Spatial optimization as a generative technique
for sustainable multiobjective land use allocation

Running head title: Spatial optimization as a generative technique

ARIKA LIGMANN-ZIELINSKA*,‡, RICHARD L. CHURCH‡ and PIOTR JANKOWSKI†
†Department of Geography, San Diego State University, USA
‡ Department of Geography, University of California Santa Barbara, USA

* Corresponding author, 5500 Campanile Dr., San Diego, CA 92182-4493, USA. Tel. +619 594-5437
Fax. +619 594-4938 Email: ligmannz@rohan.sdsu.edu
In this paper, we examine the applicability of spatial optimization as a generative modeling technique for sustainable land use allocation. Specifically, we test whether spatial optimization can be used to generate a number of compromise spatial alternatives that are both feasible and different from each other. We present a new spatial multiobjective optimization model, which encourages efficient utilization of urban space through infill development, compatibility of adjacent land uses, and defensible redevelopment. The model uses a density based design constraint developed by the authors. The constraint imposes a predefined level of consistent neighborhood development to promote contiguity and compactness of urban areas. First, the model is tested on a hypothetical example. Further, we demonstrate a real world application of the model to land use planning in Chelan, a small environmental amenity town in the north-central region of the State of Washington, USA. The results indicate that spatial optimization is a promising method for generating land use alternatives for further consideration in spatial decision-making.

*Keywords:* Multiobjective land use allocation modeling; Generative modeling; Spatial optimization; Sustainable land use
1. Introduction: generating sustainable land use patterns

Suburban and exurban land uses are characterized by inefficient low-density and fragmented patterns, which cause a number of negative externalities to the current urban reality. Examples of these societal drawbacks include ethnical and economic separation, deterioration of the environment, and loss of agricultural land and wilderness (Leccese and McCormick 2000, Newman and Kenworthy 1999, Silberstein and Maser 2000, Randolph 2004, Williams et al. 2000). A number of studies related to sustainable development suggest that inefficient resource use and high traffic congestion are more due to the pattern of growth than the amount of growth itself (Barton 1990, Randolph 2004, Roseland 1998). As a consequence, there is a clear need to increase land use efficiency by a careful design of alternative land use patterns.

In this paper, we demonstrate the applicability of spatial optimization as a modeling technology for generating compact, compatible, and contiguous residential patterns. We describe a multiobjective land use allocation model, which minimizes incompatibility of adjacent land uses, minimizes distance to urban areas, and balances new development with infill and redevelopment. Furthermore, we enhance the model with a Hop-Skip-Jump (HSJ) method, developed by Brill et al. (1982), which encourages solutions that are potentially different from each other. We test the model on a hypothetical example of a 10x10 raster, and then report on a real-world application of the model to an exurban community of Chelan City and its vicinity in Chelan County, Washington, USA.
To address the issues of inefficiently allocated growth, a new concept of sustainable development has recently begun to take focus in the planning literature. Comprehensive sustainability in urban planning can be termed as a long-term balance between economic development, environmental protection, efficient resource use, and social equity. Leccese and McCormick (2000), in their ‘Charter of the New Urbanism’, described a sustainable land use planning agenda. Their manifesto emphasizes infill development, mixed uses, compactness, and local geography as the main constituents of a balanced urban development. Following their argument, we defined ‘sustainable land use allocation’ as a normative modeling methodology, which focuses on evaluating current land use patterns, and introduces changes leading to increased compatibility of adjacent land uses, infill development, land use compactness, and politically defensible redevelopment (Ligmann-Zielinska et al. 2006). It is apparent that the outlined principles are spatially explicit and therefore linking GIS with optimization models provides a useful technology in planning for land use pattern sustainability (Camagni 1998, Greenhuizen and Nijkamp 1998).

The utility of optimization as a normative tool for spatial problems is widely recognized (Church 1999, Church 2002, Cromley and Hanink 1999, Malczewski 1999). These generative techniques allow for multiple scenario analysis, where the outcomes obtained are non-inferior (Pareto-optimal) to the objectives contained in the model. In essence, for a given set of model objectives, we cannot improve the outcome of any objective without compromising the other objectives, and thus our analysis focuses on those options that are not dominated by any other alternatives (Cohon 1978).
Due to multifaceted nature of land use allocation, spatial optimization modeling should aim at finding a set of high-performing alternatives instead of just one solution (Bankes 1993, Church 1999, Harris 2001) and allow the stakeholders to choose from a number of scenarios that are both good and different from each other, since not every planning objective of interest may be introduced into the model in the form of mathematical formulation (Brill et al. 1990, Chang et al. 1983). The decision makers should be provided with alternatives that allow for consideration of other – less defined – planning goals, or recognition of overlooked issues and innovative solutions (Chang et al. 1983). Consequently, the generated alternatives should not be treated as ultimate solutions but rather as propositions for further analysis by stakeholders.

The remainder of this article is organized as follows. Section two summarizes the existing approaches to modeling land use contiguity and compactness. We develop a framework, which serves as a basis for a new spatial constraint – Density Based Design Constraint (DBDC) – that encourages infill development based on contiguity and compactness principles. Section three outlines the methodology of using spatial optimization to generate diverse land use alternatives. In section four we formulate and describe the model. Section five reports the results from model evaluation based on a hypothetical example and a real world case. The final section summarizes the findings and concludes on the utility of the proposed model.
2. Encouraging contiguity and compactness in land use allocation

One of the most recognized attempts to integrate multiobjective land use allocation with GIS is a module in IDRISI (www.clarklabs.org), called MOLA and developed by Eastman et al. (1995). MOLA built-in heuristics allows for conflict resolution between competing land uses, which are being allocated to a given land. The obtained patterns stem from rank maps which comprise a number of allocation factors. However, due to the aspatial nature of cell ranking and the lack of explicitly spatial objectives and constraints, this procedure has a potential downside of producing scattered and disjoint patterns (Brookes 1997, Herwijnen and Rietveld 1999). In reality, a given land use, e.g. single family residential, tends to cluster in land use plans. Such zoning or clustering, defined as compact or contiguous, is used to minimize conflicts of neighboring land uses. We define compactness as an allocation of a given land use to sites that are in direct proximity of each other resulting in circular patches. Furthermore, we define contiguity as the degree to which a specific use has been allocated to land in an unbroken fashion. Contiguity may result in elongated development shapes (Ligmann-Zielinska et al. 2005). In the discussion that follows, we analyze the existing methods that encourage these two spatial characteristics in multiobjective land use allocation outputs.

We can classify land allocation models in terms of the method used to encourage connectedness and contiguity within two types of constructs: solution-based and explicit constraint-based. The explicit constraint approach may be further divided into adjacency-based clustering, perimeter-based compactness, and block aggregation.
One of the easiest ways in which to ensure contiguity is to develop a solution process that incorporates connectedness as an embedded rule. For example, Gilbert et al. (1985) developed a model that optimizes the selection and layout of a set of connected cells, such that the cost of purchase and development is minimized, the distance to ‘detractor’ cells is maximized, the distance to ‘amenity’ cells is minimized, and the ‘diameter’ of the layout is minimized. They did not suggest explicit constraints to ensure connectedness, but embedded the search for connected sites within the implicit enumeration algorithm that they developed. Another example of incorporating connectedness within the solution process is the parameterized region-growing program of Brookes (1997) and the patch growing process of Church et al. (2003). Ensuring connectedness within an optimization model tends to require a large number of constraints and variables expressly devoted to the task.

There are several ways in which explicit constraints on connectivity can be structured. Virtually all of these techniques are based upon the development of a network, where nodes represent cells/units and arcs represent shared edges between the adjacent cells/units. For each shared edge there is an arc, which crosses the shared edge connecting the nodes representing the cells/units that are adjacent. Cova and Church (2000) defined a set of ordered variables for each cell about a starting cell. An ordered cell variable could equal one only when an adjacent cell was selected with an order that was one less. Constraints of ‘order’ were based upon the arcs of the network. The ordered constraints and variables ensured connectedness of selected cells to the starting cell.
For a given set of selected cells/units, one can define an adjacency sub-network, which consists of only those arcs associated with shared edges of the selected set. If, for any pair of selected cells, a pathway exists on the adjacency sub-network between the two cells, then the set of selected cells is connected. This path-based approach was used by Church and Cova (2000) to select a connected cluster of neighborhoods that maximize demand versus exit capacity, thereby identifying a potential evacuation hazard. Shirabe (2005) used a similar network structure to force contiguity of areas being selected as a part of a larger region, where contiguity was ensured by a set of network flows emanating from a ‘source’ area and flowing to other selected areas on the adjacency sub-network. Williams (2002) defined a dual graph to the primal network of adjacencies. He observed that if the arcs on the primal graph are defined for only adjacent selected cells/units, then connected trees on both graphs ensure contiguity of the selected cells/units. This observation leads to a very elegant model formulation for ensuring contiguity of selected units. Unfortunately, the dual graph approach of Williams (2002), the ordered variable approach of Cova and Church (2000), the path-based constraint of Church and Cova (2000) and the method of Shirabe (2005) are limited in application to solving small problems.

Within a model it is possible to include variables and associated constraints that define the length of the perimeter associated with the set of selected units/cells. Wright et al. (1983), Benabdallah and Wright (1992), Minor and Jacobs (1994), and Fischer and Church (2003) have all used an objective of minimizing the length of perimeter to encourage connectedness. Although this construct does not ensure connectedness, it can be used to generate solutions that tend to be more connected and less scattered and disjoint.
Another method to encourage compactness and connectedness is the building block concept of Aerts et al. (2003). The building block model is based upon the selection of cells, and promotes solutions in which all cells in a building block are selected. If two overlapping building blocks are used, then the area associated with the two building blocks is connected. Again, using building blocks in a model can be used to encourage contiguity, but not to require it. Overall, each of the presented approaches that encourage connectedness and/or contiguity has a weakness. This has motivated us to develop a new approach to promote connectedness in land use optimization.

3. Spatial optimization as a method for generating location alternatives

Generative land use modeling has been gaining momentum in systems that support planning activities. These techniques, known as exclusionary screening procedures, are based on the concept of weighted map overlay and non-spatial site suitability scoring (Dobson 1979, Mc-Harg 1969, Jankowski and Nyerges 2001). As stated earlier, such rank based allocation may result in non-clustered regions and is potentially insufficient for siting problems. In this paper, we focused on another search technique for land use allocation, namely spatial optimization. The rationale behind the use of multiobjective spatial optimization as a generative tool lies in its ability to find non-inferior solutions. In other words, properly defined optimization models help to identify non-dominated solutions (within the context of modeled problem), which can frame a policy implication debate and help avoid making poor decisions. Moreover, the use of objectives and constraints that
account for spatial association (compactness, proximity etc.) allows for generation of clustered patches of a given size and shape (Brookes 1997, Herwijnen and Rietveld 1999).

Harris (2001) stressed the importance of time efficiency in generating spatial alternatives and claimed that overemphasis on details is counterproductive. Instead of traditional ‘just-in-case’ planning, we should therefore provide modeling tools for planning ‘just-in-time’ (Portugali and Alfasi 2000). Due to the fact that spatial problems are inherently multidimensional and involve repetitive analysis, spatial ‘sketches’ should be prepared in a semiautomatic way (Harris 2001). This requirement may be fulfilled with ease by using spatial optimization, which allows for running a number of experiments that differ in model parameter values. From the normative standpoint, spatial optimization is not the final step in decision-making process but only a technique for generating alternatives for further consideration and human judgment (Harris 2001, Nalle et al. 2002). Moreover, optimization in land use allocation does not need to be perceived as an extremely precise method – in fact, both data and assumptions used in optimization modeling should be good enough to give better results than those obtained by common sense (Simon 1977).

3.1 Generating different land use plans

Traditional multiobjective modeling deals with a weighting method for creating a composite score, where each objective has an importance (weight) assigned, and the composite objective value is calculated using a simple additive weighting technique.
Sensitivity analysis is then implemented by varying the values of the importance weights of each objective. Such an approach, however, is very unlikely to generate significantly different land use arrangements (Ligmann-Zielinska et al. 2006).

Brill et al. (1982) and Chang et al. (1983) conceptualized a Modeling to Generate Alternatives (MGA) optimization approach for obtaining sufficiently different alternative solutions. Their methodology starts from generating an initial non-inferior solution using one of the traditional approaches (e.g. constraint or weighting methods). This solution provides two types of information: an initial set of positive variables and model objective values, which can be used to set acceptable targets for solution performance. As a next step, a new model is constructed based upon the original model where a constraint is added for each of the original objectives. Each new constraint specifies that any subsequent solution must perform at least as good as some preset value (usually set as a percentage of the original objective performance, say at least 90% of optimal). The objective target values are then used in the following model iterations to ensure that the objectives will not perform worse than an acceptable level. The set of initial variables that were positive may be then configured into an objective to be optimized (e.g. minimized), encouraging the model to pick other variables to become positive in order to obtain the best objective score. In this sense the model seeks a solution, which is as different as possible from previous solutions, and yet achieves acceptable values in terms of objective performance. In this paper, we will use a specific conceptualization of MGA called Hop-Skip-Jump (HSJ) developed by Brill et al. (1982).
4. Model formulation

We begin by describing the Sustainable Multibjective Land Use Allocation model (SMOLA) (Ligmann-Zielinska et al. 2005, Ligmann-Zielinska et al. 2006). The model utilizes raster data format, where the land use of each cell is homogenous. Also, the model does not allow for urban redevelopment leading back to open space, which is in practice very unlikely (Nalle et al. 2002, Silberstein and Maser 2000). The SMOLA model utilizes a design constraint that is based upon a defined neighborhood about each cell being considered for development or redevelopment (called a focus cell). This concept will be described at greater length after we formulate the overall model. In the formulation that follows, the focus cell neighborhood is a 3x3 rectangle.

4.1 SMOLA Notation

\[ i,j \quad 1,2,\ldots,n; \text{cell locations, where } n \text{ is the total number of cells in the study area} \]
\[ l,m \quad 1,2,\ldots,k; \text{types of urban land uses} \]
\[ u \quad \text{undeveloped land use} \]
\[ D_l \quad \text{set of cells that already have land use } l \]
\[ D \quad \text{set of developed cells; all subsets of } D \text{ are mutually disjoint} \]
\[ U \quad \text{set of cells of undeveloped land; } U \cup D = \bigcup, \text{where } \bigcup \text{ is the universal set of all cells} \]
\[ B_j \quad \text{set of } j \text{'s neighbors that are undeveloped} \]
\[ e_j \quad \text{existing land use of cell } j \]
\[ t_l \quad \text{number of cells that initially have land use } l, \text{where } l = 1,2,\ldots,k \]
\[ c_{lm} \quad \text{estimated compatibility index between land uses } l \text{ and } m \text{ (the higher this coefficient, the more compatible the land uses); in the model } l \text{ is represented by } d_j \]
The dominant land use type within a neighborhood is the one that covers the maximum neighborhood area. In our model, $d_j$ is this land use which has the maximum number of neighboring cells (including the focus cell); for ties, the dominant land use is chosen randomly. The dominant land use type is set to ‘undeveloped’ if and only if all neighbors (including the focus cell) are undeveloped.

$d_j$ dominant urban land use type within the neighborhood of $j$. The dominant land use type is the preferred (most compatible) land use to be allocated at $j$.

$s_j$ number of initially developed cells within $j$’s neighborhood.

$a_j$ attractiveness of undeveloped location $j$, the higher the coefficient, the more probable that new development occurs.

$r_j$ resistance to change for already developed location $j$, the higher the coefficient, the less probable that redevelopment occurs.

$dist_j$ distance of location $j$ to its nearest developed area (in cells).

$v_l$ estimated demand for land use $l$.

$b$ minimum required number of neighboring cells that are developed after allocation.

Variables

$x_{jum}$ = 1, if undeveloped land at location $j$ is changed to $m$; and 0 otherwise.

$x_{je,m}$ = 1, if current land use $e_j$ at location $j$ is changed to $m$; where $m \neq e_j$, and 0 otherwise.
4.2 Sustainable multiobjective land use allocation model

Minimize
\[ \sum_{j \in U} \sum_{m} (1 - a_j) x_{jum} \]
\[ \sum_{j \in D, m \neq e_j} r_j x_{jpm} \]
\[ \sum_{j \in U} (1 - c_{d,m}) x_{jum} + \sum_{j \in D, m \neq e_j} (1 - c_{d,m}) x_{jpm} \]
\[ \sum_{j \in U} \text{dist}_j x_{jum} \]

Subject to
\[ \sum_{m \neq e_j} x_{jpm} \leq 1; \forall j \in D \]
\[ \sum_{m} x_{jum} \leq 1; \forall j \in U \]
\[ t_i - \sum_{j \in D, m \notin I} x_{jum} + \sum_{j \in (D - D_i)} x_{jpm} + \sum_{j \in U} x_{jum} \geq v_i; \forall i \]
\[ s_j + \sum_{m} x_{jum} \geq b \sum_{m} x_{jum}; \forall j \in U \]
\[ x_{jum} \in \{0,1\}; x_{jpm} \in \{0,1\} \]

Objective (1) minimizes development of open space areas. It assures that if we build outside of urbanized areas, the developed sites will have the maximum possible attractiveness (figure 1). The attractiveness coefficient is subjectively defined by the planner. For example, it may represent a composite measure that combines scenic views, slope, distance to water etc. As a consequence, objective (1) is structured so that if we allocate to open space, we maximize the number of the most attractive areas from a given planning perspective. Objective (2) minimizes redevelopment of urban areas. It assures that if redevelopment is selected, the locations picked will have the minimum possible resistance to change (figure 1). These two objectives allow for analyzing the tradeoff...
between new development and redevelopment. Objective (3) minimizes incompatibilities between land use of site \( j \) and its neighborhood (figure 1). Finally, objective (4) minimizes the distance of new development to already developed sites (figure 1). Constraints (5) and (6) ensure that only one land use may be allocated to each cell \( j \). Equation (7) guarantees that the demand for land use \( l \) is satisfied. It permits allocation of undeveloped land but also land use reallocation for already built-up areas, which is appropriate for urban siting problems (Ligmann-Zielinska et al. 2006).

4.3 Density based design constraint

Equation (8) represents a contiguity and compactness constraint called Density Based Design Constraint (DBDC), which forces a user-specified neighborhood infill development (figure 1). It assures that we will allocate to cell \( j \) if and only if the sum of the \( j \)’s initially and newly developed neighbors is at least equal to threshold \( b \).

The value of \( b \) is constrained by the depth of the neighborhood considered. We can define a simple neighborhood centered about a ‘focus’ cell \( j \) as those cells that are cardinally and diagonally adjacent to the focus cell (i.e. a neighborhood of a 3x3 centered at \( j \)). If we conceived of a neighborhood that included an additional set of cells that were cardinal and diagonal to the 3x3 neighborhood, we would then define a neighborhood of a 5x5 centered at the focus cell. We could expand this to a 7x7 or a 9x9 in the same manner.
We define the radius of a neighborhood, $p$, as the number of adjacent cell bands about the focus cell $j$. For example a 3x3 neighborhood has the $p$-value of one. A 5x5 neighborhood has the $p$-value of two. In general, the neighborhood about a focus cell can be defined at $(2p+1) \times (2p+1)$ in size.

Assume for now that a specific cell will be allocated to land use $m$. To cluster development activities, it would be desirable to have as many neighbors as possible about this cell to be also allocated to the same land use $m$. We do not know in advance whether the cell will be totally surrounded by cells that are allocated to land use $m$ or whether the cell is on the edge of a zone of cells allocated to land use $m$. Let us say for now, that the cell is on the perimeter of a zone of cells allocated to land use $m$. If the zone is compact and connected, then the case in which the cell has the fewest neighbor cells of the same type occurs when the cell serves as a corner of the zone. If it is a corner cell, then at best it would have 3 neighbors and itself (for a total of 4) in the 3x3 neighborhood centered about this cell. Thus, when counting the cell of focus, and its neighbors within the 3x3 neighborhood, we can require that up to 4 cells be of the same type as allocated to the cell of focus. Since we do not know whether the cell will be on the perimeter or a corner of the perimeter of allocation zone, we cannot in general require more than 4 cells of the same type in a 3x3 neighborhood. For a 5x5 neighborhood, the maximum limit could be increased to 9. In general, if a neighborhood is defined as $(2p+1) \times (2p+1)$ in size, then the maximum number of neighbors of the same type (should the focus cell be a corner of the perimeter) would be $p+1$ by $p+1$ where $p$ equals the radius of the neighborhood about the focus cell.
The value of parameter $b$ in the model represents the requirement to allocate a minimum number of neighbors along with the cell of focus to the same land use type. Therefore, the value of $b$ represents the requirement for a density of the same land use assignment in the neighborhood about a given cell. Given that the focus cell could be a corner of a perimeter, the model can then be applied for values of $b$ equal or less than $(p+1)^2$ for a neighborhood of $(2p+1) \times (2p+1)$. The higher the density of the same land use, the more the pattern of assignment will be clustered. The lower the density of the same land use, the more scattered and disjoint becomes the pattern of assignment. In the experiments presented in section 5, we varied the value of $b$ from 0 to 3.

### 4.4 Generating different alternatives

Figure (2) represents the algorithm for SMOLA enhanced with the Brill’s et al. (1982) HSJ technique for modeling different alternatives. The process starts from solving SMOLA for an equal weight case where the same importance is assigned to each objective. This solution provides an initial land use configuration ($Map_{ini}$), which is composed of those cells that received a land use allocation (i.e. $x_{jum}=1$ or $x_{je,m}=1$ for land use $m$). Moreover, for each of the minimization objectives we obtain an initial solution value ($V_j$), which may be further relaxed (parameter $u_j$) and used as a target value ($T_j$) for the objectives when they become constraints. In all of the examples that follow, we implemented a 0% relaxation (i.e. $T_j=V_j$), which is equivalent to an alternative noninferior solution for
SMOLA, followed by experiments with a 10% relaxation (i.e. $T_j=1.1*V_j$) to obtain ‘close-to-optimal’ solutions.

As the next step, the HSJ method is applied. We use the following formula for encouraging a different alternative to be generated:

$$MinZ = \sum_{k \in K} (w*i_k)x_k$$

subject to all original model constraints and new constraints for all original model objectives, where $K$ is a set of decision variables ($x_k$) that were positive in previous model iterations. In other words, $K$ comprises only those binary variables which equal one ($x_{jum}=1$ or $x_{jv,m}=1$) in every completed model iteration. Objective ($Z$) minimizes the set of those positive variables ($K$) leaving a possibility for these variables to become ‘zero’ in the next iteration. This trend may be enhanced by the coefficient ($w*i_k$), where $w$ is an arbitrarily selected ‘discouragement’ weight, which penalizes $K$-set variables even more, whereas ($i_k$) is an iteration factor, which captures the cases in which $x_k$ appears in more than one HSJ iteration. For example, if $x_k$ appeared 3 times in 4 iterations, then $i_k=3$. For every HSJ iteration, all new positive variables ($K_{new}$) are added to the $K$-set. The procedure stops when the number of new nonzero variables is close to zero and therefore the difference between $Map_{i-1}$ and $Map_i$ becomes negligible.
5. Model evaluation

Model evaluation took into consideration the following questions:

1. What configurations of DBDC $b$-value and what percentage of HSJ relaxation (parameter $u_r$ in figure 2) provide the most compact and contiguous patterns?

2. What is the level of difference between the generated spatial alternatives?

Elsewhere (Ligmann-Zielinska et al. 2005, 2006), we analyzed SMOLA from the perspective of sustainability focusing on the tradeoff between new development and redevelopment, and the level of compatibility of adjacent land uses. There, we analyzed the influence of different weighting schemes (planning scenarios) on the resulting patterns. This paper places the model in a different context – as a GIS-coupled tool for land use design of manifold options (Brill 1979, Duh and Brown 2005).

The problems presented below were formulated using the MPS file format (standard format for Linear and Integer Programming), and generated with Python scripting. The Python script was executed within the ArcMap GIS software. The model was solved using CPLEX Mixed Integer Optimizer (ILOG, www.ilog.org) on a SunBlade 2500 dual processor (467 MHz each) workstation with the Solaris 7.1 operating system. Default settings were used as no attempt was made to optimize performance.
5.1 Testing the model on a hypothetical grid

At first, the model was tested against a hypothetical example that covered an area of 10x10 cells (the upper-left corner in figure 3). This experiment was aimed at determining the influence of the HSJ technique and the parameter $b$ in DBDC on generating different and clustered land uses. As the input, we used a map that was composed of two totally incompatible land uses, and the rest of the area represented undeveloped land. Also, we used homogenous layers of attractiveness ($a_j$) and resistance to change ($r_j$), which were set to 1 (maximum possible) and 0 (minimum possible), respectively. Therefore, for the sake of simplicity, we eliminated any potential distortion of results that might be caused by these model parameters. The model contained 191 binary variables for a total demand of 24 cells (9 already developed that could be redeveloped, and 15 to be developed after allocation).

[Insert figure 3 about here]

For all the results that follow, we used the following formula to calculate difference between two output maps (in %):

\[
Map_{dif} = 2 \times \frac{d_c}{\sum_i (v_i - t_i)}
\]

(11)
Where \( l \), \( v_l \) and \( t_l \) are the same as in model formulation (section 4.1), and \( d_c \) is the number of cells that initially belong to \( U \) and have different land uses in both maps. This equation is valid for a case when the level of redevelopment equals zero, which is ‘true’ for all the experiments presented\(^\text{a}\).

The average difference between the outcome maps in the hypothetical case equals 50\%, with a maximum of 97\% between maps: \( b=0 \) & iteration=2 and \( b=0 \) & iteration=3 (Table 1). The improvement in development fragmentation, which is calculated using the ratio of development perimeter to the square root of development area, varies considerably between model runs (Figure 3). On average, fragmentation increases rather than decreases as compared to the input pattern (upper left corner in Figure 3). However, we observe that the average increase for \( b=0 \) equals 34\% whereas for \( b=3 \) it is only 1\%, which validates the rationale behind using DBDC for infill development. As a result of the initial analysis, we conclude that DBDC does enhance compact and contiguous development. We also observe a plausible behavior of HSJ in producing different land use arrangements.

\[ \text{[Insert table 1 about here]} \]

5.2 Applying the model to Chelan City and its vicinity

\(^\text{a}\) Example: Suppose that the demand for new development equals 24-9=15 i.e. total demand (24) minus land that is already urban (9). Assuming redevelopment of zero, the maximum difference between two maps may be 30, which is equivalent to the case where every land use unit is allocated to two different cells in the maps. Assuming that we obtained 18 cells of difference between maps, \( Map_{diff} \) equals \( \frac{18}{30} \times 100 = 60\% \)
The integrated SMOLA-HSJ model was tested on a small exurban community of Chelan City, in Chelan County, Washington, USA (figure 4). The reason for choosing Chelan was twofold. Firstly, Chelan’s urban growth has been impacted by two opposing and competing forces, which have a direct consequence on the future development of the locality: endangered salmon protection and recreational/retirement influx of people from the Puget Sound metropolitan area (Ligmann-Zielinska and Jankowski, 2007). Therefore, sustainable land use patterns of Chelan urban area are crucial for local environmental and economic planning. Secondly, the selected study area represents a realistic example of a self-contained community, which is computationally tractable.

[Insert figure 4 about here]

5.2.1 Datasets. The analysis was performed using a 2 acre cell resolution, which resulted in the analysis area of 73,396 cells (594 km²). We generalized current land use of the area into five categories: residential, commercial, industrial, undeveloped and restricted i.e. excluded from development (figure 5). The undeveloped type represents open space areas that might be considered for build-out. The database inventory contained land use, development attractiveness, resistance to change, ‘number of developed neighbors’ layer, distance to development, and the ‘dominant neighborhood land use’ layer. The attractiveness layer was derived based on weighted overlay of the following themes: buildable areas, planned development, slope, distance to water, and distance to parks, forests, and other recreational areas. Similarly, the composite resistance layer was obtained from developable areas,
building value, and renter occupied housing units per census block (Census 2000, www.census.gov) using the linear combination method as well.

[Insert figure 5 about here]

The ‘number of developed neighbors’ and ‘dominant neighborhood land use’ layers were derived from the input land use theme using Python scripting. The ‘number of developed neighbors’ was obtained by counting developed cells within the neighborhood of cell \( j \). Finally, the distance to developed cells was generated using ArcGIS 9.1 Spatial Analyst Euclidean Distance function (ESRI, http://www.esri.com/). The distance is given in ‘cell’ units.

Table (2) summarizes the allocation demand for the three urban land uses mentioned above. The demand was derived based on the ratio of population to land use area calculated for the year 2000, and then linearly extrapolated to 2025, based on population forecasts from Office of Financial Management (OFM) in the State of Washington (http://www.ofm.wa.gov/). The demand obtained in this way resulted in 115 169 binary variables and 75 861 linear structural constraints. The compatibility of land uses (table 3) was acquired based on personal communication with the Chelan County planner and was based on the guidelines of the Growth Management Act for the State of Washington (CCCP 2000).

[Insert table 2 about here]
5.2.2 Model results for the Chelan area. We started the experiments from obtaining the initial equal-weight SMOLA solutions for varying values of $b$ in DBDC (from 0 to 3) and leaving all other parameters unchanged. We compared the results by calculating the level of difference between allocation maps using equation (11). The highest difference (16%) occurred between $b=1$ and $b=3$. We anticipated that these values might be good representatives for analyzing urban land use fragmentation. Therefore, as the input to all HSJ iterations that followed, we selected the initial results for these two $b$ values.

At first, we set the discouragement factor ($w$ in figure 2) to 1. However, the difference between the obtained land use alternatives was not satisfactory (less than 10% on average). Therefore, in the following experiments, we decided to set the $w$-weight to 1000. Also, we divided the experiments into two groups: with $u_r=0\%$ (no objective relaxation) to look for alternative non-dominated solutions, and $u_r=10\%$ (the $T_j$ target increased by 10%) to relax slightly the SMOLA objective values.

Table (4) presents the optimal HSJ objective function values for different parameter sets. Since we minimize the objective (figure 2), we look for as low value as possible for a given HSJ iteration. Readers can observe that the increase in DBDC $b$-value (emphasizing infill development) slightly improves the result. Additionally, the level of HSJ relaxation ($u_r$) has even a greater impact on the improvement of the result. Altogether, for every HSJ iteration, $b=3$ and $u_r=10\%$ give the best objective function values.
We translated the optimal solutions into a collection of variables added to $K$-set of every HSJ iteration (figure 6.1). With the increasing number of HSJ iterations, the number of new variables ($K_{\text{new}}$) added to $K$-set decreases, which confirms the observations about the behavior of the HSJ method by Brill et al. (1982). From the practical perspective, it implies that a few HSJ iterations should be enough to capture the major differences in land use allocation alternatives. In the example reported here, the variability between outcomes became negligible after the 4th iteration (figure 6.1) and amounted to five output maps (including the initial HSJ input). Using a different initial land use map as the input to HSJ procedure would potentially increase the number of differing alternative solutions (Brill et al. 1982).

The relaxation of $T_j$ targets positively affects the number of new variables added to the $K$-set. As an example, consider $b=3$ and a varying value of $u_r$ (represented by a black square for $u_r=0\%$ and a hollow circle for $u_r=10\%$ in figure 6.1). For four HSJ iterations, the average number of new $K$-set variables equals 108 with $u_r=0\%$ and 350 with $u_r=10\%$. Thus, if an analyst wants to produce less similar land use maps, she should adequately relax the objective target values ($T_j$) sacrificing Pareto-optimality. On the contrary, if her focus is on producing non-dominated land use arrangements, then the $u_r$ value should be as low as possible. Under such circumstances, however, she is risking the possibility of generating very similar solutions.
We used the ratio of ‘development perimeter to the square root of development area’ as the urban land use fragmentation metrics (figure 6.2). None of the model runs improves the current land use fragmentation situation as measured by the selected metrics. However, the level of fragmentation increase is correlated with the $b$ and $u_r$ parameters. As anticipated, the higher the $b$-value in DBDC, the more compact and contiguous the pattern is (e.g. an average drop in fragmentation increase from 15% for $b=1$ to 12% for $b=3$ with $u_r=10\%$). Thus, higher values of $b$ result in smaller increase of urban land use fragmentation. Secondly, the objective value relaxation resulted in a more fragmented development e.g. for $b=3$ and $u_r=0\%$ the average is 7% whereas for $b=3$ and $u_r=10\%$ it is 12% (figure 6.2).

From the two parameters $u_r$ and $b$, the former contributes more to the difference between maps measured with equation 11 (table 5). Indeed, for all pairs of $u_r=0\%$ the difference never exceeds 30%. On the contrary, none of the pairs of maps generated for $u_r=10\%$ has a difference below 30%. Also, the best results concerning difference occur for $u_r=10\%$ between any two of the consecutive HSJ iterations regardless of $b$ (category ‘87% and more’ in table 5).
We also analyzed the relationship between the \( w \)-weight, objective value relaxation, and the difference between the resulting land use maps. Table (6) provides the results for the average difference of all pairwise comparisons between solutions generated for different \( u_r \) and \( w \) combinations. Observe that the increase of the \( w \)-weight results in a smaller rise of map pairwise difference (7\%) than the increase in objective target relaxation (40\%).

[Insert table 6 about here]

The maximum difference we obtained is presented in figure (7). The maps differ by 94.5\% as compared to the maximum possible difference (1986 cells, for the ‘no redevelopment’ case – equation 11).

[Insert figure 7 about here]

From the technical standpoint, we noticed a positive correlation between the value of \( b \) and the model solution time or the number of solver iterations. As expected, the results of the more constraining \( b=3 \) associated with longer solution times and an increased number of solver iterations. The maximum solution time of the CPLEX experiments occurred for the 5\textsuperscript{th} HSJ iteration with model parameters set to \( w=1000, \ u_r=10\% \), and \( b=3 \), and amounted to 4.7 hours. However, for the results of interest (i.e. \( w=1000, \ b=1 \ & 3, \) and \( u_r=0\% \ & 10\% \)) the solution time did not exceed 27 minutes.
6. Discussion and Conclusions

The SMOLA model introduced in this paper comprises four spatially-explicit objectives that tradeoff new development for redevelopment and minimize neighborhood incompatibility and proximity to existing urban areas. The model is equipped with a novel spatial DBDC constraint than imposes a user-defined neighborhood development level. It is further enhanced with a GIS implementation of an iterative HSJ procedure, proposed first by Brill et al. (1982), which is used in the model to generate land use patterns that differ in configuration and the level of fragmentation.

The model behavior is congenial with the research design objective of producing spatial alternatives that prevent leapfrog skittish land use. It was observed that imposing higher infill development \( (b \text{ in DBDC}) \) had a positive effect on decreasing fragmentation. Conversely, the relaxation \( (u_r) \) of the SMOLA objective function values obtained in the initial solution negatively influenced fragmentation.

HSJ-enhanced SMOLA proved to be an effective search technique for generating different spatial alternatives. The iterative procedure of minimizing the set of previously positive variables allowed for up to 95% difference in outcome maps. An increased value of infill \( (b) \) proved to be less effective than the HSJ relaxation \( (u_r) \) in reinforcing the variability between spatial outcomes. Likewise, a higher discouragement factor \( (w) \) also had a positive impact on variability.
From a practical standpoint, the SMOLA modeling approach could be enhanced by developing a heuristic, which can browse through a solution space until a user-defined map difference is met. Future research should focus on investigating the influence of scale in generating divergent and sustainable land use alternatives (Cromley and Hanink 2003). The model could also be implemented in a more dynamic setting, where land use siting is a continuous process rather than a one-time allocation (Shirabe 2004). The research reported in this article assumes that all four land use objectives are of equal importance. This assumption was used as the basis for all HSJ experiments that followed. A potentially intriguing future research topic involves analysis of the influence of different objective weights on the HSJ outcomes.

The integrated SMOLA-HSJ model demonstrated that spatial optimization might be employed as a normative landscape design tool for generating efficient maps for various land use policy scenarios. Presenting a set of alternative scenarios in a collaborative setting may enhance the role of descriptive non-numerical planning objectives in spatial decision making. Furthermore, the presented model supports a tradeoff evaluation between optimizing spatial objectives (like clustering) and generating divergent solutions. Obtaining more diversified patterns requires a relaxation of the spatial objectives programmed into SMOLA. On the other hand, if we want to reinforce compactness, contiguity, or proximity we must forfeit option diversity.

7. Acknowledgements
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8. References


SHIRABE, T., 2005, A model of contiguity for spatial unit allocation. Geographical Analysis, 37, pp. 2-16.


**Figure Captions**

**Figure 1** The graphical depiction of how SMOLA model objectives and the Density Based Design Constraint (DBDC) operate (*black* represents residential land use and in each of the (1) through (4) maps ‘A’ is better than ‘B’): (1) Minimization of new development; (2) Minimization of redevelopment; (3) Minimization of incompatibility of adjacent land uses; (4) Minimization of distance to already developed areas; and (5) DBDC (refer to section 4.2 and section 4.3 for details).

**Figure 2** Sustainable Multiobjective Land Use Allocation (SMOLA) enhanced with the Hop-Skip-Jump (HSJ) technique (adapted from Brill et al. 1982): $u_r$ – user input parameter for objective value relaxation, $i$ – HSJ iteration, $x_k$ – variables that were positive in at least one $i$, $i_k$ – the number of times $x_k$ appeared in $i$ iterations, $w$ – variable discouragement factor; the higher the value of this factor the more the variable is penalized if selected at the nonzero level, $f_j(x)$ – initial $j$th objective function converted to a constraint, $T_j$ – target value for the $j$th objective, $X$ - set of feasible solutions based on initial model constraints.

**Figure 3** Spatial solutions of the sample input land use map (upper left corner): grey and *black* – two hypothetical incompatible land uses, Iteration $n$ – HSJ $n$th model iteration, $b$ – DBDC threshold value. HSJ relaxation was set to 0%, $Fra$ – fragmentation statistics (the development perimeter to the square root of the development area).

**Figure 4** Chelan City (dark grey) and vicinity, Chelan County, Washington State, USA.

**Figure 5** Input land use map.

**Figure 6** Chelan model results for DBDC $b=1$ and $b=3$ and HSJ relaxation ($u_r$) of 0% and 10%: [1] number of new variables ($K_{new}$) added to the $K$-set, [2] fragmentation increase as compared to the input pattern in % (the initial solution is the HSJ iteration = 0).

**Figure 7** The most different outcome maps where $w=1000$, $u_r=10\%$, and $b=3$: (1) HSJ iteration 3, (2) HSJ iteration 4.
Table 1 Pairwise difference between output maps in the hypothetical example. The difference is measured in % according to equation (11): $i$ – HSJ iteration ($i=0$ – initial solution), $b$ – the value of $b$ in DBDC

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<th>$i=0$</th>
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<th>$b=0$</th>
<th>$i=2$</th>
<th>$b=0$</th>
<th>$i=3$</th>
<th>$b=0$</th>
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Table 2 Demand for allocation. Each cell covers an area of 2 acres

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<tr>
<th>Land use</th>
<th>Current number of cells (2000)</th>
<th>Demand (2025)</th>
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</thead>
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<td>Residential</td>
<td>2618</td>
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<tr>
<td>Commercial</td>
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<td>122</td>
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<tr>
<td>Industrial</td>
<td>68</td>
<td>92</td>
</tr>
</tbody>
</table>

Table 3 Compatibility of adjacent land uses. Value range is [0.0, 1.0], where higher values indicate more compatibility

<table>
<thead>
<tr>
<th>Land use</th>
<th>Restricted</th>
<th>Residential</th>
<th>Commercial</th>
<th>Industrial</th>
</tr>
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<td>0.5</td>
<td>0.0</td>
</tr>
<tr>
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<td>1.0</td>
<td>0.7</td>
<td>0.0</td>
</tr>
<tr>
<td>Commercial</td>
<td>0.5</td>
<td>0.7</td>
<td>1.0</td>
<td>0.2</td>
</tr>
<tr>
<td>Industrial</td>
<td>0.0</td>
<td>0.0</td>
<td>0.2</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 4 The HSJ objective function value [in 1000]

<table>
<thead>
<tr>
<th>HSJ iteration</th>
<th>$u_r=0%$ &amp; $b=1$</th>
<th>$u_r=0%$ &amp; $b=3$</th>
<th>$u_r=10%$ &amp; $b=1$</th>
<th>$u_r=10%$ &amp; $b=3$</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>874</td>
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<td>259</td>
<td>251</td>
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<tr>
<td>2</td>
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<td>1652</td>
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<td>648</td>
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<tr>
<td>3</td>
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<td>2522</td>
<td>1083</td>
<td>1075</td>
</tr>
<tr>
<td>4</td>
<td>infeasible</td>
<td>3418</td>
<td>1523</td>
<td>1487</td>
</tr>
</tbody>
</table>
Table 5 Pairwise difference between output maps in the Chelan example. The difference is measured in % according to equation (11): $i$ – HSI iteration ($i=0$ – initial solution), $b$ – the value of $b$ in DBDC, $u_r$ - user input parameter for objective value relaxation $i$

<table>
<thead>
<tr>
<th>map</th>
<th>ini</th>
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<th>$u_r = 10%$</th>
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<tr>
<td></td>
<td>$b=3$</td>
<td>16</td>
<td>12</td>
</tr>
</tbody>
</table>

Table 6 Difference between outcome maps as the % of maximum possible difference (average for all pairwise combinations)

<table>
<thead>
<tr>
<th>Weight $w$</th>
<th>Relaxation $u_r$ [%]</th>
<th>Average difference [%]</th>
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<tr>
<td>1</td>
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</tr>
<tr>
<td>1000</td>
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<td>20</td>
</tr>
<tr>
<td>1000</td>
<td>10</td>
<td>60</td>
</tr>
</tbody>
</table>
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<table>
<thead>
<tr>
<th>Iteration</th>
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<th>$b = 3$</th>
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<tbody>
<tr>
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<td>$Fra = 6.94$</td>
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<tr>
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<td>$Fra = 6.12$</td>
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<td>Iteration 2</td>
<td>$Fra = 9.80$</td>
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<td>Iteration 3</td>
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<td>$Fra = 7.35$</td>
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Figure 3 Spatial solutions of the sample input land use map (upper left corner): grey and black – two hypothetical incompatible land uses, Iteration $n$ – HSJ $n$th model iteration, $b$ – DBDC threshold value. HSJ relaxation was set to 0%, $Fra$ – fragmentation statistics (the development perimeter to the square root of the development area)
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