A Pilot-Aided Non-Resampling Sequential Monte Carlo Detector for Coded MIMO-OFDM Systems

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Abstract- In this paper, we propose a sequential Monte Carlo (SMC) symbol detector for differentially encoded multi-input multi-output orthogonal frequency division multiplexing (MIMO-OFDM) systems. We first propose the periodic termination of differential phase trellis and it is seen that this method yields advantages that allow us to simplify the pilot-aided SMC detector. Specifically, the resampling step that is present in conventional SMC methods is circumvented, and the multi-dimensional distribution required for sequence imputations reduces to a simple product of one-dimensional likelihood functions and a priori probabilities that are easily obtained from the decoder. This non-resampling SMC detector is applied to both convolutional-coded and low-density parity check (LDPC)-coded MIMO-OFDM systems and simulation results suggest the promising performance of our proposed detector.

Keywords— Iterative receivers, Orthogonal frequency division multiplexing (OFDM), multiple-input multiple-output (MIMO), Sequential Monte Carlo methods

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) [1]-[2] is a potential scheme for high data rate wireless transmission. OFDM may be combined with multiple antennas at the transmitter and receiver to increase the system capacity on time-variant and frequency-selective channels, resulting in a multi-input multi-output (MIMO) configuration [3]. A blind OFDM receiver based on sequential Monte Carlo (SMC) methods [4]-[5] was proposed in [6]. This receiver, like all conventional SMC detectors, was built on the techniques of sequential importance sampling and resampling. Resampling procedures, however, introduce theoretical and practical problems [5], most notably impoverished trajectory diversity and loss of independence among imputed trajectories. In order to circumvent the resampling step, we recently proposed forcing the phase trellis of a differentially encoded single-input single-output OFDM system to terminate at predetermined indices [7]. In this paper, we show how our proposed method can be used for pilot-aided symbol detection in a MIMO-OFDM system as well.

The remainder of this paper is organized as follows. Section II describes the coded MIMO-OFDM system that employs periodically terminated differential phase modulation. Section III introduces the non-resampling SMC algorithm for MIMO-OFDM. Finally, Section IV presents computer simulations that show the promising performance of the proposed detector for both convolutional-coded and low-density parity check (LDPC)-coded MIMO-OFDM systems.

II. SYSTEM MODEL

A. Baseband Coded MIMO-OFDM Signal Model

Consider a MIMO-OFDM system employing $N_t$ transmit antennas, $N_r$ receive antennas, and $N$ sub-carriers. A source produces binary information bits $\{c_i\}$ that are encoded by an outer channel encoder to yield code bits $\{b_i\}$. The bits $\{b_i\}$ are mapped to $M$-PSK modulated symbol stream $\{d_{i}\}$ taking values from a finite alphabet $A_M = \{a_1, a_2, \ldots, a_M\}$. The sequence $\{d_{i}\}$ is subsequently fed into a differential encoder that has knowledge of $N_t$ and $N$, and operates at a termination period $K$, i.e., at every $K$th transition through the differential $M$-phase trellis, $\log_2 M$ bits are inserted into the differential encoder to terminate it at the desired state. This terminated state then acts as the initial state for the next $K-1$ information symbols before being terminated once again on the $K$th transition. This continues until the entire sequence $\{d_{i}\}$ has been encoded. To illustrate, consider $K-1$ $M$-PSK symbols $\{d_{p,j}\}_{j=1}^{K-1}$ that are differentially encoded to yield the $p$th complete sub-trellis:

$$
\left\{ \begin{array}{ll}
|d_{p,\text{initial}}| & ; j = 0 \\
|d_{p,j-1}d_{p,j}| & ; j = 1, K-1
\end{array} \right.
$$

(1)

where $a_{p,\text{initial}} \in A_M$ is the initial state of the $p$th complete sub-trellis, or equivalently, the terminated state of the $(p-1)$th complete sub-trellis. It is easy to see that periodic termination of the differential encoder results in a serial concatenation of sub-trellises. We further define $T_p = \{s_{p,i}\}_{i=0}^{K-1}$ as the $p$th complete sub-trellis that consists of $K$ symbols. The final sub-trellis $T_{final}$ in the concatenated series, however, may not be of depth $K$. For example, when there is only one transmitter, $T_{final}$ is of depth $K$ if $K$ divides $N$, otherwise, it has depth less than $K$. To generalize this to the case of multiple transmitters, note that trellis concatenation in a single MIMO-OFDM word (one time slot) consists of $N_r N_t / K$ complete sub-trellises only,
\[ \{ S_{it} \}_{t=0}^{N_s - 1} = H_{it} S_{it} + \mathbf{V}_{it}, \]
\[ i = 0, N - 1; t = 0, 1, \]
where SNR is the signal-to-noise ratio and \( \mathbf{V}_{it} \in \mathbb{C} \) is the ambient noise, which is circularly symmetric complex Gaussian noise with zero mean and variance \( \sigma^2 \). \( S_{it} \in \mathbb{C} \) and \( Y_{it} \in \mathbb{C} \) are the transmitted signals and received signals at the \( i^{th} \) sub-carrier and \( t^{th} \) time slot, respectively. Further, \( H_{it} \in \mathbb{C}^{N_s \times N_t} \) is the matrix of complex channel frequency responses at the \( i^{th} \) sub-carrier and \( t^{th} \) time slot. It is assumed that the fading process \( H_{it} \) remains static during each MIMO-OFDM word but varies from one word to another and that the fading and noise processes associated with different transmitter-receiver antenna pairs are uncorrelated. The samples \( \{ Y_{it} \}_{t=0}^{N_s - 1} \) are then sent to an iterative receiver. In the following, we describe a generic pilot-aided iterative receiver that processes the set \( \{ Y_{it} \}_{t=0}^{N_s - 1} \).

B. Pilot-Aided Iterative Receiver

Since the combination of M-PSK modulation and differential encoding may be viewed as an inner encoder, the aforementioned system is therefore a serial concatenation of two encoders. Generically, a turbo receiver consists of two stages: a soft-input soft-output (SISO) demodulator followed by a SISO channel decoder. These two stages may be implemented using any appropriate method. In this paper, the method chosen for soft demodulation is the proposed non-resampling SMC algorithm for a MIMO-OFDM system which will be explained in Section III.

The pilot-aided SISO demodulator utilizes the a priori symbol probabilities \( \Pr(d_m = a_m) \), the channel estimates \( \{ \hat{H}_{it} \}_{t=0}^{N_s - 1} \), and the samples \( \{ Y_{it} \}_{t=0}^{N_s - 1} \) as inputs to produce an a posteriori symbol probabilities \( \Pr(d_m = a_m | \{ Y_{it} \}_{t=0}^{N_s - 1}) \) as its output. The channel decoder on the other hand, uses the a priori log-likelihood ratios (LLR) of the code bits and delivers an update of the a posteriori LLRs of the code bits. Being SISO in nature, the demodulator and the channel decoder exchange extrinsic information with each other to improve the receiver’s performance iteratively. Although the extrinsic information exchanged are statistically independent at the first iteration, they become progressively more correlated as the iterations proceed. This translates to diminishing performance improvements as the number of iterations increases. The reader is referred to [8] for details on the computation of the LLRs and extrinsic information.

III. PILOT-AIDED NON-RESAMPLING SMC DETECTOR FOR MIMO-OFDM SYSTEM

In this section, we consider pilot-aided symbol detection in a coded MIMO-OFDM system employing differential M-PSK modulation over a frequency-selective multipath-fading channel.

A. Preliminaries

Recall the \( N \cdot N_t \) symbols \( \{ S_{it} \}_{t=0}^{N_s - 1} \) in (2) that constitute the output of the differential encoder during one MIMO-OFDM symbol. For a termination period \( K \), assuming \( K \) does not divide \( N \cdot N_t \), the result of the demultiplexing step on \( \{ S_{it} \}_{t=0}^{N_s - 1} \) is the transmission grid shown in Fig. 1. The entries in this grid correspond to the set \( \{ S_{it} \}_{t=0}^{N_s - 1}, c = 0, N - 1 \) mentioned in Section IIA and each column of a given grid corresponds to the transmitted signal vector \( \mathbf{S}_{it} \) at the \( i^{th} \) subcarrier (see (3)). From Fig. 1, we see that the indices \( \omega = 0, N \cdot N_t - 1, i = 0, N - 1 \), and \( c = 0, N - 1 \) are related via the relation

\[ \omega = c \cdot N + i \]  

(4)

Using (4), any set \( \{ X_{it} \}_{t=0}^{N_s - 1} \) may be represented as \( \{ X_{it} \}_{t=0}^{N_s - 1}, c = 0, i = N - 1 \) and vice versa. When the termination period \( K \) does not divide \( N \cdot N_t \), we see that there is at least one termination state at any frequency \( i \) (see Fig. 1). These terminated states may serve as the pilot symbols in the estimation of the MIMO-OFDM channel parameters. Furthermore, the phase of these pilots could be made to cycle through each of the \( M \) states \( a_1, a_2, a_3, \ldots, a_M \) in sequence.
This method has the advantage of simplicity for both the transmitter and receiver since only the termination period $K$ and the initial state $\zeta_{0,0}$, or equivalently $a_{0,\text{initial}}$, of the $0^\text{th}$ sub-trellis, need to be known. Note that pilots will occur at values of $\omega$ which are multiples of $K$. Another issue is the arrangement of pilots from one time slot to the next. A possible method to make the set of pilot sub-carriers at the $(t−1)^{\text{th}}$ time slot different from that at the $t^\text{th}$ time slot is to vary the termination period $K$ from one time slot to the next, in a predetermined fashion. This results in pilots that are staggered from one MIMO–OFDM symbol to the next.

In Fig. 1, each column corresponds to the transmitted signals $S_{i,j}$ at the $i^{\text{th}}$ sub-carrier [cf. (3)]. Henceforth, we shall omit the index $t$ for time, since each MIMO–OFDM symbol is independent of all others. Thus, we simply write $S_{i,0}$, $Y_{i,0}$, and $H_{i,0}$, respectively. At each frequency $i$, at least one transmit antenna is associated with pilots while the other antennas transmit data symbols. Assuming there are $I_i$ pilots at the $i^{\text{th}}$ sub-carrier, we now have $N_T−I_I$ data symbols transmitted simultaneously over the $i^{\text{th}}$ sub-carrier. When the pilot symbols at all sub-carriers are known (which is usually the case), we see that the $N_T−I_I$ data symbols are independent of each other since each sub-trellis is independent of all others. Thus, when $K$ does not divide $N \cdot N_T$, periodic termination yields two main effects which we exploit: (i) a reduced number of interfering symbols at any frequency $i$, and (ii) independence among these interferers.

Before introducing our non–resampling SMC detector for the MIMO–OFDM system mentioned in Section II, we introduce several quantities. Let $\Phi_{i,\text{pilot}}$ and $\Phi_{i,\text{data}}$ be the sets containing the indices of the antennas that transmit the $i^{th}$ pilot and $N_T−I_I$ data symbols, respectively, over the $i^{th}$ sub-carrier. Clearly, $\Phi_{i,\text{pilot}} \cup \Phi_{i,\text{data}} = \{0,1, \ldots, N_T−1\}$. Suppose the data symbol $S_{i,c}$, $c \in \Phi_{i,\text{data}}$ is of interest to us, we define the following vectors:

$$S_{i,pilot} \begin{bmatrix} S_{i,pilot_0} \ S_{i,pilot_1} \ \cdots \ S_{i,pilot_{I_i−1}} \end{bmatrix}^T$$

(5a)

$$S_{i,data} \begin{bmatrix} S_{i,0} \ S_{i,1} \ \cdots \ S_{i,\text{data}_c−1} \ \cdots \ S_{i,\text{data}_{I_i−1}} \end{bmatrix}^T$$

(5b)

where $[ \ ]^T$ denotes the transpose of a vector. The elements of these vectors are drawn from $\{S_{i,d}\}_{Y_i \in \Phi_{i,\text{pilot}}}$ and $\{S_{i,d}\}_{\Omega_{i,c} \in \Phi_{i,\text{data}}}$, respectively. We denote the values taken by $S_{i,pilot}$ and $S_{i,data}$ by $a_{i,pilot}$ and $a_{i,data}$, respectively. It is clear that only one permutation of $[S_{i,pilot} \ S_{i,data}]^T$ will yield vector $S_i = [S_{i,0} \ S_{i,1} \ S_{i,\text{data}_c−1}]^T$ in (3). Since (i) any value of $\omega$ is associated with a unique pair of $i$ and $c$ through (4), and (ii) pilots occur only at values of $\omega$ which are multiples of $K$, the receiver has knowledge of this permutation.

### B. Pilot–Aided Non–Resampling SMC Algorithm

We consider the problem of estimating the a posteriori symbol probabilities of $d_{N+c}$ based on knowledge of the samples $Y_i = \{Y_{i,0}, Y_{i,1}, \ldots, Y_{i,T}\}$ up to the $i^{th}$ sub-carrier, the pilots $S_{i,pilot}$, the channel estimates $\{\hat{H}_i\}_{i=0}^{N−1}$, and the a priori symbol probabilities of $d_{c,N+i}$.

When the receiver knows the channel estimates $\{\hat{H}_i\}_{i=0}^{N−1}$ and $S_{i,pilot}$, the predictive distribution of the states is

$$\Pr(S_{i,c} = a_m | S_{i−1,c}, Y_{i−1}, \hat{H}_{i−1}, S_{i−1,pilot} = a_{i−1,pilot}) \propto \Pr(S_{i,c} = a_m | S_{i−1,c}, Y_{i−1}, \hat{H}_i, S_{i,pilot} = a_{i,pilot}) \cdot \Pr(S_{i−1,c} = a_m | S_{i−1,c})$$

(6a)

$$\alpha^{(i,c)}_{i−1,m}$$

(6b)

$a_m \in A_{Y_i}; i = 0, 1, \ldots, N−I_I; c \in \Phi_{i,\text{data}}; q = 1, \ldots, \Psi$

The second term on the right–hand–side of (6a) is the $a$ priori probability of symbol $d_{c,N+i}$ since $S_{i,c}$ is independent of $Y_{i−1}$ and $\hat{H}_i$, and $\{S_{i,j}\}$ is a Markov chain. We denote the right–hand–side of (6a) by $\alpha^{(i,c)}_{i,m}$ to simplify subsequent notations. To obtain $f(Y_i | S_{i−1,c}, Y_{i−1}, \hat{H}_{i−1}, S_{i−1,pilot} = a_{i−1,pilot}, S_{i,c} = a_m)$ in (6a), we first note that the term

$$f \begin{bmatrix} Y_i \ S_{i−1,c} \ Y_{i−1} \ \hat{H}_{i−1} \ S_{i−1,pilot} \ a_{i−1,pilot} \ S_{i,c} \ a_m \end{bmatrix}$$

is the multivariate Gaussian distribution with mean vector $\mu^{(a)}_{i,c}$ and covariance matrix $A^{(a)}_{i,c}$ given by:

$$\mu^{(a)}_{i,c} \in \epsilon \left[ Y_i, S_{i−1,c}, Y_{i−1}, \hat{H}_{i−1}, S_{i−1,pilot} \right]$$

$$A^{(a)}_{i,c} = \{ \mu, \mu, \mu_{i−1,c}, \mu_{i−1,c}, \mu_{i−1,c}, \mu_{i−1,c}, \mu_{i−1,c}, \mu_{i−1,c} \}$$

(7)
\[ \Lambda^{(q)} = \text{diag} \left[ \sigma^2_{i,0}, \sigma^2_{i,1}, \ldots, \sigma^2_{i,k_{\text{max}}-1} \right] \quad (8) \]

where \( \text{diag}[\cdot] \) denotes a diagonal matrix, and \( \sigma^2_{i,r} \) is the variance of the random sample \( Y_{i,r} \). Therefore, \( f \left( Y_{i,r}, S_{i,\text{pilot}}^{(q)} \right) \) is simply the product of \( N_g \) one-dimensional Gaussian likelihood functions, each with mean \( \mu_{i,r} \), \( r = 0,1, \ldots, N_g-1 \) and variance \( \sigma^2_{i,r} \). From (3), it is easy to see that each \( \sigma^2_{i,r} \), \( r = 0,1, \ldots, N_g-1 \) is equal to \( \sigma^2 \). The quantity of interest \( f(Y_i, S_{i,\text{pilot}}^{(q)} \mid Y_{\text{r}}, \hat{H}, S_{\text{pilot}}, S_c = a_m) \) is easily obtained from \( f(Y_i, S_{i,\text{pilot}}^{(q)} \mid Y_{\text{r}}, \hat{H}, S_{\text{pilot}}, S_c = a_m) \) by:

\[
f(Y_i, S_{i,\text{pilot}}^{(q)} \mid Y_{\text{r}}, \hat{H}, S_{\text{pilot}}, S_c = a_m) = \sum_{a_m, a_q} a_q^{(q)} \cdot f(Y_i, S_{i,\text{pilot}}^{(q)} \mid Y_{\text{r}}, \hat{H}, S_{\text{pilot}} = a_m) \quad (9a)
\]

\[
eq \sum_{a_m, a_q} a_q^{(q)} \cdot \prod_{d=0}^{N_g-2} \Pr( S_c^{(q)} = a_{c,d} \mid Y_{i,d}) \quad (9b)
\]

\[
eq \sum_{a_m, a_q} a_q^{(q)} \cdot \prod_{d=0}^{N_g-2} \Pr( S_c^{(q)} = a_{c,d} \mid Y_{i,d}) \quad (9c)
\]

Equation (9c) is the result of the independence among sub–trellises due to knowledge of the termination states. The terms \( \Pr( S_c^{(q)} = a_{c,d} \mid Y_{i,d}) \) in (9c) are the \( a \) \textit{priori} probabilities of the data symbols that are obtained from the SISO channel decoder. Given the state predictive distribution \( \Pr(S_{i,c} = a_m \mid S_{i-1,c}^{(q)} = a_{i-1,c}, Y_{i-1}, \hat{H}, S_{\text{pilot}} = a_{i-1,pilot}) \), the importance weights are updated according to:

\[
w_{i,c}^{(q)} \propto w_{i,c-1}^{(q)} \cdot f(Y_i, S_{i,\text{pilot}}^{(q)} \mid Y_{\text{r}}, \hat{H}, S_{\text{pilot}} = a_m) \quad (10)
\]

The function \( f(Y_i, S_{i,\text{pilot}}^{(q)} \mid Y_{\text{r}}, \hat{H}, S_{\text{pilot}} = a_m) \) in (10) is easily obtained from \( a_{q,c,m}^{(q)} \) by:

\[
f(Y_i, S_{i,\text{pilot}}^{(q)} \mid Y_{\text{r}}, \hat{H}, S_{\text{pilot}} = a_m) = \sum_{a_m, a_q} a_q^{(q)} \quad (11)
\]

With both the state predictive distribution and weight update accounted for, the pilot–aided non–resampling SMC algorithm for the MIMO–OFDM system described in Section II proceeds as outlined in Table I. The \( a \) \textit{posteriori} probability of symbol \( d_{i,c} \) is then made according to:

\[
\Pr(d_{i,c} = a_m \mid Y_i) = \Pr(S_{i,c} = a_m \mid Y_i) = \sum_{a_m} \frac{1}{W_{i,c}} \sum_{q=1}^{Q} \left( \prod_{d=0}^{N_g-2} \Pr(S_{c,d}^{(q)} = a_{c,d} \mid Y_{i,d}) \right) w_{i,c}^{(q)} \quad (12)
\]

\[
\begin{array}{|c|c|c|}
\hline
0. Initialization: Obtain channel estimates \( \hat{H}_i \in \mathbb{R}_{N_g \times k_{\text{max}}} \), \( i = 0, \ldots, N-1 \) and initialize all importance weights to \( w_{i,c}^{(q)} = 1 \), \( q = 1, \ldots, Q \). & \\
\hline
1. For each \( a_m \in A_{\text{data}} \), compute the following using the results in (6a)–(9c):
\begin{align*}
&f(Y_i, S_{i,\text{pilot}}^{(q)} \mid Y_{\text{r}}, \hat{H}, S_{\text{pilot}} = a_m) \cdot \Pr(S_{i,c} = a_m \mid S_{i-1,c}^{(q)}) \\
&\alpha_i^{(q,m)}
\end{align*}
\end{array}
\]

2. Update the symbol \( S_{i,c}^{(q)} \in A_{\text{env}} \) with probability:

\[
\Pr(S_{i,c} = a_m \mid S_{i,c}^{(q)} = a_m) \cdot \alpha_i^{(q,m)}
\]

3. Compute the importance weight recursively: \( w_{i,c}^{(q)} = w_{i,c}^{(q-1)} \cdot \sum_{a_m} \alpha_i^{(q,m)} \)

where \( a_m \in A_{\text{data}} \), \( \sum_{a_m} \alpha_i^{(q,m)} = 1 \), \( \sum_{a_m} w_{i,c}^{(q)} = 1 \)

The \( a \) \textit{posteriori} probabilities are then passed to the SISO channel decoder. After the desired number of outer (turbo) iterations between the decoder and detector, the SISO decoder makes a hard decision on the symbols received.

\[ I(X = a) = \begin{cases} 1, & \text{if } X = a \\ 0, & \text{otherwise} \end{cases} \quad (13) \]

The IV. Simulation Results

In this section, simulation results are provided to show the performance of our non–resampling SMC detector for the coded MIMO–OFDM system described in Section II. We consider a system with \( N_r = N_g = 4 \), modulation is QPSK (thus \( M = 4 \)), and where the termination period \( K \) is set to 3. Each of the \( N_r \cdot N_g \) channels has a bandwidth of 800 kHz that is divided into \( N = 64 \) sub–channels. To make the tones orthogonal to each other, a symbol duration of 80\( \mu s \) and an additional 20\( \mu s \) guard interval is used to provide protection from ISI due to multipath delay spread. This results in a total block length of 100\( \mu s \) and a sub–channel symbol rate of 10 kbits per second. Channels with uniform (UNI), typical urban (TU), and hilly terrain (HT) delay profiles [9], and a Doppler frequency of 40 Hz are used to represent various mobile environments. Each of these channels has order \( L = 3 \) and delay spreads of UNI, HT and TU profiles are \( \tau_d = 1.06 \mu s \), \( \tau_d = 5.04 \mu s \), and \( \tau_d = 1.06 \mu s \), respectively. It is also assumed that the coefficients of the channel’s tap delay line are wide sense stationary narrowband complex Gaussian random processes with uncorrelated scattering (WSSUS) of band–limited Doppler power spectral density following Jakes’ model [10]. In this paper, the channel estimates \( \hat{H}_i \) are obtained using the minimum mean square error parameter estimator of [11].

\[ \hat{H}_i \] are obtained using the minimum mean square error parameter estimator of [11].
We first consider a convolutional coded system where the outer channel encoder is of rate half and constraint length 5 with generators $[23\ 35]$, while the SISO channel decoder is the maximum a posteriori (MAP) decoding algorithm [12]. This channel encoder is fed with a user frame of 170 bits which spans a single MIMO–OFDM symbol. We then make Bayesian inference of the unknown data symbols using our detector. The number of turbo iterations is set to 4 and Fig. 2 shows the performance of the UNI, TU, and HT profiles. It is observed that the delay profiles perform within 1dB of their respective channel bounds. These bounds are obtained by providing perfect channel state information (CSI) to the receiver. An LDPC–coded MIMO–OFDM system is also investigated. In this paper, the LDPC code is of rate half and encodes a block of 682 information bits. The column weight of the parity check matrix is 3 and the row weights are randomized. Care is taken to ensure that the equivalent factor graph representation of the code has no cycles less than or equal to 4. The SISO decoder is the message passing algorithm [13], which iterates at most 100 times. The number of turbo iterations, on the other hand, is set to 6. Fig. 3 shows that the performance of all profiles is approximately 0.75dB away from their respective channel bounds.

V. CONCLUSIONS

In this paper, we show how the SMC methodology can be used for pilot–aided symbol detection in a MIMO–OFDM system. Simulation results suggest the promising performance of our detector for both convolutional–coded and low–density parity check (LDPC)–coded MIMO–OFDM systems.

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