Abstract

In future generation wireless access networks, the users will have the chance of choosing among multiple connectivity opportunities provided by different access networks (network selection problem). Moreover, the network operators themselves will have to implement effective resource allocation strategies taking wise decisions on the used technologies, frequencies, power levels, etc.

This paper proposes a game-theoretic framework to model the problems of network selection and resource allocation, capturing the interdependencies of decisions taken by different players (users vs networks). Namely, we cast the problem as a non-cooperative game where users and access networks act selfishly according to their specific objectives: maximization of the perceived quality of service for the end users, maximization of the number of customers for the access networks. We characterize the equilibria of the game by resorting to mathematical programming, and we derive numerical results to assess the "quality" of the equilibria.

1. Introduction

The proliferation of wireless access technologies, and the evolution of the end-user terminals are leading fast towards a ubiquitous, pervasive and rich connectivity offer, such that the end users will be always connected/covered by multiple access networks/technologies [8]. Such process will be further sped up by the diffusion of spectrum agile, cognitive radios devices, able to dynamically adapt transmission parameters in an opportunistic way depending on the quality of the current connectivity opportunities [2].

This scenario creates new opportunities and poses novel challenges at different networking levels. On the network’s side, there exists a resource allocation problem dealing with the development of effective strategies to allocate and dynamically manage the radio resources when different networks operated by different and potentially competing actors coexist. On the end-user’s side, the main challenge deals with network selection, that is, the development of strategies to automatically select the “best” connectivity opportunity to match the user Quality Of Service (QoS) constraints.

A common approach in the published literature is to handle the aforementioned problems of resource allocation and network selection separately. Referring to wireless access networks, the problem of allocating resources deals with the configuration of several parameters of the specific radio technology used for providing access. As an example, the problem of allocating frequency channels does exist in homogenous access scenario, e.g., in WiFi-based WLANs [5], Wireless Mesh Networks [17], and 3G systems [3], but also in heterogeneous wireless access networks where different technologies coexist [20].

On the other hand, the topic of network selection is often addressed through the definition of different metrics to measure the end users Quality of Service (QoS), which can be consequently used to steer the selection phase [6, 15]. In this field, Song and Jamalipour [19] leverage Grey Relational Analysis and Analytic Hierarchy Processing to determine the utility related to different selection choices.

Game theory has been widely used to tackle both problems, since it provides powerful modelling tools to capture the dynamics and the equilibria of multi-agents situations. In [13], a game-theoretic approach for studying bandwidth allocation in heterogeneous wireless networks is proposed, while a non-cooperative game is used in [14] to model the interactions of different access technologies. In [4], the authors introduce a game between access networks competing in a non-cooperative manner to maximize their payoffs. References [9] and [12] focus on the problem of network selection and propose game theoretic formulation for the selection problem in multi-base station wireless networks and multi-access point WLANs respectively.

Different from our work, the aforementioned manuscripts focus on a single problem only, either dealing with resource allocation (network’s side), or with network selection (user’s side). Since however the two problems are
strictly related, in this paper we consider them jointly and propose a game-theoretic framework to assess the performance of strategies for network selection (user’s side) and resource allocation (network’s side). We model the problem as a non-cooperative game where end users and access networks act selfishly according to their specific objectives. We characterize the equilibria of the game by resorting to mathematical programming models, and we thoroughly comment on the quality of the game equilibria through numerical results.

The paper is organized as follows: in Section 2 we define the reference scenario. Section 3 introduces the Interference-based Network Selection Game (INSG) and Section 4 formalizes the joint Network Selection and Resource Allocation Game (NSRAG). Concluding remarks and comments on ongoing related activities are reported in Section 5.

2. Reference Scenario

The reference network scenario considered in this work is composed of a set of wireless Access Networks (ANs) \( \mathcal{N} = \{1, \ldots, n\} \), and a set of end users \( \mathcal{U} = \{1, \ldots, m\} \). As a first step of analysis we further assume an homogeneous network scenario where all the access networks feature the same nominal offered bandwidth. This may well represent the case when multiple WiFi hot spots geared with the same technology cover a given area (e.g., in airports, railway stations, community neighborhood, etc.). Each access network is assigned specific radio resources (e.g., frequency channels) chosen within a given set \( \mathcal{F} = \{1, \ldots, Q\} \). Each user can select among different covering access networks.

Coverage is defined in the following way: let \( P_{tx}^i \) be the power transmitted by AN \( i \); user \( j \) is covered by AN \( i \) if the following inequality is verified:

\[
P_{tx}^i \alpha_{tx} d_{ij}^\eta \geq P_{th},
\]

where \( d_{ij} \) is the distance between position \( i \) and position \( j \), \( \eta \) the attenuation factor and \( 10^\eta \) accounts for the loss due to slow shadowing, being \( \epsilon \) a normal variate with zero mean and \( \sigma^2 \) variance. \( P_{th} \) is a threshold value for the required received power.

In this scenario, we can now cast two different problems:

**Network Selection, NS** The first problem we tackle is pure network selection, where radio resources are statically assigned to the access networks. In Section 3 we model the user decision problem as a non-cooperative game in which the players are the end users, the feasible strategies are the available access networks, and the pay-off of each user depends on the number of interfering stations in the chosen access network. The game is non-cooperative since it is reasonable to assume that each end user adopts a strategy which maximizes her own pay-off, regardless of the status of the other users.

**Network Selection and Resource Allocation, NSRA** The radio resource allocation is not pre-determined, but the access networks can decide a specific radio resource allocation strategy to maximize their own utility. Such utility depends, in turn, on the specific decision of the end users on where to associate. Therefore, in Section 4, we model this scenario as a bi-level stage game where the payoffs for the resource allocation game played by the access networks depend on the outcome of a nested network selection game played at the lower level by the end users.

3. Interference Based Network Selection Game

To formally define the Interference based Network Selection Game (INSG) we introduce the following definitions:

**Definition** The strategy space of user \( i \), \( \mathcal{U}_i \) (with \( i \in \mathcal{U} \)) is the set of all the possible combinations of strategies being played by the users, that is \( \mathcal{U}_i = \mathcal{F}_1 \times \mathcal{F}_2 \times \ldots \times \mathcal{F}_m \).

**Definition** An element \( S_u \in \mathcal{U}_u \), \( S_u = (S_1^1, S_2^1, \ldots, S_m^1) \) with \( S_1^1 \in \mathcal{F}_1, \ldots, S_m^1 \in \mathcal{F}_m \), is a strategy profile of the game.

**Definition** The interference level of user \( i \) in the strategy profile \( S_u \), is the total number of end users covered by the same ANs covering \( i \), and using the same radio resources, that is, \( n_i(S_u) = |U_K| \), being \( U_K \) the set of users covered by all the ANs covering user \( i \) and using the frequency user \( i \) is using.

With this interference level definition we are assuming the user quality of service to be a function of interference perceived by the user itself. This is quite common when modelling problem of wireless access, and has the advantage to provide tractable but consistent expression of the user perceived saturation throughput [5].

The INSG can now be formally defined as:

\[
\text{INSG} = \langle \mathcal{U}, \mathcal{N}, \mathcal{U}_u, [n_i(S_u)]_{i \in \mathcal{U}}. \rangle
\]

By definition, the generic end user \( i \) selfishly plays the strategy \( S_i^1 \) which minimizes her experienced interference level, that is: \( S_i^1 = \text{argmin}_{S_i^1} n_i(S_u), S_i^1 \in \mathcal{F}_i \).

It is easy to see that the INSG belongs to the class of congestion games [18], where a finite number of resources can be shared among multiple users, the strategy of each user being the set of resources the user can choose. It can be further observed that INSG is a particular type of congestion game known as crowding game [10]. Crowding games are single-choice congestion games where the payoff is player specific. From the consideration above, we can state the following:

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Theorem 3.1 The INSG admits at least one pure-strategy Nash Equilibrium (NE).

Proof comes directly from the consideration that every congestion game with player specific pay-off function always admits a pure-strategy NE (see Theorem (1) in [11]).

The concept of Nash equilibrium is widely adopted in game theory to characterize the stability of games. Indeed, a Nash equilibrium is a status of the game where each player has not any incentive in deviating from the played strategy unilaterally [16]. However, provided that a game admits Nash equilibria, it becomes fundamental to find such equilibria and characterize them. In fact, a game can possess several equilibria of different “quality”. In the next section we comment on the “quality” of the Nash equilibria of the INSG, and we provide an operational method to find them.

3.1. Characterizing INSG Equilibria

We derive hereafter a mathematical programming formulation of the INSG, which can be used to find and characterize different Nash equilibria of the game. Namely, we design an Integer Linear Programming (ILP) formulation of an admissibility problem whose constraints enforce equilibria situations. To this end, we introduce the parameters $c_{im}$ to represent coverage and interference among users, where $c_{im} = 1$ if Eq. (1) holds true for user $i$ and AN $m$, otherwise $c_{im} = 0$.

The interference perceived by the users depends on the specific resource allocation among the access networks. Thus, we define such resource allocation through the following parameters: $d_{mk} = 1$ if AN $m$ works on frequency $k$, otherwise $d_{mk} = 0$; $a_{ik} = 1$ if user $i$ can associate to frequency $k$, otherwise $a_{ik} = 0$; $b_{ijk} = 1$ if users $i$ and $j$ may interfere on frequency $k$, otherwise $b_{ijk} = 0$.

Binary decision variables are used to define the association of the users to specific ANs, and to specific frequency channels, consequently. Namely, we introduce: $y_{im} = 1$ if user $i$ chooses AN $m$, otherwise $y_{im} = 0$; $x_{ik} = 1$ if user $i$ is associated to frequency $k$, otherwise $x_{ik} = 0$.

We can now state the admissibility problem $P$ whose constraints are:

\[
\sum_{k \in \mathcal{F}} x_{ik} = 1 \quad \forall i \in \mathcal{U} \tag{3}
\]

\[
\sum_{m \in \mathcal{N}} y_{im} = 1 \quad \forall i \in \mathcal{U} \tag{4}
\]

\[
x_{ik} \leq a_{ik} \quad \forall i \in \mathcal{U}, k \in \mathcal{F} \tag{5}
\]

\[
y_{im} \leq c_{im} \quad \forall i \in \mathcal{U}, m \in \mathcal{N} \tag{6}
\]

\[
y_{im} d_{mk} \leq x_{ik} \quad \forall i \in \mathcal{U}, m \in \mathcal{N}, k \in \mathcal{F} \tag{7}
\]

\[
M (x_{ik} + a_{il} - 2) + \sum_{j \in \mathcal{U}, j \neq i} x_{jk} b_{ijk} \leq \sum_{j \in \mathcal{U}, j \neq i} x_{jl} b_{ijl} \quad \forall i \in \mathcal{U}, k \in \mathcal{F}, l \in \mathcal{F} \tag{8}
\]

Constraints (3) and (4) ensure that each user chooses only one frequency, and one access network, respectively. Constraints (5), (6), and (7) guarantee the feasibility of the association. Constraints (8) force each user to choose the strategy (access network) which leads to the minimum interference condition, that is, they ensure that if the single user unilaterally changes her strategy, the change does not improve her own payoff\(^1\), which is the definition of Nash equilibrium.

We can now state the following:

Proposition 3.2 Any feasible solution of the admissibility problem $P$ is a Nash Equilibrium (NE) for INSG.

The proof holds by construction of the problem $P$.

As previously stated, Nash equilibria can be multiple and of different quality. One of the most used concepts in game theory to classify equilibria is Pareto optimality. An equilibrium identified by a given strategy profile is said to be Pareto optimal if does not exist any other strategy profile which leads to an enhancement in the payoffs of all the users [16].

To find Pareto optimal NE, we introduce the concept of equilibrium efficiency defined as the average number of interferers per user. Then, we introduce an optimization version $P_1$ of the admissibility problem $P$ which features the following objective function:

\[
f_{\text{opt}}(X) = \min \sum_{i \in \mathcal{U}} \sum_{k \in \mathcal{F}} \sum_{j \in \mathcal{U}, j \neq i} x_{ik} b_{ijk} x_{jk} \tag{9}
\]

The non-linear objective function in Eq. (9) can be easily linearized with standard techniques to get back to a linear formulation of $P_1$. Details on how to linearize Eq. (9) are skipped for the sake of brevity.

The following result holds.

Theorem 3.3 Any solution of the optimization problem $P_1$ is a Pareto-optimal Nash Equilibrium for INSG.

Proof Sketch We can prove the theorem by contradiction, supposing that the solution provided by $P_1$, $X^*$, is not Pareto-optimal. If such proposition is true, it must exists another solution, $\bar{X}$, which gives lower interference for at least one player but at most the same interference for every other player. This implies that the value of the objective function of $\bar{X}$ is strictly lower than the one provided by $X^*$, which means that $X^*$ is not a solution to $P_1$, contradicting the starting proposition.

Besides determining the optimal equilibria, it is interesting to derive and characterize the worst equilibria, also. To this end, we define a further optimization model with the same constraints as $P$, but with the following objective function aiming at maximizing the total (average) number of interferers.

\[
f_{\text{worst}}(X) = \max \sum_{i \in \mathcal{U}} \sum_{k \in \mathcal{F}} \sum_{j \in \mathcal{U}, j \neq i} x_{ik} b_{ijk} x_{jk}, \tag{10}
\]

\(^1\)the interference level is non-decreasing
Table 1. Quality of INSG Nash equilibria when varying the number of end users (m) compared to random and best-received association policies. Uniform network scenario with n =10 access networks, L =500m, and r =50m.

<table>
<thead>
<tr>
<th>m</th>
<th>INSG NE (Best-Worst)</th>
<th>RANDOM</th>
<th>BEST RECEIVED</th>
<th>γ</th>
</tr>
</thead>
<tbody>
<tr>
<td>m=100</td>
<td>9.78-9.786</td>
<td>9.948</td>
<td>10.022</td>
<td>0.9993</td>
</tr>
<tr>
<td>m=120</td>
<td>11.813-11.823</td>
<td>12.02</td>
<td>12.025</td>
<td>0.9991</td>
</tr>
<tr>
<td>m=140</td>
<td>13.905-13.911</td>
<td>14.125</td>
<td>14.118</td>
<td>0.9993</td>
</tr>
<tr>
<td>m=160</td>
<td>16.006-16.017</td>
<td>16.313</td>
<td>16.28</td>
<td>0.9997</td>
</tr>
<tr>
<td>m=180</td>
<td>17.94-17.944</td>
<td>18.189</td>
<td>18.141</td>
<td>0.9997</td>
</tr>
<tr>
<td>m=200</td>
<td>20.053-20.056</td>
<td>20.3</td>
<td>20.3</td>
<td>0.9998</td>
</tr>
</tbody>
</table>

Let $P_2$ be this new optimization problem. Any solution of $P_2$ is a NE (see Proposition 3.2) in which the average number of interferers per end users is maximized. The ratio: $\gamma = f_{opt}/f_{worst}$ provides a quality measure of the equilibria space of INSG. Roughly speaking, the closer to 1 $\gamma$ is, the more “similar” are the Nash equilibria.

3.2. Numerical Results

In order to test the quality of INSG equilibria, we have implemented an instance generator able to create synthetic instances representing multi-access network scenarios. The software takes as input the following parameters: the edge of the square area to be simulated ($L$), the number of end users ($m$), the number of access networks ($n$), and the coverage range of each access network, expressed in meters ($r$).

In a basic set of instances, each network is assumed to have a circular coverage region with radius $r$.

According to the above parameters, the generating tool randomly draws the positions of the $n$ access networks and of the $m$ end users, so that each user is covered by at least one access network.

All the results reported in the remainder have been obtained formulating the ILP problems $P$, $P_1$, and $P_2$ in AMPL [7] and solving them with CPLEX commercial solver [1]. Unless differently specified, the reported results are average values on 100 randomly generated instances.

Table 1 reports the results obtained on a uniform topology with $n=10$ randomly deployed access networks, when varying the number of end users ($m$), in case $L=500$ meters, and $r=50$ meters. The average number of interferers is reported for Pareto optimal and non optimal Nash equilibria (NE) of the INSG, compared to the cases where the end users choose randomly the access network to associate to, and where the end users choose the closest network.

The main result coming from this analysis is that the NE of the INSG are almost equivalent from the efficiency point of view. In fact, the ratio $\gamma$ between the average number of interferers in the best NE and the one in the worst NE is very close to one. Moreover, it is also worth noting that there is a slight difference in terms of efficiency between the three network selection policies. Namely, random network selection and best received network selection lead to higher number of interferers per user on average.

Table 2 shows similar results as those in Table 1 when varying the number of access networks available in the area. The ratio between best and worst equilibria remains close to unity, even if a slight decrease can be observed as the number of access networks increases, i.e., the strategy space gets bigger. Furthermore, it is interesting to note that as the number of access networks increases, the difference between the three assignment policies becomes more relevant.

Besides the difference in the average values of interferers, it is also worth getting some insights in the distribution of per-user interferers. In Figure 1 reports the p.d.f. of the number of interferers for a specific network scenario with $n=10$ access networks and $m=100$, in case $L=500$ meters, and $r=50$ meters. The average number of interferers, as well as the corresponding standard deviation are also reported in the legend. As clear from the figure, the Nash equilibrium of the INSG is characterized by a more compact p.d.f. of interferers than the associated p.d.f. of the number of interferers (lower standard deviation), which means that the selfish behavior of the users trying to minimize their experienced interference leads globally to a fair situation, where the interference status of all the users is similar.

4. Network Selection and Resource Allocation

The focus of previous section was on the users’ side. Hereafter, we drop the assumption of a fixed frequency.
assignment and let the single access network take part in the game. Namely, whilst the end users keep playing to minimize their experience congestion (that is, interference), each access network plays the frequency assignment strategy which maximizes its own revenues. Referring back to the definitions introduced in Section 3, we can extend the concepts of strategy set, $\mathcal{S}_n$, strategy space, $\mathcal{F}_n$, and strategy profile, $S_n$, to the access networks.

The payoff for the generic access network $j$ associated to the access network strategy profile $S_n$, $p_j(S_n, \text{INSG})$, can be defined as the number of end users which decide to associate to the access network $j$ under the access network’s strategy profile $S_n$. Consequently, the payoffs of the resource allocation game among access networks depend on the underlying game of network selection played by the end users. If we decouple the decision time of end users and access networks and assume the access networks and end users play their strategies in different steps, this leads to the definition of a bi-level stage game; the lower level game seeks to allocate the frequencies among access networks given the responses of the end users to these assignment relationships among end users and access networks.

Formally, the bi-level game of Network Selection and Resource Allocation Game (NSRAG) can be defined as:

$$\text{NSRAG} = (\mathcal{N}, \mathcal{F}_n, \{p_j(S_n, \text{INSG})\}_{j \in \mathcal{N}})$$

(11)

where $\mathcal{N} = \{1, \ldots, n\}$ is the set of access networks, and $p_j(S_n, \text{INSG})$ is the payoff associated to access network $j$ playing the strategy profile $S_n$, which depends on the nested INSG.

The following property holds for the NSRAG:

**Theorem 4.1** Any NE for the NSRAG is Pareto-Optimal from the access networks point of view.

The proof comes from the observation that the payoffs of the access networks sum up to a constant value, which is the number of users. Consequently, given a NE point for NSRAG, it cannot exist another strategy profile which leads one or more networks to improve their payoffs, without decreasing the payoff of other networks.

4.1. Characterizing NSRAG Equilibria

To characterize the equilibria of the NSRAG, we develop thereafter a solution method based on a steered enumeration of the access networks strategy space, $\mathcal{F}_n$. The solution procedure at first identifies strategy profiles of the access network strategy space which are equivalent from the end users and access networks point of view. Two strategy profiles are equivalent if they lead to equivalent NE in the NSRAG.

Strategy profiles equivalent to a given strategy profile can be pruned from the strategy space of the access networks; thus reducing the dimension of the strategy space itself.

Algorithm 1

1: $\mathcal{F}_n \text{ Reduced} = \text{Prune Strategy Space}$
2: for $S_n(i) \in \mathcal{F}_n \text{ Reduced}$ do
3: Solve $\text{INSG}(S_n(i))$
4: end for
5: Search for Network Equilibria

Algorithm (1) requires the solution of the INSG which leads to an assignment relationships among end users and access networks. The quality of such assignment is then evaluated with respect to the access networks utility, thus determining the NE of the overall NSRAG.

From the access network point of view, a strategy profile is a Nash equilibrium if there is no player (i.e., access network) which has advantage in modifying its strategy unilaterally. Therefore, a strategy profile $S = (S_j, S_{-j})$ is a Nash equilibrium for the access networks if and only if:

$$p_j((S_j, S_{-j}), \text{INSG}) \geq p_j((S'_j, S_{-j}), \text{INSG})$$

$\forall S'_j \in \mathcal{F}_n$ and $\forall j \in \mathcal{N}$,

where $S_j$ is the strategy played by access network $j$, and $S_{-j}$ indicates the set of strategies played by all the other access networks but $j$.

We have applied the solution method to the network scenario reported in Figure 2, featuring 5 access networks with coverage radius of 50 m providing access to 50 users with 3 available frequencies to be allocated. The strategy space of the access networks is therefore composed of $3^5 = 243$ strategy profiles. After reducing the strategy space by identifying equivalent strategies, the number of remaining strategy profiles to be analyzed scales down to 41, in this case. Thus, we can solve 41 instances of the INSG and identify the equilibria of the NSRAG.

Table 3 reports the characteristics of the Nash equilibria, by reporting the frequency allocation pattern, the number of
Table 3. Characteristics of the NSRAG Nash equilibria in the network scenario represented in Fig. 2.

<table>
<thead>
<tr>
<th>Frequency Allocation</th>
<th>Number of Customers per AN</th>
<th>Average Interferers per user</th>
</tr>
</thead>
<tbody>
<tr>
<td>NE 1</td>
<td>&lt;8,7,8,19,8&gt;</td>
<td>11.04</td>
</tr>
<tr>
<td>NE 2</td>
<td>&lt;8,7,8,19,8&gt;</td>
<td>11.04</td>
</tr>
<tr>
<td>NE 3</td>
<td>&lt;8,7,8,19,8&gt;</td>
<td>11.04</td>
</tr>
<tr>
<td>NE 4</td>
<td>&lt;8,7,8,19,8&gt;</td>
<td>11.04</td>
</tr>
<tr>
<td>NE 5</td>
<td>&lt;8,7,8,19,8&gt;</td>
<td>11.04</td>
</tr>
<tr>
<td>NE 6</td>
<td>&lt;8,7,8,19,8&gt;</td>
<td>11.04</td>
</tr>
</tbody>
</table>

captured customers for each access network and the average number of interferers per user. As clear from the table, the reduced NSRAG has 6 equivalent equilibria where the access networks tend to choose frequencies as diverse as possible with respect to surrounding access networks. Namely, AN1, AN2, and AN3 always choose non-overlapping frequencies in every equilibrium, whereas AN 5 always picks a frequency which is non interfering with AN 3. On the other side, the choice of AN 4 is independent on the other AN’s frequency pattern. Similar results, not reported here for the sake of brevity, have been obtained also in other network scenarios.

The general concept coming form the aforementioned results is that even if the game is non-cooperative and the players (ANs and users) potentially can act selfishly and competitively, the NSRAG features equilibria situation where both the end users and the access networks tend to play strategies which do not hinder the other players. In fact, the users tend to spread uniformly amongst the different access networks, and the access networks themselves tend to choose resource allocation strategies which lower the perceived/induced interference to other access networks.

5. Concluding Remarks

Motivated by the proliferation and widespread deployment of heterogeneous wireless access technologies, we have addressed in this paper the problems of network selection and radio resource allocation resorting to game theory. Different from other approaches, we have tackled the two problems jointly by casting a non-cooperative game where end users and access networks play selfishly strategy profiles to achieve maximum utility (quality of service, for the end users, and number of customers, for the access networks).

We have formalized the joint Network Selection and Resource Allocation Game (NSRAG) as a non-cooperative bi-level stage game, and we have characterized its Nash equilibria. Moreover, we have proposed a solution method to obtain the Nash equilibria based on mathematical programming. Finally we have commented on the quality of the Nash equilibria of the NSRAG in case of synthetic network instances representing realistic network scenarios.

References