Abstract—Analog Circuit sizing is the task to determine the sizes of all components in the circuit during automated synthesis. Randomized combinatorial optimization algorithms are desired for quicker determination of a set of optimal sizes of the components. These algorithms require set of multiple performance parameters, for a very large number of sized circuits. Therefore the reduction in time required to estimate these performance parameters is also highly desired. For the purpose of estimation of performance parameters, we employ Support Vector Machine (SVM) based macro-models of analog circuits, instead of using SPICE simulation. These SVM macro-models are not only faster to evaluate, but use of efficient kernel functions has also made them almost as accurate as SPICE. In this paper, we report multi-objective genetic algorithm for simultaneous optimization of multiple performance parameters. We compute the Pareto optimal points for various performance parameters of a two-stage op-amp circuit in 180 nm technology. We perform SVM classification and regression using Least Square SVM toolbox [1] with MATLAB. HSPICE was used to generate data-set from simulation of two-stage op-amp, which was used to train the SVM macro-model. The results pertaining to total time consumed in sizing task are very encouraging. We observed 'time taken' in one evaluation by SVM macromodel as compared to HSPICE is upt to two order smaller, resulting in speed-up of approximately 20.

I. INTRODUCTION

Circuit synthesis of analog circuit is the task to determine the size of all components in the circuit so that it is able to meet the set of performance specifications. Optimum sizes of the components are determined using stochastic combinatorial optimization method such as simulated annealing and genetic algorithms. Since, performance parameters for great number of circuit sizing values are needed by these algorithms, the reduction in time is highly desired to estimate these performances.

Many macromodeling techniques have been proposed to match the non-linear performance functions to design parameters [2]–[4]. In this paper, we have used Support Vector Machine (SVM) [5]–[7] based macro-models to provide robust and accurate estimate of performance parameters for two stage op amps. The utility of these models is demonstrated in circuit sizing methodology using multi-objective genetic algorithm optimization. The SVM models used in this work were trained using data generated directly from SPICE and therefore are able to provide SPICE level accuracy. Since, the evaluation time taken by the SVM models is much less than the time needed for a full SPICE simulation, the models can be used inside an optimization loop as cost function during circuit synthesis.

Performance Macromodeling usually consist of two steps: feasibility design space identification and performance macromodels generation. A feasibility design space is defined as a multidimensional space in which every design satisfies all the design constraints. The minimum set of constraints is the one that ensures the correct functionality of the given circuit topology. Performance macromodels are only constructed and thereby valid in the functionally correct design space. Support vector machines (SVMs) are used as classifier to identify the feasible design space of analog circuits and then as regressor to model performance function of the circuits. Once SVM models are developed for different performance parameters of op-amp, one model for each of the parameter, the op amp can then be synthesized for different performance specifications.

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Multi-objective Genetic Approach for Analog Circuit Sizing using SVM Macro-model

D. Boolchandani  
Dept. of ECE  
NIT Jaipur, India - 302017  
Email: dbool@ieee.org

Anupam Kumar  
Centre for Development of Telematics  
Mehrauli Road, New Delhi, India - 110 016  
Email: anupam.mnit@yahoo.co.in

Vineet Sahula  
Dept. of ECE  
NIT Jaipur, India - 302017  
Email: sahula@ieee.org
II. BACKGROUND & RELATED WORK

A. SVM Classification

The procedure is a separation out infeasible design points from feasible ones. In this section, our further discussion is based on [5] [8]. All such points together constitute input data to classifier. We consider each of N data points \( x_k \in \mathbb{R}^n \), \( k = 1, \ldots, N \) to be associated with a label \( y_k \in \{+1, -1\} \) thus classifying the data into one of two sets. In the simplest SVM formulation, the problem of finding a general representation of the classifier \( y(x) \) becomes that of the construction of a hyper-plane \( \omega^T x + b \) which provides 'maximal separation' \( \frac{2}{\|\omega\|^2} \) between points \( x_k \) belonging to the two classes. This give rise to an optimization problem of the form given by (1), and its dual in (2) using Lagrangian form. Here the \( \frac{1}{2} \omega^T \omega \) term represents a cost function to be minimized in order to maximize separation. The constraints are formulated such that the nearest points \( x_k \) with labels [either +1 or -1] are (with appropriate input space scaling) at least \( \frac{1}{\|\omega\|^2} \) distant, from the separating hyper-plane.

\[
P : \min_{\omega, b} \quad \frac{1}{2} \omega^T \omega \quad \text{s.t.} \quad y_k[\omega^T x_k + b] \geq 1 \quad (1)
\]

\[
D : \max_{\alpha} \mathcal{L}(w; b; \alpha) \quad (2)
\]

Here, \( \mathcal{L}(\omega; b; \alpha) = \frac{1}{2} \omega^T \omega - \sum_{k=1}^{N} \alpha_k (y_k[\omega^T x_k + b] - 1) \) and \( \alpha_k \) are the Lagrange multipliers. After applying the conditions as in (3) for optimality, kernel trick [7] and eliminating \( \omega \) by expressing it in terms of \( \alpha = [\alpha_1, \ldots, \alpha_N] \), we arrive at a Quadratic Programming (QP) problem as in (4).

\[
\frac{\partial \mathcal{L}}{\partial \omega} = 0, \quad \frac{\partial \mathcal{L}}{\partial b} = 0, \quad \frac{\partial \mathcal{L}}{\partial \alpha_k} = 0,
\]

\[
\min(\alpha Q\alpha + B\alpha), \quad (3)
\]

for suitably defined matrices \( Q, B \). Having solved for \( \alpha \), the following classifier representation is obtained

\[
y(x) = \text{sign} \left( \sum_{k=1}^{\#SV} \alpha_k y_k x_k^T x + b \right). \quad (5)
\]

Here \( \#SV \) represents the number of non-zero Lagrange multipliers \( \alpha_k \), called support vectors, corresponding to input data \( x_k \). A key feature of the Support Vector Machines is the ability to replace the input data by a non-linear function \( \phi(x) \) operating on the input data. This may be viewed as mapping the input data to higher dimensional space, to enable classification of data that is not linearly separable in the original input space. To do this, we formally replace \( x_k^T x \) (the dot product between a support vector \( x_k \) and any point \( x \) of the input space) in (5) by \( \phi(x_k)^T \phi(x) \) to represent the action of this mapping, obtaining

\[
y(x) = \text{sign} \left( \sum_{k=1}^{\#SV} \alpha_k y_k \phi(x_k)^T \phi(x) + b \right). \quad (6)
\]

The expression \( \phi(x_k)^T \phi(x) \) may under certain conditions be replaced by a Kernel function \( K(x_k, x) \). There are different kernel functions that provide the SVM, the ability to model complicated separation hyperplanes. We use the efficient kernels proposed in [9] to built SVM model for classifying feasible design space.

B. SVM Regression

In the case of function regression, the labels \( y_k \in \{-1, 1\} \) represented by the \( \{+1, -1\} \) valued function \( y(x) \) are replaced by the real valued \( y_k \in \mathbb{R} \). Further by a certain nonlinear mapping \( \phi \), the training pattern \( x_k \) is mapped into some feature space, in which a real valued function \( y(x) \) is defined as follows.

\[
y(x) = \omega^T \phi(x) + b \quad \text{with} \quad \omega \in \mathbb{R}^n, b \in \mathbb{R} \quad (7)
\]

Here, \( \phi(\cdot) : \mathbb{R}^n \rightarrow \mathbb{R}^m \) is the mapping to the high dimensional and potentially infinite dimensional feature space. For the Least-Squares SVM regression error variables for the fitting problem are as given in (8).

\[
e_k = w^T \phi(x_k) + b - y_k \quad k = 1, \ldots, N \quad (8)
\]

Given a training set \( \{x_k, y_k\}_{k=1}^{N} \) following optimization problem is formulated in the primal weight space together with the N constraints of (8).

\[
P : \min_{w, b, e} \quad J_p(w, e) = \frac{1}{2} w^T w + \gamma \frac{1}{2} \sum_{k=1}^{N} e_k^2 \quad (9)
\]

This formulation involves the trade off between a cost function term and a sum of squared errors governed by the trade-off parameter \( \gamma \). In the regression formalism the term \( \frac{1}{2} w^T w \) is no longer related to hyper-plane separation, but instead determines the smoothness of the resulting model. In fact, the primal problem in the LS-SVM formalism is wholly equivalent to a ridge regression problem formulated in the feature space, with parameter \( \gamma \) performing the role of smoothing parameter. The dual Lagrangian-based formulation is given in (10), where \( \mathcal{L} = J_p(w, e) - \sum_{k=1}^{N} \alpha_k \{w^T \phi(x_k) + b + e_k - y_k\} \) and \( \alpha_k \) are Lagrange multipliers.

\[
D : \max_{\alpha} \mathcal{L}(w; b; e; \alpha) \quad (10)
\]

\[
\mathcal{L} = J_p(w, e) - \sum_{k=1}^{N} \alpha_k \{w^T \phi(x_k) + b + e_k - y_k\} \quad (11)
\]

The conditions for optimality are given by (12).

\[
\begin{align*}
\frac{\partial \mathcal{L}}{\partial w} &= 0 \quad \rightarrow \quad w = \sum_{k=1}^{N} \alpha_k \phi(x_k) \\
\frac{\partial \mathcal{L}}{\partial b} &= 0 \quad \rightarrow \quad \sum_{k=1}^{N} \alpha_k = 0 \\
\frac{\partial \mathcal{L}}{\partial \alpha_k} &= 0 \quad \rightarrow \quad \alpha_k = \gamma e_k, k = 1, \ldots, N \\
\frac{\partial \mathcal{L}}{\partial e_k} &= 0 \quad \rightarrow \quad w^T \phi(x_k) + b + e_k - y_k = 0
\end{align*} \quad (12)
\]
After elimination of the variables \( w \) and \( e \) one gets the solution as in (13), where \( y = [y_1; \ldots; y_N] \), \( w = [1; \ldots; 1] \) and \( \alpha = [\alpha_1; \ldots; \alpha_N] \). The kernel trick is applied as in (14).

\[
\begin{bmatrix}
0 \\
1_N \\
\Omega + I/\gamma \\
\end{bmatrix}
\begin{bmatrix}
b \\
\alpha \\
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
y \\
\end{bmatrix}
\quad (13)
\]

\[
\Omega_{kl} = \phi(x_k)^T \phi(x_l) = K(x_k, x_l) \quad k, l = 1, \ldots, N \quad (14)
\]

The resulting LS-SVM model, after elimination of variables and applying kernel trick [7], for function estimation becomes

\[
y(x) = \sum_{k=1}^{N} \alpha_k K(x, x_k) + b
\quad (15)
\]

C. Pareto optimal surface

Concept of Pareto optimality is used to characterize competing objectives. For a problem, where multiple performances need be optimized, let there be vector of objectives \( F(x) = [F_1(x), F_2(x), \ldots, F_m(x)] \) that must be traded off. Multi-objective optimization is concerned with the minimization of a vector of objectives \( F(x) \) that can be the subject of a number of constraints or bounds as given in (16). The performance vector \( F(x) \) maps parameter space into objective function.

\[
\min_{x \in \mathbb{R}^n} F(x) \quad \text{subject to} G_i(x) = 0, \text{ for } i = 1, \ldots, k;
\]

\[
G_i(x) \leq 0, \text{ } i = k_{c+1}, \ldots, k; \text{ and } l \leq x \leq u
\quad (16)
\]

D. Pareto optimal solution

Let us consider a feasible region, \( \Omega \), in the parameter space. \( X \) is an element of the \( n \)-dimensional real numbers \( x \in \mathbb{R}^n \) that satisfies all the constraints i.e., \( \Omega = \{x \in \mathbb{R}^n\} \), subject to \( G_i(x) = 0, i = 1, \ldots, k_c \), and \( G_i(x) \leq 0, i = k_{c+1}, \ldots, k, \) \( l \leq x \leq u \). Point \( x^* \in \Omega \) is a Pareto optimal solution if for some neighborhood of \( x^* \) there’s not exist a \( \Delta x \) such that \( (x^* + \Delta x) \in \Omega \) and \( F_i(x^* + \Delta x) \leq F_i(x^*) \), \( i = 1, \ldots, m \), and \( F_j(x^* + \Delta x) \leq F_j(x^*) \) for at least one \( j \). In the two dimensional representation of the Fig. 1, the set of Pareto optimal solution lies on the curve between C and D. Points A and B represent specific Pareto optimal points because an improvement in one objective, \( F_1 \), requires a degradation in the other objective, \( F_2 \), i.e., \( F_{1B} < F_{1A}, F_{2B} > F_{2A} \).

The Pareto curve is the set of \( x^* \) where there are no other solutions for which simultaneous improvement in all objectives can occur. A general goal in multi-objective optimization is constructing the Pareto optima. We have used GA-multi-objective tool box in MATLAB to create a set of Pareto optima. It uses genetic algorithms to identify Pareto points.

III. EXPERIMENTAL SETUP & RESULTS

A known instance of all the design variable is considered a tuple. HSPICE is used on each of these tuples of design variables to evaluate performance and verify functional and performance constraints. For given set of tuples which satisfy both functional and performance constraints, output is taken as '1' otherwise as '-1'. This forms set of input and output data pair. Some of these data pairs are used to train SVM classifier and while the remaining are used for validation i.e. to check the accuracy of classifier.

The circuit simulator HSPICE is also used to compute several standard performance parameters describing the functionality of operational amplifier circuit. By doing SPICE simulation for many combinations of op amp transistor sizes, training and validation data sets are produced. These are then used to generate SVM model for the performance parameter. A general iterative methodology as suggested in [9] and reproduced in Fig. 2, is used for SVM model development.

![Figure 2. Flow diagram of SVM model development.](image-url)

After training, the SVM model for given op amp topology is used to provide estimates of op amp performance parameter...
during optimization. Op amp under various performance constraints is then synthesized using a genetic algorithm. Genetic algorithm is chosen since it is robust in the presence of multiple constraints and is insensitive to the nature of the cost function. Genetic algorithm have been successfully used previously for synthesis of analog circuits [10], [11]. The block diagram in Fig. 3 depicts the synthesis methodology using the SVM models.

A. Nominal Sizing of Two Stage op-amp

A single output two-stage op-amp is shown in Fig. 4. The circuit has 8 transistors, compensation and load capacitance and a reference bias current. The length of all transistors are fixed to a 1 μm. This immediately eliminates nearly half of the free design parameters. Further imposing sizing rule mentioned in [12], results in the 5-dimensional parametric configuration for the two-stage op-amp. The design variables and geometry constraints are shown in Table I. Functional and Performance constraints are shown in Table II. The functional constraints ensure all the transistors are on and in saturation region with some margin. We set $V_{on,min}$ and $V_{sat,min}$ to 0.1V.

Least Square SVM toolbox interfaced with MATLAB was used for developing the classifier to identify feasible design space. Data in feasible design space was used to develop SVM regressor models for different performance parameters like open loop gain, phase margin and unity gain frequency. The respective correlation coefficients between outputs of SVM model and SPICE for various performance parameters of two stage op-amp are computed and illustrated in Fig. 5. The correlation coefficient between outputs of SVM model and SPICE of various performance parameters of two stage op-amp are found close to one, which implies that models are quite accurate. Further from the Table IV we see that these models are also quite efficient as the time taken to compute performance parameters is very less as compared to HSPICE.

![Figure 3. Block diagram of circuit synthesis via a genetic optimization engine with SVM models](image)

![Figure 4. Two stage op amp](image)

![Figure 5. Correlation factors for (a) Gain and (b) UGF](image)

**Table I**

<table>
<thead>
<tr>
<th>Design variables</th>
<th>Geometric constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_1 = W_2$</td>
<td>[1μm, 100μm]</td>
</tr>
<tr>
<td>$W_3 = W_4$</td>
<td>[1μm, 50μm]</td>
</tr>
<tr>
<td>$W_5$</td>
<td>[1μm, 100μm]</td>
</tr>
<tr>
<td>$W_7$</td>
<td>[1μm, 100μm]</td>
</tr>
<tr>
<td>$C_c$</td>
<td>[5pF, 20pF]</td>
</tr>
</tbody>
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**Table II**

<table>
<thead>
<tr>
<th>Performance constraints</th>
<th>Functional constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{gs} - V_{th} \geq V_{on,min}$</td>
<td></td>
</tr>
<tr>
<td>$V_{ds} \geq V_{gs} - V_{th} + V_{sat,min}$</td>
<td></td>
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</tbody>
</table>

**Table III**

<table>
<thead>
<tr>
<th>Performance specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open-loop gain $\geq 60 \text{dB}$</td>
</tr>
<tr>
<td>Phase-margin $\geq 70^\circ$</td>
</tr>
<tr>
<td>UGF $\geq 2 \text{MHz}$</td>
</tr>
</tbody>
</table>

**Table IV**

<table>
<thead>
<tr>
<th>Performance parameter</th>
<th>SVM-Model</th>
<th>HSPICE</th>
<th>Speed-up</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open loop Gain</td>
<td>0.4449 Sec</td>
<td>10.0 Sec</td>
<td>22.48</td>
</tr>
<tr>
<td>Unity Gain Frequency</td>
<td>0.4450 Sec</td>
<td>10.0 Sec</td>
<td>22.47</td>
</tr>
<tr>
<td>Phase Margin</td>
<td>0.4457 Sec</td>
<td>10.0 Sec</td>
<td>22.44</td>
</tr>
</tbody>
</table>
Table V

<table>
<thead>
<tr>
<th>Design Variable</th>
<th>Optimal Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_1 = W_2$</td>
<td>$6.48 \times 10^{-13} \mu m$</td>
</tr>
<tr>
<td>$W_3 = W_4$</td>
<td>$9.63 \times 10^{-13} \mu m$</td>
</tr>
<tr>
<td>$W_5$</td>
<td>$2.89 \times 10^{-13} \mu m$</td>
</tr>
<tr>
<td>$W_7$</td>
<td>$6.50 \times 10^{-13} \mu m$</td>
</tr>
<tr>
<td>$C_c$</td>
<td>$6.79 \times 10^{-12} F$</td>
</tr>
</tbody>
</table>

Table VI

<table>
<thead>
<tr>
<th>Performance Parameters</th>
<th>HSPICE</th>
<th>SVM Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open-loop gain</td>
<td>61.71 dB</td>
<td>61.23 dB</td>
</tr>
<tr>
<td>Phase-margin</td>
<td>122°</td>
<td>135°</td>
</tr>
<tr>
<td>UGF</td>
<td>2.75 MHz</td>
<td>3.05 MHz</td>
</tr>
</tbody>
</table>

Table VII

<table>
<thead>
<tr>
<th>Design Variables for Cascode Op-amp</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_1 = W_2$</td>
</tr>
<tr>
<td>$W_3 = W_4$</td>
</tr>
<tr>
<td>$W_5 = W_6$</td>
</tr>
<tr>
<td>$I_{bias}$</td>
</tr>
<tr>
<td>$C_L$</td>
</tr>
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Table VIII

<table>
<thead>
<tr>
<th>Functional constraints</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{gs} - V_{th} \geq V_{on,min}$</td>
<td></td>
</tr>
<tr>
<td>Performance constraints</td>
<td>Phase Margin $\geq 60^\circ$</td>
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</tbody>
</table>

Table IX

<table>
<thead>
<tr>
<th>Performance Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>CMRR $\geq 100 dB$</td>
</tr>
<tr>
<td>PSRR $\geq 120 dB$</td>
</tr>
<tr>
<td>Phase margin $\geq 70^\circ$</td>
</tr>
<tr>
<td>Slew rate $\geq 1.25 \times 10^6 V/sec$</td>
</tr>
<tr>
<td>Unity-gain frequency (UGF) $\geq 1 \times 10^8 Hz$</td>
</tr>
</tbody>
</table>

Table X

<table>
<thead>
<tr>
<th>Design Variables obtained from optimal sizing of cascode op-amp</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_1 = W_2$</td>
</tr>
<tr>
<td>$W_3 = W_4$</td>
</tr>
<tr>
<td>$W_5 = W_6$</td>
</tr>
<tr>
<td>$I_{bias}$</td>
</tr>
<tr>
<td>$C_L$</td>
</tr>
</tbody>
</table>

All the performance parameter satisfy the constraint specified in sizing algorithm.

C. Multi-objective Sizing of op amp

Using multi-objective genetic optimization [13], the surface of Pareto-optimal design point is computed for three performance parameters of two stage op amp. Performance parameters that are considered are open-loop gain, phase-margin and unity gain frequency. Multivariate regression model is developed using SVM of two stage op amp for the above three output performance function of op amp. The multivariate model has SPICE level accuracy as it is trained by data obtained through SPICE simulation and is quite fast in evaluation of circuit performances for given set of design variables. This models is then used within multi-objective genetic algorithm to generate set of Pareto-optimal points. Pareto optimal points obtained for two stage op amp are shown in Table XII. It was run for 122 generations with population size of 60. Crossover function used was scattered with crossover fraction of 0.8. Pareto-optimal plots for two stage op amp are shown in Fig. 7 and Fig. 8. In the two figures, we observe that there is an obvious trade off between gain and phase margin and also between gain and unity gain frequency. Pareto-optimal points
of three performance parameters of two stage op amp is shown in Fig. 9. Total computation time taken by GA-multi-objective algorithm was 437.63 seconds.

![Figure 7. Pareto optimal front for UGF with Gain](image1)

![Figure 8. Pareto optimal fronts for Gain with Phase margin](image2)

### IV. Conclusions

We have proposed multi-objective genetic algorithm formulation based on SVM macro models for analog circuit sizing. The analog macro-models replace Spice simulators, used for circuit fitness evaluation inside GA, consuming a very little time and are almost as accurate as Spice. We have formulated the SVM macro-model for 2-stage op-amp and cascode op amp using multi-variate regression using efficient kernel functions for SVM. Multi-objective sizing provides much desired Pareto optimal points from the large design space. Input parameters corresponding to these Pareto points in turn provide appropriate sizes of devices. Further work is being carried out for tuning the SVM kernel parameters, which would enhance the optimality of solution.

### REFERENCES


![Figure 9. Pareto points on surface for three performance parameters](image3)