% err</td>
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<td>116</td>
<td>14.6</td>
<td>8.2</td>
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<tr>
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<td>506</td>
<td>6.0</td>
<td>41.1</td>
<td>44.6</td>
<td>8.2</td>
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<tr>
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<td>5.2</td>
<td>55.4</td>
<td>51.9</td>
<td>6.7</td>
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<tr>
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<td>419</td>
<td>10.3</td>
<td>37.1</td>
<td>33.7</td>
<td>9.6</td>
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</table>
classification (figure 13 a, c) the decomposed aneurysm sac was extracted. We were able to detect changes in aneurysm volumes (4.5% error) and surface areas (8.4% error) that were close to that segmented by an expert.

Fig. 13. In (a) WACD is applied to case 3 and in (c) the decomposition of aneurysm section is shown. Reconstruction of aneurysm sections from preliminary and follow up II data sets are shown in (c, d). In (e) these sections are aligned and in (f) surface correspondence is obtained between aneurysm sacs.

IV. DISCUSSION

The quantitative results above validate the feasibility of the proposed vascular decomposition and classification methodology. Results obtained for vascular decomposition suggest that we attain near optimal decomposition close to expert segmented sections, with approximately 81.5% of the vessel sections decomposed into the correct number of components. This is an essential step for vessel classification. Since classification of large sections of vasculature relies on vessel decomposition to simplify the problem.

Vessel classification is performed in the spectral domain. We obtain a 3D shape representation that is invariant under natural deformations, and at the same time, contains enough information to perform effective shape characterization. RFE was performed to find the most compact and informative set of features. We showed that the selected sub-set of eigenvalues produces minimum error and improved classifier precision.

The method was also validated for computer-assisted detection (CADe) on a longitudinal study of four cases having internal cerebral aneurysms. The main reason for the follow up study analysis was to demonstrate the effectiveness of this approach in automated detection of cerebral aneurysms. Results show we were able classify aneurysms and detect changes in aneurysm sac volumes and surface areas that were close to expert segmented sections.

V. CONCLUSION

Our primary objective was the decomposition of the vasculature into a geometrically consistent (meaningful) set of convex regions, and to characterize these decomposed sections for computer-aided detection. We have demonstrated that the proposed approach is a promising method for vascular analysis capable of producing robust, effective representations and classification of complex vessel structures which include bifurcations and aneurysms. Form our initial longitudinal study findings suggest that our methodology can provide a basis for vascular analysis. However, a much larger follow up study, and also a comparative study with patients with ruptured and unruptured aneurysms, would be necessary to determine the clinical efficacy of this methodology on identifying and labeling cerebrovascular aneurysms.

REFERENCES


