A procedure for an integrated network and vehicle routing optimisation problem

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Abstract

In this paper, an integrated Network and Vehicle Routing Optimisation Problem (NVROP) for freight vehicles is proposed. The variables involved in the problem are discrete (link topology, routes for freight vehicles) and continuous (link capacity in term of signal settings). The discrete variables are related to the link direction, lane numbers, lane use and sequence of retailers reached by the vehicles. The continuous variables, assuming that the link capacity is a function of the characteristics of the final node in the link, are related to the regulations at the intersection. The NVROP is, hence, formulated as a mixed (discrete/continuous) problem in a congested network. A heuristic procedure is used both to find the best network configuration and the best vehicle route.

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1. Introduction

The Network Optimisation Problem (NOP) consists of finding the optimal configuration of an existing network while optimising certain criteria (i.e. total delay minimisation). The NOP ranges from the link direction and lane allocation optimisation (discrete problems) to capacity optimisation (continuous problems). In this paper, the capacity optimisation is considered as signal setting optimisation at the intersections.

The Vehicle Routing Problem (VRP) consists of finding the optimal set of routes while respecting some constraints. The VRP for optimising the routes for a fleet of petrol delivery truck was first formulated by Dantzig and Ramser (1959). After its initial introduction, it has been extended to other fields (i.e. emergency vehicles).

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In most cases, the NOP and the VRP are treated separately, but they can be connected since network optimisation influences the path of vehicle. In this paper, a formulation trying to link the two problems is proposed. The result is a two stages problem (network and route optimisation).

The NOP formulated considers link orientation, lane allocation and signal setting optimisation to regulate the intersections and, hence, optimise the link capacity. The objective function is related to the total travel time, linked with a user equilibrium approach, spent on the network. As formulated, the problem has both discrete variables (link orientation and lane allocation) and continuous variables (capacity).

The VRP formulated considers that the costs involved in the problem (path costs) are those obtained solving the NOP.

In this paper, the whole problem (NOP and VRP) is considered with the NVROP (Network and Vehicle Routing Optimisation Problem).

The solution procedures proposed to solve NOP and VRP are both heuristic.

In this paper, the original contributions are as follows:
- a mixed (discrete and continuous variables) network optimisation problem highlighting the link capacity optimisation is defined;
- the network optimisation problem is connected with the vehicle routing problem.

The paper is structured in six sections. In section 2, a literature review is discussed. In section 3, the NROP is formulated, and in section 4, the solution procedures are discussed. In section 5, an application on a test network is performed. Finally, in section 6, the conclusions and the future developments are considered.

2. State of the art

In this section, for simplicity sake, the literature reviews related to NOP and VRP are treated separately.

The NOPs can be classified, for example, considering the decision variables (continuous, discrete, mixed), the objectives (mono-objective, multi-objective) and the demand (rigid or elastic).

Referring to the variables type, the following items are taken into consideration in the reviewed literature:
- problems with discrete variables (discrete NOP);
- problems with continuous variables (continuous NOP);
- mixed problems with discrete and continuous variables, (mixed NOP).

The continuous case is treated in controversial ways in the literature. In fact, frequently the problem considers the capacity as it depends on link width and does not consider the following: 1) the intersection optimisation, 2) the fact that the link width depends on lane allocation and hence is a discrete problem. Therefore, the problems concerning signal setting optimisation are considered continuous.

Concerning the discrete NOPs, Billheimer and Gray (1973) propose a heuristic algorithm to solve the problem on uncongested networks; Chen and Alfa (1991) propose a heuristic branching algorithm for solving discrete NOP on congested networks. Foulds (1981) proposes an approach to generate a set of possible network configurations; within this set, the best solution must be selected by the researcher. Gao et al. (2005) propose a discrete NOP in which the supply is modifiable by allowing the building of new infrastructures (within a budget constraint). Similarly, Poorzahedy and Abulghasemi (2005) propose a problem in which the network optimisation is made with the addition (building) of new lanes. In Xie and Turnquist, an approach for reserving some lanes for emergency vehicles and for giving them priority at intersections is proposed. Others discrete NOPs are considered in Hermann et al. (1996), Solanky et al. (1998) and Kalafatas and Peeta (2009).

Concerning the continuous NOPs, a sub-classification (Cantarella et al., 2006) involving the path choice (rigid or elastic) and the intersection optimisation (isolated intersection or interacting intersections) can be made.

With rigid path choice, the following items are taken into consideration in the reviewed literature:
- isolated intersection, in the work of Webster (1958), Webster and Cobb (1966), Allsop (1971, 1976) and
With elastic path choice, the following items are taken into consideration in the reviewed literature:


Concerning the mixed NOPs, Cantarella et al. (2006) propose some heuristic approaches (tabu search, simulated annealing, genetic algorithm) for optimising the link topology and the intersections while considering rigid supply and demand. Russo and Vitetta (2006) propose a three-steps method (topological similarity, cluster analysis, solution selection). Poorzahedy and Rohuani (2007) propose several hybrid algorithms that consider rigid demand and the possibility to build new lanes (a budget constraint is defined). Gallo et al. (2010) propose a two level approach where first the network topology is optimised and then the intersection optimisation is made. Finally, Caggiani and Ottomanelli (2011) propose a fuzzy approach for the uncertainty evaluation in the constraints.

The classification of VRPs can be made in various manners (i.e. problem specification, temporal constraints, capacity and so on). Referring to the solution approach, the following items are taken into consideration in the reviewed literature:

- exact approaches;
- heuristic approaches.

Considering the exact approaches, two main fields are taken into consideration: problems based on lagrangian relaxations; problems based on Dantzig-Wolfe decomposition.

In the first field, Fisher (1994) proposes a branch-and-bound approach, solving a lagrangian problem that provides the lower bounds. Similarly, Kallehauge et al. (2006) propose a method based on lagrangian relaxations and branch algorithm. Other papers based on the lagrangian approach are Desrosiers et al. (1984), Fisher et al. (1997).


Considering the heuristic approaches, various algorithms are proposed to solve the VRPs (genetic, tabu search, simulated annealing, and so on). Baker and Ayechew (2003) use a standard genetic algorithm to solve the VRP and propose a selection procedure to fit the solutions. Berger and Barkaoui (2004) propose a parallel genetic approach, by evolving two solution populations simultaneously. Finally, Hanshar and Ombuki-Berman (2007) propose a genetic algorithm with the aim to optimise the travel cost.

Gribkovskaia et al. (2008) propose a tabu search algorithm to solve the pick-up and delivery problem, and Brandão (2009) proposes a tabu search using some strategies to favour the convergence.

Osman (1993) proposes a simulated annealing algorithm to minimise the total travelled distance. Similarly Tavakkoli-Moghaddam et al. (2006) use simulated annealing to minimise the vehicles used and the management cost.

In other works, such as Dorigo e Stützle (2004), Ando e Taniguchi (2006) and Côté e Potvin (2009), heuristic approaches are proposed.

An extended review concerning the VRP and several variant and solution approaches is reported in Laporte (2007). Furthermore, in recent years many commercial tools based on both exact algorithms and heuristic algorithms have been developed with a view to solving routing problems. Vitetta et al. (2009) lists the main tools
and their characteristics.

3. Models

Considering the previous classification, a mixed NOP is developed. The mixed NOP is then integrated with a VRP defining a mixed Network and Vehicle Routing Optimisation Problem (NVROP). The problem is formulated in two stages: the first is the mixed NOP, the second the VRP. In Figure 1, the flowchart of entire problem is shown.

Fig. 1. Network and vehicle routing optimisation problem: holistic approach

The inputs of the mixed NOP are the infrastructural supply and the demand. The infrastructural supply is represented by the streets available. The demand is the travel demand that moves on the network during a typical day. The hypothesis is that some links in the network can be reserved for freight vehicles (preferential lanes) to improve delivery/pick-up service.

The outputs are as follows:

- the road network (topology and optimal regulation at the intersections) for freight vehicles, with some lanes reserved for freight vehicles;
- the road network (topology and optimal regulation at the intersections) obtained by deleting the reserved lanes from the previous network for private use vehicles.

The inputs of the VRP are the road network for freight vehicles and the retailer position where the delivery/pick-up operations are needed. The network and the link costs are those obtained by solving the network optimisation problem.

The output of the VRP is the best node sequence associated with a freight vehicle. The link costs are flow dependent if the link is used for both private use vehicles and freight vehicles, and it is constant if the link is only used by freight vehicles.

The NVROP is formulated as a constrained problem, with the route optimisation (VRP) performed after the network optimisation (NOP).

The NVROP specified below in the equation (1) has the following variables:

- \( y^{(TO)} \) and \( y^{(SG)} \) vectors, the first being related to the network topology (lane allocation), and the second being related to the signal setting (intersection regulation, capacity);
- \( \Omega \) matrix, related to the paths used by the vehicles in VRP.

The NVROP objective function is reported in equation (1):
min \sum_{v} \sum_{i} \sum_{j} h_{ij}^* \cdot \omega_{ij,v} \quad \text{i, j} \in \Lambda, \text{ v} \in \mathbf{V} \quad (2)

subject to
\sum_{v} \sum_{j} \omega_{ij,v} = 1 \quad \forall \text{i} \in \Lambda, \text{i} \neq j \quad (3)
\sum_{v} \sum_{j} \omega_{ij,v} = m \quad \forall \text{i} \in \Lambda, \text{i} \neq j \quad (4)
\sum_{v} \sum_{j} \omega_{ij,v} = m \quad \forall \text{i} \in \Lambda, \text{i} \neq j \quad (5)
\sum_{i} \sum_{j} r_{j} \cdot \omega_{ij,v} \leq b_{v} \quad \forall \text{ v} \in \mathbf{V}, \text{i} \neq j \quad (6)

where

\begin{itemize}
  \item \mathbf{G}(\text{N, E}), a graph with \text{N} node set and \text{E} link set;
  \item \Lambda \subseteq \text{N}, the user set (user that needs the delivery and/or pick-up operations);
  \item h_{ij}, the path cost between the nodes \text{i} and \text{j} (i, j \in \Lambda);
  \item \mathbf{V} = \{1, 2, \ldots, \text{m}\}, the set of freight vehicles;
  \item r_{i}, the delivery (or pick-up) demand at node \text{i};
  \item b_{v}, the vehicle \text{v} capacity.
\end{itemize}

The objective function (2) allows minimising the total travel cost for the vehicle fleet which is understood as the sum of the travel time related at each route and which depends on the node sequence and path cost. The output is the matrix \Omega^*, which associates the nodes to the vehicles.

The path cost \( h_{ij}^* \) is given by:
\begin{equation}
\bar{h}_{ij}^* = \sum_{i} \bar{\delta}_{ij} \cdot c^* (w) \quad (7)
\end{equation}

where
\begin{itemize}
  \item \( c^* \), the equilibrium cost on link \( \tau \) (the cost \( c \) depends on the characteristics of link \( \tau \) and, in general, on flow vector \( w \));
  \item \( \bar{\delta}_{ij} \), a binary variable equal to 1 if link \( \tau \) belongs to the path between \( i \) and \( j \), otherwise equal to 0;
\end{itemize}

Detailed analysis of the link costs are reported in Quatrone and Vitetta (2011) and Russo and Vitetta (2011).

The NOP is reported in equation (8) and the link cost is obtained from the solution of same problem:
\begin{equation}
\min_{y^{(TO)}, y^{(SG)}, \Omega, w} \sum_{\tau} c(\omega, y^{(T)}, y^{(SG)}) \cdot (w_{\tau}) \quad (8)
\end{equation}

subject to
\begin{itemize}
  \item structural (the lanes allocated do not exceed the available lanes, (9));
  \item network connection (for each centroid pair, at least a path exists, (10));
  \item user behaviour (11);
\end{itemize}
where
- \( x_C \), the centroid set;
- \( x_L, \) reverse link \( L \) with initial node \( j \) and final node \( i \);
- \( x_{nl} \), the number of lanes of infrastructural resource that include the links \( L \) and \( L_R \);
- \( x_{wSUE} \), the stochastic user equilibrium function, the function that for each network configuration \((y_{TO}, y_{SG})\) gives the SUE flow.

The problem (8) is expressed as minimisation of total travel time spent on the network; the costs and the flows considered are those obtained from the equilibrium point of the assigned problem.

4. Procedures

Both the network optimisation and the route optimisation problems are solved using heuristic procedures.

**Network optimisation**

The genetic algorithm implemented to solve the NOP is that proposed in Cantarella et al. (2006).

The genetic algorithm, starting from an initial population, evolves for a fixed number of iterations. The crossover and the mutation operators are implemented to perform the population evolution. Each solution represents a network configuration; each solution generated is tested to verify its admissibility (the network must be connected). The connection test is performed using the Dijkstra algorithm. The solution is considered connected if at least one path exists between each o/d pair. Given an admissible solution, a procedure based on the Webster approach is applied to optimise the signals at intersections.

**Route optimisation**

The route optimisation is performed using a genetic algorithm proposed previously in Polimeni et al. (2010). In this case, the genetic algorithm is used to solve a discrete problem (the variable \( \omega_{i,v} \) of the problem (2) is a binary variable). A solution represents a set of routes; each route is a sequence of nodes.

5. Application

In this section, an application of a test network (Figure 2) was performed. In this preliminary test, in the first stage, the network topology and intersection capacity were optimised. In this stage, the location of the reserved lanes was also established. In the second stage, the routes for freight vehicles were optimised by directing these vehicles to the reserved lanes.

The study area consisted of three zones, identified by centroids A, B, C. The demand was supposed note and un-elastic at emission, departure clock time, distribution and modal split levels.
The supply consisted of 20 nodes and 36 infrastructural resources. The freight vehicles were required to deliver (or pick-up) goods at the users located from 2 to 7 starting at node 1.

Two scenarios were considered, as follows:

1. all of the lanes of all the links in the network are for mixed use: the links are used both by the freight vehicles and by the private vehicles;
2. some lanes are reserved for freight vehicles only.

For example, when reviewing intersection 18 (Figure 3), four streets must be evaluated (18-13, 18-19, 18-23, 18-17); in this example, three lanes were allocated to each street.

In the case of point 1 above, the freight vehicles move on the network mixing with private vehicles, in this case the freight vehicles are subject to the congestion due to vehicle circulation: the link cost function used in equation (7) depend on flow vector.
In the case of point 2 above, several possible configurations are possible. For street 18-13, two lanes were allocated for private vehicles, and one lane was allocated for freight vehicles; the same situation was applied for street 18-23. For street 18-17, two lanes for freight vehicles were allocated, and one lane for private vehicles was allocated; the same situation was applied for street 18-19. Other configurations, not considered here, are also possible. In this case, when the freight vehicles move on reserved link, the link cost function not depend on flow vector.

Regarding the demand, the variation is tied to the demand level (users) that moves on the network. Regarding the supply, the street can be optimised for vehicles or for pedestrian/parking use while still considering the link reservation for freight vehicles (it is the output of the first stage).

The NVROP solutions vary in topology and cost depending on whether lane reservation is considered or not. In effect, lane reservation reduces the cost of the routes for freight vehicles (a 57% reduction in this test, see Table 1), but it increases the total cost spent in the network. The discussed approach could be used to analyse the impact of the transport system at urban level (Musolino and Russo, 2012).

In Table 1, a summary of the route cost comparison, with and without lane reservation, is reported for this example.

Table 1. Comparing route cost with and without lane reservation

<table>
<thead>
<tr>
<th></th>
<th>No lane reservation</th>
<th>Lane reservation</th>
<th>No reservation-reservation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cost (s)</td>
<td>Cost (s)</td>
<td></td>
</tr>
<tr>
<td>mixed NOP</td>
<td>592.62</td>
<td>1292.12</td>
<td>+118</td>
</tr>
<tr>
<td>VRP</td>
<td>1987.28</td>
<td>847.44</td>
<td>-57</td>
</tr>
<tr>
<td>Total</td>
<td>2579.90</td>
<td>2139.56</td>
<td>-17</td>
</tr>
</tbody>
</table>

In both of the cases, with and without lane reservation, the objective was to minimise the total time spent in the network (considered as the sum of private use and freight vehicle travel time).

In the first case, the freight vehicles move on a congested network. In the second case, although the private user travel time increased (by approximately 118%), the freight vehicles moved on some reserved lanes which resulted in a reduction in the route travel time (by approximately 57%).

Moreover, it can be note that by reserving some links for freight vehicles that a reduction of 17% in the total delivery (pick-up) time is possible in this example.

6. Conclusions

In this paper, a model for network and route optimisation for freight vehicles is reported. A holistic approach, consisting of two stages, is proposed. In the first stage, the route optimisation (user sequence) is performed; in the second stage, the network optimisation (lane allocation and link capacity) is performed. A genetic algorithm is used to solve both the problems. Moreover, two scenarios are considered, regarding the lane reservation for freight vehicles.

Future developments to be considered are the specification of a multi-objective problem with the total travel time for private users (that increases with lane reservation) and the routes cost (that decreases with lane reservation). Moreover, there are plans to extend the application to a real-world case.

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