Semi-Blind Channel Estimation for Space-Time Coded Amplify-and-Forward Relay Networks

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Abstract—In this paper, we propose a semi-blind channel estimation algorithm for amplify-and-forward (AF) relay networks. The algorithm fits well for the recently developed space-time coding (STC) technique in AF relay network that serves for small size terminal and achieve the transmission diversity. Compared to the optimal training based estimators, e.g., maximum likelihood (ML) or linear minimum mean square (MMSE), the proposed semi-blind approach requires less training for successful channel estimation, or it yields better estimates if the same amount of training is used. The channel ambiguity issue as well as its relationship with the traditional semi-blind method is discussed in detail. We then provide various numerical examples to corroborate the proposed studies.

I. INTRODUCTION

Wireless relay network is emerged as a solution to realizing the long distance transmission and achieving the additional diversity for small-size terminals, e.g., cell phone, laptop, where only one or two antennas can be equipped [1]–[3].

The system configuration contains one source terminal, one destination terminal, as well as multiple relay nodes staying in between. The transmission process is usually divided into two phases. In Phase I, the source broadcasts its information signals to all relays. In Phase II, the relays would either choose to purely amplify and retransmit the information to the destination, or to decode the information first and then transmit these information bits to the destination. The former process is referred to as amplify-and-forward (AF) while the latter is referred to as decode-and-forward (DF). Although the DF protocol works similar to the traditional point-to-point multiple-input multiple-output (MIMO) system, the AF one demands careful study due to the mixture of the two phase during the overall transmission.

To realize multiple access and to achieve the spatial diversity in AF relay network, one could resort to the recent work [4], where a space-time coding (STC) technique is applied during the second phase. The training based channel estimation that serves for exactly the same scheme has been proposed in [5], where it points out an important concept that the channels used for data detection in AF relay network is the cascaded channels from the first and the second phase, while the way how to cascade the channels depends on the STC protocols.

Therefore, it is crucial that the estimation be embedded into the data transmission so that the estimated channels are the same as those used for data detection.

Similar to the traditional point-to-point MIMO, sending training frequently in AF relay networks will reduce the bandwidth efficiency. Therefore, we develop a semi-blind approach for channel estimation. Compared to the training based algorithm [5], our proposed approach can produce effective channel estimation with a smaller amount of training, or it can yield better channel estimation with the same amount of training as long as the number of the received data blocks within channel coherence time is large.

II. SYSTEM MODEL

Consider a wireless network with $M$ randomly placed relay nodes $R_i$, $i = 1, \cdots, M$, one source node $S$, and one destination with $N$ co-located receive nodes $D_j$, $j = 1, \cdots, N$, as shown in Fig. 1. Each node has only one antenna that cannot transmit and receive simultaneously. The channel between each node pair is assumed quasi-stationary Rayleigh flat fading; namely, channels do not change during one frame of transmission, where a frame usually contains tens or several hundred of data blocks. Denote the channel from $S$ to $D_j$ as $f_j$, etc.

Fig. 1. Wireless relay networks with one source, $M$ relays, and $N$ destinations.
from $\mathbb{S}$ to $\mathbb{R}_j$ as $g_i$, from $\mathbb{R}_j$ to $\mathbb{D}_j$ as $h_{i,j}$, respectively, namely $f_j \in \mathcal{C}\mathcal{N}(0, \sigma_f^2)$, $g_i \in \mathcal{C}\mathcal{N}(0, \sigma_g^2)$, and $h_{i,j} \in \mathcal{C}\mathcal{N}(0, \sigma_{h_{i,j}}^2)$. The transmission is completed in two phases, containing $T_1$ and $T_2$ consecutive time slots, respectively. Note that $T_2$ is greater than or equal to $T_1$, and the the value of $T_1/T_2$ depends on the rate of the STC that will be used during the second phase.

For phase I, $\mathbb{S}$ broadcasts the signal block $s = [s_1, \ldots, s_{T_1}]^T$ to $\mathbb{D}_j$ to both relays and the destinations. The received signal at $\mathbb{R}_j$ and $\mathbb{D}_j$ could be expressed as

$$r_i = g_i s + n_{ri}, \quad i = 1, \ldots, M,$$

(1)

$$d_{ij} = f_j s + n_{dij}, \quad j = 1, \ldots, N,$$

(2)

where $n_{ri}$ and $n_{dij}$ are the vectors representing the circularly complex white Gaussian noise at the relays and the destination, respectively. For simplicity, All noise variances are assumed as $\sigma_n^2$, namely $n_{ri}, n_{dij} \in \mathcal{C}\mathcal{N}(0, \sigma_n^2 I)$. The power constraint of the transmission is $\mathbb{E}[|s|^2] = TP_s$, where $P_s$ is the average transmitting power of the source. To exploit the diversity of the relay system, $r_i$ is first precoded by a random matrix $P_i$ with $\text{tr}(P_i P_i^H) = T_2$, and is then scaled by a real factor $\alpha_i$ to keep the average power of $r_i$ as $P_r$. To enable complex space time code (STC), the precoding scheme is [4]

$$t_i = \alpha_i P_i r_i^{(s)}$$

(3)

where

$$\alpha_i = \sqrt{\frac{P_r}{\sigma_n^2 + \sum \sigma_g^2}}$$

(4)

and $(\cdot)^{(s)}$ represents the item itself if the $i$th relay operates on $r_i^s$; otherwise it represents the conjugate of the item if the $i$th relay operates on $r_i^s$. Note that, one relay node should exclusively work on $r_i^s$ or $r_i^r$ to fit in the STC scheme in [4], [6].

III. SUBSPACE BASED SEMI-BLIND CHANNEL ESTIMATION

The semi-blind channel estimation for $f_j$’s can be readily obtained from [7], [8] since it is exactly the same as the traditional point-to-point system. The algorithm will be omitted here. We will only focus on the estimation of the cascaded channels.

A. Remodeling of the received signal

The destination $\mathbb{D}_j$ in phase II then receives

$$d_{2j} = \sum_{i=1}^{M} h_{i,j} t_i + n_{d2j}$$

(5)

where $n_{d2j} \in \mathcal{C}\mathcal{N}(0, \sigma_n^2 I)$ represents the noise vector at $\mathbb{D}_j$ during the second phase, and $w_{i,j} = h_{i,j} g_i^{(s)}$ denotes the cascaded channels from $\mathbb{S}$ to $\mathbb{R}_i$ and then to $\mathbb{D}_j$. From [5], we know $w_{i,j}$’s are the channels required for data detection. We further define $w_j = [w_{1,j}, \ldots, w_{M,j}]^T$ for future use. Furthermore, $s_i \triangleq s$ is defined for notation clarity that specifically indicate the data at the $i$th relay.

Rewrite $d_{2j}$ in the following equivalent form

$$d_{2j} = \begin{bmatrix} X_{1,j} & X_{2,j} \end{bmatrix} \begin{bmatrix} (s)R \\ (s)_I \end{bmatrix} + n_{dj},$$

(6)

where $(\cdot)R$ and $(\cdot)_I$ denotes the real and the imaginary part of the argument. Moreover

$$X_{1,j} = \sum_{i=1}^{M} w_{i,j} \alpha_i P_i, \quad X_{2,j} = J \sum_{i=1}^{M} \mathcal{I}(i) w_{i,j} \alpha_i P_i$$

$$n_{dj} = \sum_{i=1}^{M} h_{i,j} \alpha_i P_i n_{ri}^{(s)} + n_{d2j}$$

where $J = \sqrt{-1}$ is the imaginary unit, and $\mathcal{I}(i)$ is the sign indicator, defined as

$$\mathcal{I}(i) = \begin{cases} +1, & \text{if the } i\text{th relay operates on } r_i \\ -1, & \text{if the } i\text{th relay operates on } r_i^* \end{cases}$$

(7)

We then stack $(d_{2j})_R$ and $(d_{2j})_I$ together to create a real vector $\tilde{d}_{2j}$ as

$$\tilde{d}_{2j} = \begin{bmatrix} (d_{2j})_R \\ (d_{2j})_I \end{bmatrix} = \begin{bmatrix} (X_{1,j})_R \\ (X_{1,j})_I \\ (X_{2,j})_R \\ (X_{2,j})_I \end{bmatrix} \begin{bmatrix} (s)R \\ (s)_I \end{bmatrix} + \begin{bmatrix} (n_{dj})_R \\ (n_{dj})_I \end{bmatrix},$$

(8)

where

$$(X_{1,j})_R = \sum_{i=1}^{M} \alpha_i (w_{i,j} P_i)_R,$$

$$(X_{1,j})_I = \sum_{i=1}^{M} \alpha_i (w_{i,j} P_i)_I,$$

$$(X_{2,j})_R = -\sum_{i=1}^{M} \alpha_i \mathcal{I}(i)(w_{i,j} P_i)_R,$$

$$(X_{2,j})_I = \sum_{i=1}^{M} \alpha_i \mathcal{I}(i)(w_{i,j} P_i)_I,$$

and $\tilde{X}_{1,j}$, $\tilde{X}_{2,j}$ denote the corresponding items. Meanwhile, it can be readily checked that

$$(w_{i,j} P_i)_R = (w_{i,j} P_i)_R - (w_{i,j})_I (P_i)_I,$$

(9)

$$(w_{i,j} P_i)_I = (w_{i,j} P_i)_I + (w_{i,j})_I (P_i)_R.$$
Define $Q_t = [\alpha_1 P_1(:, t), \ldots, \alpha_M P_M(:, t)]$. The $t$-th column of $\tilde{X}_{1,j}$ can be represented as

$$\tilde{X}_{1,j}(t) = \begin{bmatrix} (Q_i)_{R} - (Q_i)_{I} \\ (Q_i)_{R} - (Q_i)_{I} \\ \vdots \\ (Q_i)_{R} - (Q_i)_{I} \end{bmatrix} \begin{bmatrix} w_i \gamma \end{bmatrix} R, \quad t = 1, \ldots, T_1.$$  

(11)

Define $F = \text{diag}\{I(1), I(2), \ldots, I(M)\}$ as the $M \times M$ diagonal matrix. The $t$-th column of $\tilde{X}_{2,j}$ can be represented as

$$\tilde{X}_{2,j}(t) = \begin{bmatrix} - (Q_i)_{I} \\ - (Q_i)_{R} \\ \vdots \\ - (Q_i)_{I} \end{bmatrix} \begin{bmatrix} \gamma \end{bmatrix} F 0 F \tilde{w}_j, \quad \text{where} \quad D_{2,j}.$$  

(12)

Finally, we stack $d_{2j}$, $j = 1, \ldots, N$, together, and get the new received data vector $\tilde{d}_2$ for one block transmission as

$$\tilde{d}_2 = \begin{bmatrix} d_{21}^T \\ \vdots \\ d_{2N}^T \end{bmatrix} = \begin{bmatrix} \tilde{C}_1 \tilde{w} + \tilde{D}_1 \tilde{w} \\ \vdots \\ \tilde{D}_t \tilde{w} \end{bmatrix} \begin{bmatrix} \gamma \end{bmatrix} + \tilde{n}_d,$$  

(13)

where

$$\tilde{C}_t = \text{diag}\{C_{t,1}, \ldots, C_{t,N}\},$$

$$\tilde{D}_t = \text{diag}\{D_{t,1}, \ldots, D_{t,N}\},$$

$$\tilde{w} = \begin{bmatrix} \tilde{w}_1^T \\ \vdots \\ \tilde{w}_N^T \end{bmatrix},$$

$$\tilde{n}_d = \begin{bmatrix} \tilde{n}_{d1}^T \\ \vdots \\ \tilde{n}_{dN}^T \end{bmatrix}.$$  

Due to the circularity of the noise vector, the covariance of $\tilde{n}_d$ is still an scaling of the identity matrix.

### B. Subspace based channel estimation

The subspace method could be applied to (13) if (1) the $2T_2 N \times 2T_1$ matrix $A$ is tall; (2) Matrix $A$ is full rank. The first condition is satisfied for any value of $N$ if $T_2 > T_1$, i.e., non-full rate STC, and is satisfied for full rate STC if $N > 1$. The second condition is satisfied most of the time since $\tilde{w}$ are random variables. The identifiability problem will be discussed in the next subsection.

Then, the covariance matrix of $\tilde{d}_2$ is derived from

$$R = E\{\tilde{d}_2 \tilde{d}_2^T\} = A R_s A^T + \sigma_n^2 I_{2MN},$$  

(14)

where $R_s = E\{\tilde{w} \tilde{w}^H\}$ is the source covariance matrix. The covariance matrix $R$ can be eigen-decomposed as

$$R = U_s A_s U_s^T + \sigma_n^2 U_n U_n^T,$$  

(15)

where $A_s$ is the $(2T_2 N - 2T_1) \times (2T_2 N - 2T_1)$ diagonal matrix that contains the largest $(2T_2 N - 2T_1)$ eigenvalues while $U_s$ is the matrix that spans the signal subspace. In turn, $U_n$ spans the noise subspace of $R$. The standard subspace method [7] says that $U_n$ is orthogonal to every column of $A$. This can be expressed as

$$U_n^T C_1 \tilde{w} = 0, \quad U_n^T D_t \tilde{w} = 0, \quad t = 1, \ldots, T_1.$$  

(16)

which is equivalent to

$$\tilde{w}^T \sum_{i=1}^{T_1} C_i^T U_n U_n^T C_i + \sum_{i=1}^{T_1} D_i^T U_n U_n^T D_i \tilde{w} = 0.$$  

(17)

The channel vector $\tilde{w}$ can be obtained from the eigenvectors of $Q$ that corresponds the zero eigen-values.

### C. Channel Identifiability

Through the simulations, we found that the multiplicity of the zero eigen-values of $Q$ is always 2 for random precoding. This result is, in fact, not surprising since we stack the real part and the imaginary and construct a real covariance. In traditional subspace based algorithm [7], a complex vector $z$ is directly estimated and the identifiability is guaranteed when the estimate values $\hat{z}$ satisfies

$$\hat{z} = \gamma z,$$  

(18)

where $\gamma$ is an unknown complex scalar. The above equation is equivalent to

$$(\hat{z})_R = (z)_R (\gamma)_R - (z)_I (\gamma)_I,$$

$$(\hat{z})_I = (z)_I (\gamma)_R + (z)_R (\gamma)_I, $$

or

$$\begin{bmatrix} (\hat{z})_R \\ (\hat{z})_I \end{bmatrix} = \begin{bmatrix} (z)_R \\ (z)_I \end{bmatrix} (\gamma)_R + \begin{bmatrix} (z)_I \\ (z)_R \end{bmatrix} (\gamma)_I.$$  

(19)

This says that the scalar ambiguity in complex estimate indicates two-dimensional ambiguity in the real equivalent.

One example is given here. Consider the specific case when $F = I$, i.e., the STC proposed in [4]. It can be readily checked that if one column of $U_n$ is $[u_1^T, u_2^T]^T$, where $u_1$ and $u_2$ both contain $NT_2$ elements, then $[-u_2^T, u_1^T]^T$ is also one column in $U_n$. Denote any vector in the noise subspace of $Q$ as $q$ and divide it as $\tilde{q} = [q_1^T, q_2^T]^T$. Bearing in mind the special structure of $U_n$, $C_t$, and $D_t$, it can be checked from (16) that $[-q_1^T, q_2^T]^T$ also stays in the noise subspace of $Q$. The derivation is quite tedious and is rather omitted here. Therefore, the dimension of the corresponding noise subspace is 2.

However, for a general $F$, the basis vectors in the noise subspace of $Q$ may not have elegant relationship. Nevertheless, the ambiguity having dimension of 2 should never be a surprising result.

**Remark:** When we remodel a complex system into a real one, the traditional definition of identifiability is no longer applicable. In fact, even when the estimated result is a scaling of the true channel, this unknown scaling factor should be solved by sending some training symbols. In this sense, it is not that important to stick to the definition of identifiability itself, because even if the severe ambiguity happens, one can still solve it via sending training symbols. Compared to purely training based estimation, the amount of training used to remove the ambiguity is usually smaller. Further discussion of the identifiability is beyond the scope of this paper and will be approached in the future works.
D. Ambiguity Removal

The multiplicity of the 0 eigen-values of Q being 2 says that the \( \hat{w} \) lies in the space spanned by the two eigenvectors that correspond to the two smallest eigen-values. Denote these two eigenvectors as \( \hat{w}_1 \) and \( \hat{w}_2 \) and define \( H = [\hat{w}_1, \hat{w}_2] \).

The true channel vector can be expressed as

\[
\hat{w} = [v_1, v_2] \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} = H\beta,
\]

where \( \beta = [\beta_1, \beta_2]^T \) contains the two unknown real scalars that will be determined via training symbols.

Rewrite the received signal in the following form:

\[
\tilde{d}_2 = \left[ C_1 H \beta, \cdots, C_T H \beta, D_1 H \beta, \cdots, D_T H \beta \right] s + \tilde{n}_d
\]

\[
= \left( \sum_{i=1}^{T_1} (s_i)_R C_i H + \sum_{i=1}^{T_1} (s_i)_I D_i H \right) \beta + \tilde{n}_d.
\]

Two approaches to determine \( \beta \) are introduced here. The first one is similar to the traditional approach where one signal block \( s \) is devoted to training [8]. Then, the least-square (LS) estimation of the ambiguity can be found as

\[
\hat{\beta} = (B^H B)^{-1} B^H.
\]

Note that there are only two unknown real scalars \( \beta_1 \) and \( \beta_2 \) while \( B \) is always a tall or square matrix if \( T_2 \geq 1 \). Therefore, theoretically it is also possible to use the minimum amount of training to resolve the ambiguity. If only one training symbol is sent from \( S \), the precoding \( P_i \)'s at relay then reduce to different phase rotation of the received symbols. The received signal at destinations could be expressed in a similar way as (21) but with corresponding terms redefined to have different dimensions. Once again, the LS estimation of \( \beta \) can be obtained.

IV. SIMULATIONS

In this section, the performance of the proposed subspace based channel estimation algorithm is examined. The channels \( g_i, h_{i,j} \) and the noises are taken as circularly symmetric complex Gaussian random variables with unit variances. The channel covariance matrices \( R_{h_{i,j}} \) and \( R_{g} \) have the following structures [9]

\[
[R_{g}]_{a,b} = \varepsilon_1^{a-b}, \quad [R_{h_{i,j}}]_{a,b} = \varepsilon_2^{a-b},
\]

where \( [A]_{i,j} \) denotes the \( (i,j) \)th entry of \( A \). In all the examples, we set \( \varepsilon_i = 0.1 \). The symbols are taken from QPSK constellations and the signal to noise ratio (SNR) is defined as, \( \text{SNR} = P_s/\sigma_n^2 = P_s \). The estimation mean square error (MSE) is defined as

\[
\text{MSE} = \frac{1}{MN} \| \hat{\beta} - \beta \|^2_F
\]

where \( \hat{\beta} \) is the estimate of \( \beta \), and \( \| \cdot \|^2_F \) denotes the Frobenius norm. Moreover, the factor \( MN \) is used for normalization. The sample covariance matrix \( \bar{R} = \frac{1}{\pi} \sum_{k=1}^{K} \tilde{d}_2 \tilde{d}_2^H \) is used for the decomposition, where \( K \) is the number of the received data blocks within the channel coherence time. In all examples, we take \( M = N = 4 \) and 10000 independent Monte-Carlo runs are adopted to average the results.

In the first example, we examine the performance of the proposed method with random precoding at different relays. All relay powers are set the same as \( P_s \). The two eigenvectors of \( Q \) that correspond to the two smallest (non-zero) eigen-values are obtained as the basis vectors for the noise subspace, whereas one training block is used to remove the estimation ambiguity. The performance MSEs versus SNR are displayed in Fig. 2 for different SNRS and different number of the received data blocks. For Fig. 2, it is seen that, the estimation accuracy improves when SNR and the number of blocks increase. This conclusion is consistent with the general one in the traditional algorithms [7], [8].

Since the relay nodes may come from different devices and
corresponding effect in channel estimation errors. We consider may have different powers, it is then of interest to check the LS method and MMSE method.

$K$ data block is as small as $K = 20$ and relay powers are equal to each other. The ambiguity is removed by four training blocks in our proposed algorithm while these 4 blocks are directly used for training with the LS and MMSE estimator. For training based estimation, the precoding involved in QOSTC structure is already optimal [5] but it may not be optimal for the proposed semi-blind approach. The estimation MSEs versus SNR are shown in Fig. 5. It is clearly seen that the proposed algorithm yields better performance than the training based algorithm. Moreover, from the first example we know the performance of the blind approach improves when $K$ increases, but the training methods will not. So, the different between the semi-blind approach and the training based approach will be enlarged as long as more data blocks can be received within the channel coherence time.

V. CONCLUSIONS

In this paper, we develop a semi-blind channel estimation for STC based AF relay networks. We reformulate the complex model into real one and discuss important issues of identifiability and the removal of estimation ambiguity. The proposed algorithm is effective with only one training symbol, or it out-performs the optimal training based channel estimations especially when the number of the received blocks is large. Numerical examples are provided to validate the proposed studies.

REFERENCES