Power Control using Steffensen Iterations for CDMA Systems with Beamforming or Multiuser Detection

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Abstract—We present an accelerated power control algorithm applicable for CDMA based communications systems employing advanced uplink receiver techniques such as beamforming or multiuser detection. The proposed algorithm operates a fixed point power control algorithm accelerated by utilizing Aitken’s $\Delta^2$-process (also known as Steffensen’s method) that is merged with linear MMSE filtering. The linear MMSE filter either performs beamforming or multiuser detection. The proposed algorithm shows asymptotically quadratic convergence and is benchmarked against Newton’s method. We further evaluate a lower bound on the Lipshitz constant which can be used to ensure convergence of the proposed algorithm. Numerical results are given for a fully deployed sectorized UMTS network with MMSE multiuser detection. Beside its employment within CDMA based communication systems, the proposed algorithm can also be utilized for the acceleration of computational intensive network simulations.

Index Terms—Power Control, Multiuser Detection, Beamforming, Steffensen Iteration, Jakobi Iteration, Aitken’s Process, Lipshitz Continuous

I. INTRODUCTION

The evolution of third generation wireless mobile communications systems such as UMTS (Universal Mobile Telecommunication System) or the CDMA2000 system provides significant improvements in network performance. Evolving techniques for the uplink of code-division-multiple-access (CDMA) based communication systems are (1) beamforming and (2) multiuser detection. Receive beamforming has been intensively analyzed in the last few years, e.g. refer to [1], [2], [3] and references therein, all indicating a performance improvement for beamforming since it spatially suppresses inter/intra-cell interference. Multiuser detection is another approach to cope with high intra-cell interference. Multiuser detection has also been subject to an intensive research, e.g. [4], [5], and indicates significant performance improvements. Nowadays cellular networks apply power control in order to guarantee a certain Quality-of-Service (QoS) for all active users in the network. A typical quantity related to the users provided QoS is the signal-to-interference-plus-noise ratio (SINR). These SINR power control schemes can be centralized, see [6] and the references therein, or distributed [7], [8], whereas the distributed controller requires only local information of a local user and is therefore more attractive for application. A framework for the convergence of the generalized uplink power control problem was proposed by Yates in [8]. Ulukus and Yates extended in [9] that fundamental contribution to minimum mean squared error (MMSE) multiuser detection. A power control analysis of jointly applying multiuser detection and beamforming has been done in [10]. All power control algorithms analyzed can be classified to be fixed-point algorithms based on Jacobi iterations, that, in general, are characterized by a linear rate of convergence resulting in a usually slow convergence to the solution seeking for. Li and Galic introduced in [11] an acceleration method utilizing Aitken’s $\Delta^2$-process to speed up convergence of power updates. The application of Aitken’s process to linearly-convergent fixed-point iterations leads to the Steffensen method [12]. Li and Galic restricted their investigations to power control only, that is, neither multiuser detection nor beamforming was covered. Moreover, acceleration methods are characterized by smaller convergence regions compared to Jakobi iterations. Thus, if we accelerate power control we have to account for smaller convergence regions. The analysis of this item is completely missed in [11].

In this paper, we present a novel iterative algorithm where we merge fast power control utilizing Aitken’s $\Delta^2$ acceleration method [13] with MMSE filtering. In order to ensure convergence due to the smaller convergence regions in case of speeding up the power updates, we derive a lower bound on the Lipshitz constant that can be used for monitoring the convergence process. Results will be given for the case of multiuser detection\(^1\) indicating the convergence and the acceleration of the proposed algorithm. For benchmarking we also applied Newton’s method that is known to have at least quadratic convergence. Note that all algorithms of interest are investigated for a fully deployed sectorized UMTS network

\(^1\)Since the application of the proposed algorithm to beamforming is straightforward we skipped discussing beamforming in this paper.
with a UMTS 64 kbps data service [14].

The paper is organized as follows. In Section II we describe the standard power control iteration as well as the MMSE filtering. In Section III we introduce and describe the proposed acceleration algorithm. Next, in Section IV we present numerical results for a fully employed sectorized UMTS network, and in Section V we finally summarize the paper.

II. POWER CONTROL & FILTERING

Linear multiuser detection as well as beamforming can be interpreted as linear filtering of received user signals. In case of multiuser detection users are separated in a CDMA manner utilizing user specific signatures also denoted as spreading codes [5]. After matched filtering, often performed by RAKE reception [15], the signal to be detected still suffers from high multiple-access interference that depends for example on the mobile channel and spreading codes applied. With linear multiuser detection this multiple-access interference can be reduced leading to an overall improved system performance [5], [14].

With uplink beamforming, the users get a finger print in space domain and the user specific signature is given by the array response (or array steering vector) of the beamforming array [2], [1], [10].

The minimum mean squared error (MMSE) filter (either in code or space domain) is often applied and used for benchmarking of receiver structures. It minimizes the expected squared error between the transmitted signal and the output of the receive filter and also maximizes the output SINR [5].

In order to set up a generic view of linear filtering we describe the MMSE filter by a coefficient matrix \( W_{MMSE} \) which can be, in general, expressed by a generic function as

\[
W_{MMSE} = f(p, R, \sigma^2, H). \tag{1}
\]

The matrix \( R \) denotes the cross-correlation matrix of user specific signatures (either in code or in spatial domain), while \( \sigma^2 \) is the noise power and the matrix \( H \) composes the mobile channel properties between base stations and users. The interested reader can find detailed MMSE descriptions e.g. in [10], [4], [9], [5] and references therein. Most importantly, note that the MMSE filter solution depends on the received power values \( p = [p_1, \ldots, p_K]^T \in \mathbb{R}^K \) where \( K \) denotes the number of active users in the network. These received power values has to be adapted by power control in order to meet the QoS requirements. The transmitted power is directly related to the received power by the log-normally distributed shadow-fading path loss coefficient \( h_i \), i.e. \( p_i = \tilde{p}_i h_i \), with \( h_i \) as transmitted power of the \( i \)-th user. In the following, we refer the evaluation to the received power.

The goal of controlling the setting of the signal power values as well as the linear filters is to assign users with power levels so as to minimize the total power while ensuring a certain QoS. We determine the common QoS in terms of a signal-to-interference-plus-noise ratio (SINR) per user measured at the output of the linear filter. This SINR of user \( i \) is defined as

\[
SINR_i = \frac{\psi_{i,i} p_i}{\sum_{k \neq i} \psi_{i,k} p_k + \sigma_n^2}. \tag{2}
\]

The elements \( \psi_{i,k} \) describe the cross-coupling between the \( i \)-th user and \( k \)-th interferer after filtering and can be collected in the matrix \( \Psi \in \mathbb{R}^{K \times K} \). This matrix \( \Psi \) includes all system characteristics like user specific signatures (code or space domain) as well as the user specific mobile radio channel profile. Note that \( \Psi \) depends on the filter \( W \). Thus, matrix \( \Psi \) is a function of the filter \( W \), e.g. \( \Psi(W) \), where \( W \) is given by (1). Finally, \( \sigma_n^2 \) is the noise power. With this, we are ready to formulate the power control problem.

In what follows, inequalities between vectors to be understood componentwise. To minimize the total power consumption of users, we face the optimization problem

\[
\min_{W,p} \sum_{k=1}^K p_k \quad \text{s.t.} \quad 0 < p_k \leq p_{k,\text{max}} \tag{3}
\]

\[
SINR_k \geq \gamma_k,
\]

with \( k = 1, \ldots, K \). Thus, the minimization process is over all power vectors \( p \in \mathbb{R}_+^K \) as well as over all filter matrices \( W \). The values \( \gamma_k \) denote the target signal-to-interference-plus-noise ratios for every user \( k = 1, \ldots, K \). Provided that an interference function

\[
I_i(p, W) = \frac{\gamma_i}{\psi_{ii}(W)} \left( \sum_{k \neq i} \psi_{ik}(W) p_k + \sigma_n^2 \right), \tag{4}
\]

is associated with every user \( i \), Ulukus und Yates [9] showed that the optimization problem (3) is equivalent to

\[
\min_{p} \sum_{k=1}^K p_k \quad \text{s.t.} \quad p_k \geq \min_{W} I_k(p, W) \tag{5}
\]

for \( k = 1, \ldots, K \). Moreover, by mathematically proving \( \min_{W} I_k(p, W) \) to be a standard interference function [8] for MMSE filters, Ulukus and Yates showed in [9] that a solution \( p^* \) to (5) is unique and uniquely characterized by the fixed-point equation

\[
p^* = I(p^*, W^*) \tag{6}
\]

as long as at least one feasible point for (5) exists. The standard iteration, defined by \( p^{(0)} := 0 \in \mathbb{R}^K \) and

\[
p^{(t+1)} := I(p^{(t)}, W^{(t)}),
\]

converges strictly monotonic to \( p^* \), i.e.

\[
\lim_{t \to \infty} p^{(t)} = p^* \tag{8}
\]

Here, \( W^{(t)} \) (\( t = 0, 1, \ldots \)) is the \( t \)-th filter matrix in use, updated according to \( p^{(t)} \). Since the MMSE maximizes SINR it minimizes the interference function, that is \( W^* = W_{MMSE} \).

The standard iteration (7) usually converges slowly to the solution sought since simple fixed-point algorithms are likely
to have a linear rate of convergence [13]. Thus, especially for the application of CDMA networks supporting users with high mobility, power control acceleration methods are required. Li and Gajic [11] have proposed a Steffensen iteration power update [13] utilizing Aitken’s $\Delta^2$ acceleration technique. In the following, we apply the Steffensen method to the iterative update of power values and filter coefficients of the multiuser detection or beamforming technique respectively. Moreover, by using Banach’s theorem [12, Chapter 5], we propose a lower bound on the Lipshitz constant to estimate the convergence progress.

III. ACCELERATION ALGORITHM

Since the standard iteration (7) converges for all starting vectors $P^{(0)} \leq p^*$ or $P^{(0)} \geq p^*$ (as long as a solution $p^*$ to the fixed point problem (6) exists!), we know by results from standard iteration that the fixed point $p^*$ is attractive, i.e. there exists a neighborhood $U \subseteq \mathbb{R}^n$ of $p^*$ $(p^* \in U)$ and a real number $L$, $0 < L < 1$, such that we have $||I(p) - I(q)|| \leq L||p - q||$ for all $p, q \in U$. In other words, $I$ is locally Lipshitz continuous with Lipshitz constant $L < 1$. For clarity reasons, we shortly list Banach’s theorem.

Definition: Let $U \subseteq \mathbb{R}^K$ be a closed set and $I : U \rightarrow U$ be a function. The function $I$ is called Lipshitz continuous with Lipshitz constant $L \in \mathbb{R}$, if

$$||I(x) - I(y)|| \leq L ||x - y||$$

holds for all $x, y \in U$.

Theorem 1: Let $U \subseteq \mathbb{R}^K$ be a closed set and let $I : U \rightarrow U$ be a Lipshitz continuous function with Lipshitz constant $L < 1$. Then, the following holds:

1. The function $I$ has exactly one fixed point $p^* \in U$.
2. For all starting vectors $P^{(0)} \in U$, the standard iteration $P^{(t+1)} := I(P^{(t)})$ ($t = 0, 1, \ldots$) converges to $p^*$.
3. We have the a posteriori as well as the a priori error estimate

$$||P^{(t)} - p^*|| \leq \frac{L^t}{1 - L} ||P^{(0)} - P^{(1)}|| \leq \frac{L^t}{1 - L} ||P^{(1)} - P^{(0)}|| \quad (9)$$

for suitably chosen $P^{(0)}$, meaning that the sequence defined by (8) converges linearly.

Proof: See [12, Chapter 5].

We now turn our attention to the question how to accelerate the convergence speed of the process. For simplicity, we consider the case of one user. Define the error in step $t$ by $e^{(t)} := p^{(t)} - p^*$. According to Ulukus and Yates [9] the interference function (4) is standard, thus it is strictly monotonic. With (7) plus assuming convergence, we can write for large $t$,

$$e^{(t+1)} = p^{(t+1)} - p^* \approx L(p^{(t)} - p^*),$$
$$e^{(t+2)} = p^{(t+2)} - p^* \approx L(p^{(t+1)} - p^*).$$

Interpreting the two approximate equal signs as equality signs (for large $t$, the error will be small), we arrive at two equations for the unknowns $L$ and $p^*$. Eliminating $L$ results in the new estimate

$$p^* \approx \frac{p^{(t+2)} - (p^{(t+1)})^2}{p^{(t+2)} - 2p^{(t+1)} + p^{(t)}}$$

which, after some easy calculations, be written as

$$p^* \approx \frac{(p^{(t+1)} - p^{(t)})^2}{p^{(t+2)} - 2p^{(t+1)} + p^{(t)}} =: z^{(t)} \quad (10)$$

As a consequence, we can, for each $t$, define a new value $z^{(t)}$, thereby defining a new sequence $(z^{(t)})_t$. The process by which this sequence is derived is usually called Aitken’s $\Delta^2$-process. For the new sequence, we have the following convergence result.

Theorem 2: If the sequence $(p^{(t)})_t$ converges linearly to $p^*$, then the sequence $(z^{(t)})_t$ defined by (10) converges faster to $p^*$ in the sense that

$$\lim_{t \rightarrow \infty} \frac{z^{(t)} - p^*}{p^{(t)} - p^*} = 0.$$

Proof: See [12].

However, Aitken’s $\Delta^2$-process can not be easily employed algorithmically. Indeed, before $z^{(t)}$ can be computed, $p^{(t)}$ must be at hand, and therefore no work has been saved. Therefore, Aitken’s process has to be coupled directly with the given iteration itself resulting in Steffensen’s method [12]. In order to apply accelerated power control for CDMA systems with MMSE filtering, we finally have to merge Steffensen’s method with MMSE filtering leading to the following algorithm:

- Choose a suitable $P^{(0)} \in \mathbb{R}^K$.
- FOR $t = 0, 1, 2, \ldots$
- By using $p^{(t)}$, compute the filter matrix $W^{(t)}$ by applying (1). Then, execute one step of the standard power control algorithm (7), thereby finding a new power vector $p^{(t)} \in \mathbb{R}^K$: $\tilde{p}^{(t)} := I(p^{(t)}, W^{(t)})$.
- By using $\tilde{p}^{(t)}$, compute the filter matrix $\tilde{W}^{(t)}$ by applying (1). Then, execute one step of the standard power control algorithm (7), thereby finding a new power vector $\tilde{p}^{(t)} \in \mathbb{R}^K$: $\tilde{p}^{(t)} := I(\tilde{p}^{(t)}, \tilde{W}^{(t)})$.
- For $k = 1, \ldots, K$, define $p^{(t+1)}_k$ by means of the Aitken process applied to $p^{(t)}_k, z^{(t)}_k$, and $p^{(t)}_k$, i.e.

$$p^{(t+1)}_k := p^{(t)}_k - \frac{\left(\tilde{p}^{(t)}_k - p^{(t)}_k\right)^2}{\tilde{p}^{(t)}_k - 2\tilde{p}^{(t)}_k + p^{(t)}_k}.$$

Theorem 3: Suppose that the standard iteration has an order of convergence of 1. Then, the sequence constructed by Steffensen’s method has an order of convergence of at least 2.

Proof: See [12, Chapter 4].

This result means that, asymptotically, the number of correct digits of the approximation $p^{(t)}$ of $p^*$ is at least doubled in each step of Steffensen’s method. Contrary to this, linear convergence means that asymptotically the number of correct digits is increased by a constant in each step of the method.
Note that, in principle, the denominator in (10) might be zero. However, in the case of the standard iteration this can only happen at the fixed point, as the next theorem shows. Accordingly, the algorithm above is well-defined.

**Theorem 4:** Let \( f : [a,b] \rightarrow [a,b] \) be continuous and \( f'(\xi) \neq 1 \) for all \( \xi \in [a,b] \). Then \( f \) has at most one fixed point. Moreover,

\[
f(f(x)) - 2f(x) + x = 0
\]

holds if and only if \( x \) is the fixed point of \( f \).

**Proof:** (Cmp. also [16, Chapter 10].) Suppose that \( x, y \in [a,b] \) are two fixed points of \( f \) with \( x \neq y \). Then

\[
1 = \frac{f(x) - f(y)}{x - y} = f'(\xi)
\]

for some \( \xi \in [a,b] \), a contradiction to \( f'(\xi) \neq 1 \). Therefore, \( f \) has at most one fixed point. If \( x \) is a fixed point of \( f \), then \( f(f(x)) - 2f(x) + x = 0 \). On the other hand, let \( x \in [a,b] \) be given with \( f(f(x)) - 2f(x) + x = 0 \), i.e.

\[
f(f(x)) - f(x) = f(x) - x.
\]

Suppose now that \( x \) is not a fixed point. Then there exists a \( \xi \in [a,b] \) with

\[
1 = \frac{f(f(x)) - f(x)}{f'(x) - x} = f'(\xi),
\]

a contradiction. Therefore, \( x \) is a fixed point.

Let us note explicitly what is not stated in the last theorem. It is not stated that from an arbitrary starting point, Steffensen’s method will converge faster than the standard power iteration. The statement in the last theorem is a purely asymptotic one. Thus we have to face the problem, that an iteration method with higher convergence rate has generally the drawback of a smaller region of convergence. In the following, we introduce a lower bound on the Lipschitz constant to ensure convergence if we speed up the power updates in the proposed algorithm.

Suppose that the sequence \( (p^{(t)}) \), is produced by the standard iteration with linear convergence. Clearly,

\[
\|p^{(t)} - p^{(t-1)}\| \leq L\|p^{(t-1)} - p^{(t-2)}\|
\]

holds for all \( t \). As a consequence,

\[
L^{(t)} := \frac{\|p^{(t)} - p^{(t-1)}\|}{\|p^{(t-1)} - p^{(t-2)}\|}
\]

is a lower bound on \( L \), converging to \( L = \|\nabla f(p^*)\| \). Therefore, the quantity \( |L^{(t)} - L^{(t-1)}| \) can be used to estimate the progress the standard iteration has made towards the fixed point. If this quantity is small, i.e.

\[
|L^{(t)} - L^{(t-1)}| \leq \epsilon
\]

holds for some prespecified tolerance \( \epsilon \), we can then starting to apply the proposed algorithm instead of the standard iteration, thereby constructing a sequence converging faster than the original one. To the rest of the author’s knowledge, employing (11) is a crucial new aspect, which has not been used before. As outlined at the start of this section, linear convergence of the sequence \( (p^{(t)}) \), holds only in a neighborhood \( U \) of the solution \( p^* \), and it is of crucial importance to iterate into this neighborhood first, before employing convergence accelerating techniques. Indeed, by not using the check (11), we were able to observe nonconvergent sequences of power vectors, produced by Steffensen’s method! Note that Li & Gajic [11] also applied Steffensen’s method to power control problems, but apparently did not check the quality of the starting point employed by something akin to (11) or some similar approach.

**IV. Numerical Results**

In our numerical experiment, we applied multiuser MMSE-detection being subject to UMTS constraints. Thus, the results we worked out reflect UMTS performance gains. According to [14], we assumed a typical UMTS data service with a data rate of 64 kbps being associated with an required SINR of \( \gamma_k = 3.7 \) dB for \( k = 1, \ldots, K \). The results were obtained via statistically sufficient Monte-Carlo-Simulations of a fully deployed UMTS sectorized network. As noted before, the application to beamforming is straight forward and no additional insight in the proposed algorithm might be gained. Detailed description of the MMSE multiuser filter can be found e.g. in [5] and its application to UMTS was analyzed in [17], [14]. The initial power value was chosen as \( p_k^{(0)} := 10^{-15} \) Watt for all users \( k = 1, \ldots, K \) as an example. Moreover, for benchmarking purposes, we also implemented Newton’s method for finding a zero of the function \( F(p) := I(p, W(p)) - p \). All algorithms stop as soon as a point \( p \) has been found with \( \max_k |p_k - I_k(p, W(p))| \leq \epsilon p^{1/2} \), where \( \epsilon p \) is the machine precision.

Fig. 1 shows the average number of iterations for the various power control methods. The x-axis depicts the system load in terms of number of users. Clearly, Newton’s method needs the fewest number of iterations, as expected, with its drawback of high computational complexity. The proposed algorithm results in a much lower number of power control iterations compared to the fixed-point method. Interestingly, the higher the load the higher the performance gain. Note that for high system loads (being the system operating points network providers are at most interested in) the accelerated algorithm requires almost half of iterations only. Now, Fig. 2 shows the computation times\(^2\) for the standard power iteration and proposed method (Steffensen’s method coupled with the standard iteration). Fig. 2 indicates that the proposed method has a small overhead in computation time for low loads. This is probably due to the fact that checking (11) takes an additional amount of time. On the other hand, the method proposed outperforms the standard algorithm for high loads.

\(^2\)Herein the times observed are for a MATLAB-PC implementation and indicate the relative performance of the algorithms only.
In this paper, we presented an algorithm that comprises an accelerated power control algorithm and MMSE filtering. With MMSE filtering we performed beamforming or multiuser detection in the uplink of a CDMA based communication system. The accelerated power control algorithm operates a fixed-point iteration (Jakobi-iteration) utilizing an Aitken $\Delta^2$-process. This iteration is known as Steffensen’s algorithm. In particular, we described the merge of Steffensen’s iteration algorithm with MMSE filtering. To gain more insight, we also derived a new lower bound on the Lipschitz constant that is used to ensure convergence of the proposed method. The proposed algorithm was benchmarked with given fixed-point iterations as well as with Newton’s method. The results clearly show the acceleration and indicate that the proposed algorithm is applicable to advanced CDMA uplink transmission with beamforming or multiuser detection.

REFERENCES


Fig. 1. Average number of iterations for the standard iteration, Newton’s method, and proposed method.

Fig. 2. Average computation time for the standard iteration and proposed method employed after a suitable starting value has been found.

V. SUMMARY