THICKNESS DETERMINATION OF A PLATE WITH VARYING THICKNESS USING AN ARTIFICIAL NEURAL NETWORK FOR TIME-FREQUENCY REPRESENTATION OF LAMB WAVES

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ABSTRACT. Thickness estimation of a varying-thickness media is carried out using an algorithm acting as an artificial neural network for time-frequency representation (TFR) of Lamb waves. Dispersion curves are reconstructed using a self adjustable network multi-input fuzzy rules emulated network (MIFREN). The uncertainty in the time-frequency determination is compared with a typical spectrogram technique. The proposed algorithm is computationally less complex than others used in the past. Experimental results were obtained by exciting Lamb waves on an aluminum plate with varying thickness; these were compared with numerical estimations.

Keywords: Lamb Waves, Dispersion Curves, Time-Frequency Representation, Fuzzy Networks

PACS: 43.58.+z, 43.60,+d

INTRODUCTION

Ultrasonic guided waves have shown potential for use in long range inspections needed for evaluation of large areas in engineering structures. In like-plate structures, corrosion can affect thickness, producing variations that can influence the wave propagation. Lamb waves can be generated by interference and mode conversion of longitudinal and shear waves, resulting in complex pulse propagation even in the simple case of a uniform thickness plate [1, 2]. The propagated signal is in most practical cases multimodal and the amplitude and dispersion properties of each mode depends on the presence of discontinuities or impedance changes. For practical operations, an adequate technique to evaluate thickness variation like the one found in degraded engineering structures using Lamb waves has not yet been established [3,4].

Recently, the problem of guided wave mode conversion in a plate with varying thickness has been addressed using finite element methods [5, 6]. The scattering problem is more complex when compared with a uniform thickness single plate problem. Transmitting modes of Lamb waves propagating on a plate with varying thickness can in principle be affected by the complex scatter field due to thickness reduction and edge reflection. The presence of reflected modes can have different wave propagation
properties. This makes it very difficult to analyze guided wave propagation analytically. A numerical solution based on finite element of the transmitted and reflected guided waves was reported in [4-6] only for the lower modes. However, finite element methods are only approximate solutions for wave propagation in a media with discontinuities or boundaries. There is a need of a more appropriate guided wave analysis to study the Lamb waves’ sensitivity to a system with varying thickness.

Successful application of Lamb waves for damage detection depends on the appropriate method used to understand guided wave propagation. An alternative solution to the model based or numerical approach is to characterize the behavior of time-frequency curves, signature of multi-mode Lamb waves. The problem has been addressed using different techniques including time-frequency characterization [7, 8]. Techniques developed to study Lamb waves’ signature, besides the basic spectral analysis are directed to overcome FFT limitations for correct analysis of the non-stationary signal properties of guided waves. Efforts have been made in different research areas to develop accurate time-frequency methods that can give a better, more precise, representation of non stationary signals. Recent applications of spectrograms, Wigner-Ville distribution, and wavelets for representation of non-stationary signals have been reported in scientific literature [9, 10]. Non-parametric methods using neural network and fuzzy networks to solve optimization problems have been reported in other research areas [11], which can be applied to optimize time-frequency representation of Lamb waves.

This work has several objectives, first to develop a self adjustable network multi-input fuzzy rules emulated network (MIFREN) on the time-frequency characterization of dispersive Lamb wave modes; second, to compare results of MIFREN with typical spectrogram Wigner-Ville Distribution (WVD) techniques; and finally, to test and implement the MIFREN algorithm on an aluminum plate with step varying thickness.

TIME-FREQUENCY REPRESENTATION

Over the past, Fourier Transform has been widely used in several realms of engineering, however, limitation of this technique are presented for nonstationary signals and nonlinear systems. To solve the problem, studies in the development of alternative time-frequency methods have been carried out. The use of methods such as short-time frequency transform (STFT), or the Wigner-Ville Distribution (WVD) for analyzing Lamb waves have been reported [7, 8].

The basic idea of STFT is to segment the signal \( s(t) \) by using a window \( w(t) \) with a selected shape and to determine the frequencies’ content in that window at a certain location. STFT can be expressed by

\[
STFT(t, \omega) = \int_{-\infty}^{\infty} s(t)w(t)e^{-j\omega \tau} d\tau.
\]

A different approach is the so called Wigner-Ville Distribution (WVD) [9] given by

\[
WVD(t, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} s(t-\frac{\tau}{2})s(t+\frac{\tau}{2})e^{-j\omega \tau} d\tau.
\]

WVD uses a quadratic operation applied to the signal and then a linear operation (FFT) is carried out. It is basically the convolution of the signal by its complex conjugate (mirror image) at every instant \( t \). Its main advantage is the absence of a window as in the case of STFT. A limitation of applicability is the presence of cross-terms or interference [9,10], which limits the interpretation of the time-frequency diagrams.
In Figure 1, examples of application of STFT and WVD are shown for a chirp signal. The results of STFT (Figure 1b) were obtained using a square window manually optimized to show the better resolution. In terms of time-frequency resolution of the simple chirp signal, WVD (Figure 1c) shows an improvement. However, the presence of interference is evident, clearly shown below the main signal distribution at begin and end. Thus, the interpretation of more complex signals will be more difficult.

**MIFREN Time-Frequency Representation Algorithm**

Here, we introduce the time-frequency presentation technique based on MIFREN. The structure of MIFREN is explained in detail. For a fuzzy inference system with n inputs where each input has r fuzzy state, the total number of fuzzy rules is then $r^n$. Each fuzzy If-Then rule can be represented by

Rule $k$: IF $(I_1$ is $A_{k1,1})$ and $(I_2$ is $A_{k2,2})$… and $(I_n$ is $A_{kn,n})$ THEN $O_k$ = $B_k$,

where $I_j$ is the value of the $j$-th input. $A_{ij}$ denotes the $i$-th fuzzy state for $j$-th input, since there is $r$ possible fuzzy state. We relate the $k$-th rule and $O_k$ is the fuzzy output of this rule which belongs to the fuzzy set $B_k$. After all rules have been processed, the crisp output $O$ is obtained from some defuzzification schemes.

MIFREN is derived based on these fuzzy rules and its structure can be decomposed into 5 layers as shown in Figure 2.

The function of each layer is as follows:

- **Layer 1**: Each input $I_j (j=1,...,n)$ in this layer is sent to the corresponding nodes in the next layer directly. Thus there is no computation in this layer.
FIGURE 2. The structure of MIFREN.

- **Layer 2**: This is called the input membership function (MF) layer. Each node in this layer contains a membership function corresponding to one linguistic level (e.g., negative, nearly zero, etc.). The output at the \( i \)-th node for the input \( I_j \) is denoted by \( \mu_{a_i,j} \).

- **Layer 3**: This layer corresponds to the fuzzy inference. The number of nodes in this layer is \( r^n \) nodes. The output signal at each node in the layer is calculated as

\[
f_k = \prod_{j=1}^{n} \mu_{a_i,j}.
\]  

(3)

- **Layer 4**: This layer may be considered as a defuzzification step. It is called the Linear Consequence (LC) layer. There are also \( r^n \) nodes in this layer. The output at the \( k \)-th node in this layer is calculated by

\[
O_k = \beta_k f_k.
\]  

(4)

- **Layer 5**: The structure of this layer is similar to the output layer of an artificial neural network with unity weight. The output of the MIFREN, \( O \), is calculated by

\[
O = \sum_{k=1}^{r^n} \beta_k f_k.
\]  

(5)

As will be seen in the computer simulation results, this decomposition into 5 layers enables the user to intuitively set the initial value of MIFREN's parameters.

FIGURE 3. Frequency-Time Representation: MIFREN.
The application of the MIFREN algorithm for the non-stationary Lamb waves is briefly detailed in Figure 3 with a block diagram.

The variable \( x(n) \) stands as the time domain signal which is discontinuous or signal with sampling interval. The output signal is represented by \( X(t, f) \) in the scale of time and frequency. Due to the system configuration, If-Then rules are needed to be defined with the human knowledge. Through these rules, the window parameter is determined for the transform unit at last. The transform equation is represented by

\[
X(t, f) = \sum_{k=0}^{N-1} x(k) w(k, t) e^{-\frac{2j\pi ft}{N}},
\]

where \( N \) denotes the number of length for the signal \( x(k) \) and \( w(k, t) \) is the window function shown in Figure 4.

Results of MIFREN for a synthetic chirp signal are given in Figure 1(d), it shows a comparable time-frequency resolution to WVD without the presence of interference.

**DESCRIPTION OF EXPERIMENTS**

Figure 5 illustrates the main features of the experimental setup used in this work. The system consists of an aluminum thin plate with a 2.00 mm thickness. Three steps with thickness reduction were machined on one side of the sample leaving a thickness of 1.8, 1.6 and 1.4mm as indicated by the figure. Two longitudinal wave 2.25 MHZ broadband transducers were used in a through-transmission arrangement. Excitation and reception of Lamb waves was carried out by pulser/receiver PANAMETRICS 5800. Signals were displayed and digitized by a digital oscilloscope tektronix TDS1012. Finally, post-processing of digitized signals FFT, STFT, WVD and MIFREN were implemented and performed using MATLAB®.

In the experiment, the distance between transducer was kept constant (23cm) while one of the transducers, the receiver, was located in location P1, P2, P3 as shown if Figure 6. Careful alignment of transducer to assure maximum signal reception was carried out.
RESULTS AND DISCUSSION

In Figure 7, dispersion curves and results of STFT are summarized for the four locations: without thickness reduction- no step, P1, P2, and P3 (see Figure 6). The results were compared with theoretical calculations estimated using corresponding thickness reduction. The STFT results for a uniform thickness plate agree quite well with theoretical results (solid lines) (Figure 7a), S1 and A1 modes are clearly recognized, and S0, A0 modes are fairly located. Additional guided wave modes are observed once one of the transducers is located in the first step with thickness 1.8 mm (Figure 7b). The expected modes for a uniform thickness plate fit only the lower modes S0, A0, and A1. S1 mode shows a shift with respect to the experimental one. It looks like a linear superposition of two S0 modes corresponding to thickness 2.0 and 1.8 mm. There is also, the presence of additional modes. These are detected at 1.8mm spectrogram but are more clearly observed in the 1.6mm spectrogram (Figure 7c). This indicates the presence of edge reflections, as the number of steps increase so do these patterns. For \( h=1.4 \) mm, there is no agreement between experimental results and the theoretical expected wave modes, with the exception of S0 at around 0.5 MHz. Edge reflections are shown in detail C (Figure 7c).

FIGURE 5. Window width calculation.

FIGURE 6. Description of experimental setup.
The results of applying MIFREN for $h=2.0\,\text{mm}$ and $1.8\,\text{mm}$ are given in Figure 8. MIFREN results (Figure 8b and d) exhibit a better resolution of the time-frequency description. The modes indicated in Figure 8b show an increase in resolution uncovering the presence of A2 and S1 modes. However, the improvement is not dramatic.

FIGURE 7. Results of test on position with thickness (a) $h=2.0\,\text{mm}$, (b) $h=1.8\,\text{mm}$, (c) $h=1.6\,\text{mm}$ and (d) $h=1.4\,\text{mm}$.

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FIGURE 8. Results of test on position with thickness (a) $h=2.0\,\text{mm}$, STFT; (b) $h=2.0\,\text{mm}$, MIFREN; (c) $h=1.8\,\text{mm}$, STFT, (d) $h=1.8\,\text{mm}$, MIFREN. Theoretical calculation for a plate thickness of 1.8mm and 2.0 mm are given by solid lines and dashed lines respectively.
An explanation of these results could be that with the proposed technique we need to define the suitable IF-THEN rules for MIFREN. In the case of a chirp signal illustrated in Figure 1, the results obtained by MIFREN are clearly improved when compared with the other techniques presented. The reason is that the chirp signal has the exact relationship between time and frequency, such that the frequency increases with the time scale. On the other hand, with experimental signals, we can not correctly guess the behavior of time and frequency relation. The suitable IF-THEN rules are not easily defined. Instead we used the energy of frequency broadcast inside the time slot to define the rules.

CONCLUSION

An algorithm based on a novel MIFREN technique was implemented on a thickness varying sample. Results of MIFREN technique gave reasonable time and frequency resolution. The incorporation of pre-processing of the signals could improve results of MIFREN. An advantage of the MIFREN is the ability to include human knowledge into the neural network. Then, MIFREN can be easily incorporated in a pattern recognition methodology which can be of benefit for practical detection of defects.

ACKNOWLEDGEMENTS

The authors want to thanks CONACYT for its financial support through project SEP-CONACYT # 58951.

REFERENCES