A New Method for Direct Computation of Critical Clearing Time for Transient Stability Analysis

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Abstract

This paper presents a new computation method for transient stability analysis for electric power systems. Different from existing methods, a minimization problem is formulated for obtaining critical clearing time (CCT). The method is based on the computation of a trajectory on the stability boundary, which is referred to as critical trajectory in this paper. The critical trajectory is defined as the trajectory that starts from a point on a fault-on trajectory and reaches a critical point of losing synchronism. The new proposal includes the critical conditions for synchronism and the unified minimization formulation using a modified trapezoidal formulation for numerical integration. It will be demonstrated that the solution of the minimization problem successfully provides the exact CCT that agrees with the conventional numerical simulation method.

I. Introduction

Transient stability analysis plays an important role for maintaining security of power system operation. The analysis is mainly performed through numerical simulations, where numerical integration is carried out step by step from an initial value to obtain dynamic response to disturbances. In general, such a numerical simulation method is effective since it can easily take into account various dynamic models for complex power systems as well as various time sequences of events. Furthermore, the method is useful in analyzing various kinds of complex nonlinear phenomena such as in [1-3]. However, the numerical simulation is usually time consuming, and therefore, it is not necessarily suited for real time stability assessment.

An alternative approach, called transient energy function methods [4-16], assesses system stability based on the transient energy. Those methods provide fast and efficient stability assessment for a number of disturbances.

Although they are practically useful, a common disadvantage is concerned with the accuracy of stability judgment. A major limitation is that they cannot deal with detailed models for power systems since the transient energy functions are available only for limited types of power system models. Another problem is that most of the methods require the evaluation of critical energy, which affects considerably the accuracy of stability assessment. The critical energy is not necessarily easily calculated.

This paper proposes a new method for transient stability analysis. In order to describe the proposed method, typical dynamic behaviors of a power system are given in Figure 1, where a single machine case is presented as an example. Three kinds of trajectories are given in phase plane starting at different points on a fault-on trajectory “1”. Trajectory “2” is for a stable case where the fault is cleared early enough and it oscillates around a stable equilibrium point. Trajectory “4” corresponds to an unstable case, where the fault clearing is too late. Trajectory “3” corresponds to a critical case for stability and is referred to as the critical trajectory in this paper. In a specific single machine case, the critical trajectory reaches an unstable equilibrium point (UEP) as shown in Figure 1. The critical trajectory is defined as the trajectory that
starts from a point on a fault-on trajectory and reaches a critical point that satisfies a set of conditions of losing synchronism. The conditions will be proposed in this paper. The critical point agrees with UEP for a single machine system as in Figure 1, although this is not the case for general multi-machine systems. It is generally difficult to compute the critical trajectory by means of conventional numerical simulations. In this paper, the problem is formulated as a minimization problem for computing the critical trajectory based on preliminary examinations in [17-21]. Critical conditions with CCT for transient stability are directly computed as the solution of the minimization problem.

II. Problem Formulation (Method A)

A. Definitions

Transient stability problem for an event disturbance may be expressed as follows: Initially, a power system is operating at a stable operating point, say $x_{pre}$, when a fault occurs at time $t = 0$. Then, the system is governed by the fault-on dynamics during the fault $[0, \tau]$ as follows:

$$\dot{x} = f_F(x), \ 0 \leq t \leq \tau, \ x(0) = x_{pre} $$

where $x \in R^N, \ t \in R, \ f_F : R^N \rightarrow R^N$

The solution curve of (1) is called fault-on trajectory and is expressed in this paper by:

$$x(t) = X_F(t; x_{pre}), \ 0 \leq t \leq \tau$$

where $X_F(\cdot; x_{pre}) : R \rightarrow R^N$

The fault is cleared at time $\tau$. The system is governed by the post-fault dynamics expressed by the following nonlinear equation.

$$\dot{x} = f(x), \ \tau \leq t \leq \infty; \ f : R^N \rightarrow R^N$$

The solution curves of (3) are called post-fault trajectory, represented by

$$x(t) = X(t; x^0), \ \tau \leq t \leq \infty; \ X(\cdot; x^0) : R \rightarrow R^N$$

Critical trajectory is defined as the post-fault trajectory (4) when the fault is cleared at $\tau = \text{CCT}$. The initial point $x^0$ for the critical trajectory is the point at CCT on the fault-on trajectory:

$$x^0 = X_F(\tau; x_{pre}), \ \tau = \text{CCT}$$

Equation (5) is referred to as initial condition for the critical trajectory hereafter, in which $\tau$ is treated as a variable to be obtained.

B. Modified Trapezoidal Formulation

In this section, we explain a modified trapezoidal form that we have proposed in [17]. Letting a solution of equation (3) at time $\hat{t}^k$ be denoted as $x^k$, the following equation holds using the conventional trapezoidal formula.

$$x^{k+1} - x^k = \frac{1}{2}(x^{k+1} + x^k)(\hat{t}^{k+1} - \hat{t}^k) $$

where,

$$\hat{x}^k = f(x^k)$$

In this paper, superscript $k$ is used for state transition number with respect to time.

As is stated in the Introduction, we pay attention to the critical trajectory, where a system fault is cleared at CCT and then the state variables converge to a critical point as stated before. In some specific case, the critical point agrees with an UEP and the trajectory reaches the UEP with infinite time. Figure 2 shows the critical trajectory, where two boundary points, $x^0$ and $x^u$, represent the initial point at CCT and the critical point. A difficulty in obtaining the critical trajectory is that infinite time may be taken when to reach UEP. An example is that a ball is moving towards the top of hill (UEP), whose kinetic energy is equal to the potential energy at UEP. The ball cannot reach the UEP within a finite time since the speed tends to zero as it becomes close to the UEP. For such computation, infinite time steps are required. To avoid the problem, we have developed a new method for numerical integration as follows. First, the distance between the two points in (6) is defined as:

$$\varepsilon = \left| x^{k+1} - x^k \right| = \frac{1}{2}|x^{k+1} + x^k|\left(\hat{t}^{k+1} - \hat{t}^k\right)$$

Thus, the time duration is replaced with the distance as follows:

$$\hat{t}^{k+1} - \hat{t}^k = \frac{2}{|x^{k+1} + x^k|} \varepsilon$$

Equation (8) is substituted into (6) to obtain the following form.

$$x^{k+1} - x^k - \frac{\hat{x}^{k+1} + \hat{x}^k}{2} \varepsilon = 0$$

By the above equation, the numerical integration with respect to time is transformed into that with distance. This transformation makes it possible to represent the critical trajectory by finite points with a same distance as shown in Figure 2.
C. Critical Condition – End Point Condition

As is given in Figure 1, the critical trajectory converges to UEP for a single machine infinite bus system. However, this is not the case in general. In this section, we propose critical condition for synchronism that suffices in general multi-machine systems. It is known in a single machine case that the synchronizing force disappears when

$$0 = \frac{\partial T}{\partial \theta}$$

or

$$0 = \frac{\partial P}{\partial \theta}.$$  

Where \(T\) and \(P\) respectively are the synchronizing torque and power, and \(\theta\) is the rotor angle. A natural extension of the condition for multi-machine case may be written based on singularity condition of synchronizing force coefficient matrix as follows:

$$v^T P = 0 \quad \text{with} \quad |v| \neq 0 \quad (10)$$

Where \(v \in R^{N_{gen}}\) is the eigenvector corresponding to zero eigenvalue of matrix \([\partial P/\partial \theta] \in R^{N_{gen} \times N_{gen}}\), and \(N_{gen}\) is the number of generators. We further intuitively assume a condition that the eigenvector must agree with change direction of \(\theta\). That is, the following equation holds with a scalar \(k_S \in R\):

$$v = k_S \cdot \dot{\theta} \quad (11)$$

It will be assumed that the above conditions (10) and (11) hold at a point (the end point) on the critical trajectory. All the variables in (11) will be treated as decision variables in the minimization problem in the next section.

Although it is not the complete proof of the stability condition for dynamic system, the equations represent the stationary conditions for the synchronizing torque or power, \(T\) or \(P\) as follows:

$$\dot{P} = 0 \quad (12)$$

Since \(P\) is basically a function of the rotor angles of generators, the following equation holds.

$$\dot{P} = \frac{\partial P}{\partial \theta} \cdot \dot{\theta} \quad (13)$$

The above equation implies that (11) and (12) are equivalent to each other under condition (10). Thus, the newly proposed conditions are to capture the stationary conditions for \(T\) or \(P\) caused by the singularity of synchronizing torque/power matrix. The conditions have been derived intuitively by the author and will be used in the paper without complete proof for instability. The conditions have successfully worked so far with no exceptions.

D. Problem Formulation (Method A)

Based on the above discussion, the problem for obtaining the critical condition for transient stability for system (3) is formulated as follows:

$$\min_{x^0, x^1, ..., x^{m+1}, \nu, k_S, \mu} \left\{ \sum_{k=0}^{m} (\mu^k)^T (\mu^k) + (\mu^{m+1})^T W_A (\mu^{m+1}) \right\}$$  

(14)

where, \(x^k \in R^N\), \(\nu \in R\), \(\tau \in R\), \(k_S \in R\), \(v \in R^{N_{gen}}\)

$$\mu^k = x^{k+1} - x^k - \frac{x^{k+1} + x^k}{2} \epsilon$$  

(15)

with boundary conditions:

$$x^0 = X_F (\tau; x_{pre}) \quad (17)$$

$$\mu^{m+1} = v - k_S \cdot \dot{\theta}^{m+1} \quad (18)$$

where \(W_A\) is a square weighting matrix with positive diagonal terms. Since the selection of \(W_A\) does not affect the convergence nor accuracy of proposed method, identity matrix will be used for \(W_A\) for all the simulations in the latter section. After the minimization of (14), \(\mu^k\) becomes ideally zero, where the proposed trapezoidal equation of (9) hold to connect all point \(x^k, k = 0 \ to \ m + 1\), in Figure 2. Equation (17), a boundary condition for the initial point, expresses a fault-on trajectory as a function of fault clearing time, \(\tau\). Equation (18) is the other boundary condition, where \(\theta^{m+1}\), sub-vector of \(x^{m+1}\), satisfies the critical conditions (10) and (11). It is noted that the latter boundary condition is additional compared with the conventional numerical integration formulated as...
an initial value problem. In such a conventional method, numerical error $\mu^k$ is accumulated as $k$ increases so that a final point in general has a considerable error. On the other hand, the proposed method specifies the final point additionally as in (18), then solves the redundant equations as a minimization problem so that the individual errors $\mu^k$ are properly distributed.

The solution of the problem, (14)-(18) is interpreted as follows. The set of points, $x^k$, $k = 0$ to $m + 1$, represent the critical trajectory, where $\varepsilon$ is automatically determined when the number of integration steps, $m$, is specified; CCT and the critical point are respectively obtained as $\tau$ and $x_{m+1}$ at the solution. Note that the proposed method is exact without major approximations in the formulation,

The problem formulation proposed in this section is referred to as method A hereafter. As will be confirmed in the latter section, method A works satisfactory.

III. Alternative Formulation (Method B)

A. Alternative End Point Condition

This section proposes an alternative formulation using a different end point condition, referred to as method B hereafter.

For a single machine to infinite bus system as stated in Figure 1, the critical trajectory converges to UEP of post-fault systems. Using theXd' generator model for the post-fault system, the UEP denoted by $x^u$ is represented as follows:

$$0 = f\left(x^u\right), \text{ with } x^u = [\theta^u, 0^u]$$ (19)

Although the critical trajectory does not converge to UEP in general multi-machine systems, we have performed exhaustive examinations and have shown that the following endpoint conditions work for obtaining the critical trajectory.

End point condition

Although all the elements of $x$ do not reach $x^u$, a specific pair of the elements in $x$ reach the corresponding elements of $x^u$. That is, the following condition holds at the end point, $m+1$:

$$\theta_{cg}^{m+1} = \theta_{cg}^u, \quad \omega_{cg}^{m+1} = \omega_{cg}^u$$ (20)

Index cg represents a specific generator number which is defined here as the critical generator. The above conditions imply that only the two state variables corresponding to the critical generator (CG) reach the corresponding UEP. The conditions (20) are simpler than conditions (10) and (11), and therefore they are potentially useful for the computation of CCT.

$\bar{x}^u$ is Controlling UEP (CUEP) that satisfies equilibrium equation (19). This CUEP, $\bar{x}^u$, has been determined separately in advance by other methods such as the boundary controlling UEP (BCU) shadowing method [12, 17-18].

Critical Generator (CG) selection plays an important role in this paper. After various examinations, we utilize the potential energy value of each generator as an index for the selection. The potential energy in the energy function corresponding to this system is given as follows:

$$Ep = \sum_i E_{pi}$$ (21)

where

$$E_{pi} = \left[P_{mi} - \left(E_i \right)^2 G_{ii} \right] \left[\theta_i - \theta_i^s\right]$$ (22)

$E_i$ is constant voltage behind the direct axis transient reactant; $G_{ii}$ is driving point conductance. Superscript $s$ represents stable equilibrium point (SEP); $E_{pi}$ is used as an index to select CG.

Based on (21) and (22), generator with the largest index value of $E_p$ is selected as the critical generator (CG). It is noted that the index works almost satisfactory in our numerical examinations, identifying critical generator very accurately.

B. Problem Formulation (Method B)

According to previous discussion, the problem for obtaining the critical condition for transient stability for system (3) is formulated as follows:

$$\min_{x^0, x^1, ..., x^m, x, \varepsilon} \left\{ \sum_{k=0}^m (\mu_k) + (\mu_{m+1})^* W_{B} (\mu_{m+1})^* \right\}$$ (23)

Where, $x^k \in \mathbb{R}^N$, $(k = 0, \ldots, m)$, $\varepsilon \in R$, $\tau \in R$

Subject to equations (15) and (16),
initial conditions (17), and end point condition as below:
\[ \mu^{m+1} = x^{m+1} - x^u \quad \text{with} \quad f(x^u) = 0 \] (24)

The difference between methods A and B lies only in the treatment of the end point condition and the related variables in the minimization.

The setting of \( W_B \) is essential in method B to set the condition (20). \( W_B \) is a square weighting matrix with positive diagonal terms for (24), where a large weight is assigned for the CG and small values for other generator terms for the UEP elements of \( x^m = x^u \). The other equations are the same as those in method A.

IV. Numerical Simulations

A. Power System Model

The center of angle (COA) or center of inertia swing equations with \( X_d' \) generator model with/without controller are used for both the conventional numerical simulation and the proposed method. The models are given in the Appendix.

B. Simulation Method for Exact Solutions

In order to confirm the effectiveness of the proposed method, we have performed numerical examinations using 30-machine 115-bus system (IEEE Japan West 30) as shown in Figure 3. It is supposed that every transmission line consists of double parallel circuits, and that a three phase fault occurs at a point very close to a bus on one of the parallel lines. After a while, the fault is cleared by opening the faulted line.

The 4-th order Runge-Kutta method is applied for numerical integration with time step of 0.001 [s]. First, the fault-on trajectory is obtained numerically, which is saved as \( x^\theta(\tau) \) as a function of time, \( \tau \) in the computer memory. Then, \( x^\theta(\tau) \) with a specified \( \tau \) is chosen as an initial condition to simulate the dynamic behavior to evaluate the stability of the system. This process is repeated by setting different value of \( \tau \). The binary search method is used to judge a critical value of \( \tau \) that is CCT. The obtained results are given in Table 1, where the fault location, CCTs, and percent error in CCT are shown. For example, in Table 1, the expression of “0.173-0.174” means that the system is stable with clearing time of 0.173 [s] but unstable with 0.174 [s] and that exact value of CCT exists between 0.173 and 0.174 [s].

C. Method A

The method A is performed as in the following procedure:
1. The fault-on trajectory is obtained as \( x^\theta(\tau) \) using the numerical simulation method and is approximated as a cubic spline data interpolation to define (16);
2. Equations (A1)-(A2) are used to define (15);
3. The least square minimization problem, (14), is solved using the Newton Raphson (NR) method with \( \max |dx| < 0.01 \) as a convergence criterion to obtain CCT.

D. Method B

The method B is executed as in the following procedure:
1. Same as step 1 for Method A;
2. The CUEP \( x_s \) is obtained by the shadowing method.
3. The CG is selected using (22) to define \( W_B \);
4. The least square minimization problem, (23), is solved by the same manner for Method A to obtain CCT.

E. Results for \( X_d' \) Generator Model

The CCT for the test system obtained by both methods A and B are shown in Table 1, where critical generator (CG) used for method B is also indicated. It is affirmed by this table that the CCTs obtained by methods A and B are exact enough compared with the conventional numerical simulation method. It is realized that the methods are also numerically robust enough to obtain the exact CCT, where no computational difficulties exist.

Figures 4 - 6 show the waveforms of rotor angles of generator \( i \), \( \theta_{gi} = \theta_i - \theta_{gi} \), where generator 30 is selected as angle reference to analyze the phenomena clearly. All those figures are for fault at point G in the 30-machine system. Figure 4 focuses on the behavior of generator 1 indicating the critical waveforms obtained by A and B, with the stable and unstable waveforms given by the conventional numerical simulations. It is confirmed that both the methods provide the sufficiently accurate critical waveform as well as the CCT.
1 for fault at point G.

Fig. 4. Critical, unstable and stable waveforms of rotor angle of generator 1 for fault at point G.

Table 1. CCTs for IEE Japan West 30 without Controllers

<table>
<thead>
<tr>
<th>Fault Point</th>
<th>Open Line</th>
<th>Method A</th>
<th>Method B</th>
<th>Simulation</th>
<th>Shadowing</th>
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<td>0.1718</td>
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<td>B</td>
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<tr>
<td>C</td>
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<td>0.3119</td>
<td>0.2894</td>
<td>0.3102</td>
<td>0.3131</td>
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<td>0.1511</td>
<td>0.1514</td>
</tr>
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</tr>
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<td>0.0000</td>
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<td>0.3033</td>
</tr>
<tr>
<td>G</td>
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<td>0.0733</td>
<td>0.2728</td>
<td>0.2734</td>
</tr>
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Fig. 5. Critically stable rotor angle curves for Clearing Time (CT) = 0.272 [s] for fault at point G.

Fig. 6. Critically unstable rotor angle curves for CT = 0.273 [s] for fault at point G.

Fig. 7. Errors in CCT w.r.t. parameter m.

Fig. 8. CPU time w.r.t. parameter m.
Figures 5 and 6 respectively depict the stable and unstable waveforms of generators 1-2, 10-11, 19-20, 28-29 obtained by the conventional numerical simulations with the critical waveforms obtained by methods A and B in common. As the system becomes larger, the unstable phenomena tend to become complex, but the both of proposed methods successfully calculate the critical condition.

It is interesting to observe that the critical trajectories almost agree to each other for methods A and B, but the method A terminates a few steps before the latter end point.

Figure 7 shows the errors in CCT for different setting of $m$. It is observed that parameter $m=10$ usually provide exact CCT with negligible error for this test system. This setting is utilized to obtain CCTs in Table 1.

Figure 8 depicts the CPU time for test system. It is observed that parameter $m=8$ for method A and $m=7$ for method B provide the CCTs with error less than 0.01 and can save more CPU time. It is noted that the computation time for method A is faster than the method B.

Figure 9 represents the objective values after the minimization, representing the minimization error of $\mu$. It is observed that the objectives for both the methods are small enough. Note that an Intel® Core™2 Duo E8500 with 3 GB of RAM is used for the numerical examinations.

As seen from the examinations, method A is superior to method B when it is used as an independent computer program. However, potential ability of method B seems superior to method A due to its simplicity of formulation, implying a large application area. An application to a more detailed power system model will be given in the next section.

**F. Results for Xd’ Gen Model with Controller**

Examinations in this section are only for method B because of easier treatment. Tables 2-4 show the obtained CCTs for the Xd’ generator model with controllers, which are given in the Appendix. It is observed that the obtained CCT for method B agrees with conventional numerical simulation method.

**V. Conclusions**

This paper proposes a new formulation for transient stability analysis as a minimization problem for electric
power systems. Different from conventional simulation methods, the formulation is not based on an initial value problem but based on boundary value problem to directly obtain a critical condition for stability such as a Critical Clearing Time (CCT). The method computes the critical trajectory that represents a critical case for stability.

It is confirmed that the proposed method can provide the exact CCTs, consistent with the conventional numerical simulations. The result is quite important since no such methods have existed so far to compute the exact CCT without major approximations. Another important point is that the proposed method is numerically robust for detecting CCTs for various patterns of complicated instability phenomena. It is expected that the method is able to be applied to larger systems. At present, the proposed method cannot analyze multiple swing instabilities, which should be studied in the future.

The Xd’ generator model is used to confirm the validity of the new formulation in this paper. Also promising results are obtained for the Xd’ model with AVR and Governor. Since the proposed method can deal with various types of power system models at least in theory, a further study is necessary in order to take into account more detailed generator models together with various types of controllers in the future.

VI. Appendix

The proposed method is in theory able to evaluate exact CCTs directly for various multi-machine power system models. In this paper, we have confirmed the performance of the proposed method for the Xd’ generator model with and without controllers. The Xd’ generator model is represented by two dimensional differential equations. The center of inertia (COI) swing equation in reference [4] is used for both the conventional numerical simulation and the proposed method as follows:

Xd’ Generator Model

\[
P_{\text{v}}(\theta) = \sum_{j=1}^{n} Y_{ij} E_{i} E_{j} \sin(\theta_{i} - \theta_{j} + \alpha_{ij})
\]

Controllers

\[
\dot{E}_{i} = \frac{1}{T_{\text{AVRi}}} \left[ (E_{\text{refi}} - E_{i}) + K_{\text{AVRi}} (V_{\text{refi}} - V_{i}) \right]
\]

\[
\dot{P}_{\text{m}} = \frac{1}{T_{\text{GOVi}}} \left[ (P_{\text{refi}} - P_{\text{m}}) + K_{\text{GOVi}} (\dot{\theta}_{i}) \right]
\]

P_{\text{m}} is mechanical power input i-th; \omega_{i} is generator rotor speed i-th; \delta_{i} is generator angle deviation i-th; M_{i} is moment of inertia i-th; D_{i} is damping coefficient i-th; E_{i} is voltage behind transient reactance i-th; V_{\text{refi}} is terminal voltage reference i-th; V_{i} is terminal voltage i-th; P_{\text{refi}} is mechanical power reference i-th; P_{\text{m}} is electric power i-th; K_{\text{AVRi}} is automatic voltage regulator (AVR) gain i-th; T_{\text{AVRi}} is AVR time constant i-th; K_{\text{GOVi}} is governor gain i-th; T_{\text{GOVi}} is governor time constant i-th.

In the main text, we use two kinds of power system models. The Xd’ model implies (A1) and (A2) to define power system models (1) and (3) in fault condition as well as post fault condition, while the Xd’ model with controllers (A1) to (A4).

References


