$H_\infty$ Robustification Control of Existing Piezoelectric-Stack Actuated Nanomanipulators

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Abstract—In current AFM-based nanomanipulation systems, the commercial position closed-loop controller for piezoelectric nanopositioning stages are implemented with success in a wide range of industrial applications. Even if these controllers operate with satisfactory nominal tracking performance, considerable attention has been focused on appropriate control strategies to compensate hysteresis, nonlinearities, drift and creep for high bandwidths and large scanning regimes. As these closed-loop controllers are very cost-effective, a special interest in robust plug-in compensators seems to be a solution. We proposed in this paper a robust plug-in compensator using the $H$-infinity loop-shaping techniques which can be plugged into the existing controller without affecting the already satisfactory nominal tracking performance of the existing closed-loop system. Dynamic modeling, identification and robust control of a 3 d.o.f. piezoelectric nanorobotic positioner are presented in this paper in order to improve the nanorobot performance under plant parameter variations and in the presence of external disturbances. Simulation and experimental results are given to validate the proposed plug-in robust compensator in the case of a nanorobotic manipulation task.

I. INTRODUCTION

In biotechnology field, the tele-nanomanipulation control of microobjects (biological cells, viruses, MEMS) with human handling is difficult due to dynamics and uncertainties which are strongly impacted by the user’s gesture. In applications where high performance and accuracy are not critical constitutive nonlinearities of piezoelectric nanopositioning stages and hysteresis can be compensated by standard Proportional-Integral (PI) or Proportional-Integral-Derivative (PID) controllers, but this can potentially lead to bandwidth limitation and inefficiencies. Even if these controllers have proved their performance in nanoscale positioning systems, challenging problems of nanoscale control remain due to nonlinear dynamics, actuator’s modeling uncertainties, instabilities and lack of robustness against external perturbations and sensor noise [1]. As these industrial closed-loop controllers are cost-effective and dedicated, a special interest in robust plug-in compensators seems to be a solution. The key idea of this paper is to robustify existing controllers by "Plug-In" attachable robust compensators for piezoelectric nanopositioning systems. Principle of the new methods is to settle on real plants as their nominal models with local compensators. Since the additional local compensators are designed independent of previously designed controllers, they are applicable for any existing control systems including nonlinear and/or non-closed form control scheme. A brief analysis of robust control techniques shows that a considerable number of feedback design schemes based on linear robust control techniques have been already proposed for nanopositioning systems. Development of inversion-based feedforward control with robust feedback control have proved their efficacy in output tracking in Atomic Force Microscope [2], [3]. The performance of the inverse feedforward control, however, is strongly limited by modeling errors/uncertainties and disturbances [4]. Closed-loop linear $H_\infty$ control technique seems to be an efficient alternative technique. These schemes have provided improvements in bandwidth and robustness to non linearities and hysteresis. In [5], the authors proposed a Smith predictor-based $H_\infty$ controller for a piezoactuator with an emphasis in reducing the hysteresis. In [6], a $H_\infty$ controller design for one-dimensional nanopositioning system performing high closedloop bandwidths and robustness against nonlinearities has been synthesized with success. Finally, a robust Glover-McFarlane $H_\infty$ scheme [7] to simultaneously achieve performance and robustness where neither specific tracking requirement nor a characterization of uncertainty are available a priori. Considering these robust control schemes, we proposed a framework for increasing the robustness of existing industrial control schemes with a quantifiable compromise on performances. We proposed in this paper a robust plug-in compensator using the $H_\infty$ loop-shaping techniques which can be plugged into the existing controller without affecting the already satisfactory nominal tracking performance of the existing closed-loop system. A similar approach has been introduced by Salapaka in [8]. In the present study, the $H_\infty$ loop-shaping technique is applied for robustification of complex industrial controllers achieving robustness with marginal reduction in performance. Dynamic modeling, identification and robust control of a 3 degree of freedom piezoelectric nanorobotic positioner are presented in this paper in order to improve the nanorobot performance under plant parameter variations and in the presence of external disturbances. The paper is organized in the following way. In Section 2, a description of the nanorobotic device is given. This is followed by the frequency-domain-based system identification of the existing controller and nanorobotic positioner. The control design and the experimental results are then presented in Section 4. Experimental results are given in Section 5 to validate the proposed plug-in robust compensator in the case of a nanorobotic manipulation task.

II. PIEZOELECTRIC NANOMANIPULATOR DESCRIPTION

The nanomanipulator structure is composed of three linear translation stages (x,y,z) driven by DC motors for coarse motion (range: 8 mm, accuracy: 15 nm) combined with a
3 d.o.f ultra-high-resolution piezomanipulator (x,y,z) for fine positioning (range: 100 μm, accuracy: 1 nm). This hybrid nanomanipulating system combines the advantages of ultra-low inertia, high-speed and long travel range (Fig.1(a)). The micro-end effector is constituted by a piezoresistive AFM cantilever integrating a full-bridge strain gauge sensors(Fig.1(b)).

![Hybrid Piezo/DC Servo Nanopositioner](image)

**Fig. 1.** Structure of the hybrid 6 dof AFM-based nanomanipulator and its sensorized force cantilever.

The 3 d.o.f ultra-high-resolution piezomanipulator (P-611.3S NanoCube from Physics Instruments) is a versatile, multi-axis piezo-nanopositioning system. Its 100 x 100 x 100 μm positioning and scanning range comes in an extremely compact package. Equipped with a zero-stiction, zero-friction guiding system, this piezomanipulator provides motion with ultra-high resolution and settling times of only a few milliseconds. It is composed of single-axis nanopositioning stage with anti-accurate-motion flexure design. The best flexure designs provide guiding precision in the low nanometer range. In open-loop operation, the platform's position is roughly proportional to the drive voltage. In the closed-loop version, the Proportional-Integral controller allows absolute position control. However, existing closed-loop controller is designed to achieve specific tracking and performance requirements (such as zero steady-state tracking) in a narrow frequency closed-loop bandwidth. During nanomanipulation tasks, the x-y-z nanopositioner servo-controller lacks in robustness against:

a) Modeling uncertainties:
1) Mechanical nonlinearities of mechanical-guiding systems for large travel ranges;
2) Modeling uncertainties due to operating point, temperature effects and time execution;
3) Hysteresis and creep effects due to piezoelectric ceramics.

b) External perturbations:
1) Noise measurement perturbations;
2) Force/torque perturbations during nanomanipulation.

Considering these limitations, we proposed in the following a robust plug-in compensator using the $H_\infty$ loop-shaping technique which can be plugged into the existing PI-controller without affecting the already satisfactory nominal tracking performance.

### III. Controller Design and Identification

#### A. Controller Design

In this paper, we simply assume that the controller has been already designed by the manufacturer and is in operation in a real micromanipulation system. However, it is often the case that this controller may not work well when the plant is perturbed and/or external disturbances are present. In this situation, an additional controller is needed to improve the robustness of the overall system against plant uncertainties. It is desirable that this additional controller can be plugged into the existing controller and without affecting the already satisfactory tracking performance. Considering these limitations, we proposed in the following a robust plug-in compensator using the $H_\infty$ loop-shaping technique which can be plugged into the existing PI-controller without affecting the already satisfactory nominal tracking. This the reason why we called that controller a plug-in controller.

#### B. Identification

The digital system to be identified is constituted of both components: the piezoelectric nanopositioner and the PI-controller embedded into the acquisition board. The acquisition board is composed of digital inputs (DAC) and outputs (ADC), the identification should be considered as a discrete system. Among the various modes of identification for numerical systems ([9], [10]), we choose the least squares simple method. This choice is justified by the simplicity of the method during implementation, accuracy of the identified parameters and off-line parameters adjustment. The least square method is based on the determination of a vector of parameters so as to minimize an error vector $\varepsilon$:

$$J(\hat{\vartheta}) = ||\varepsilon(k)||^2 = (y(k) - \phi(k)\hat{\vartheta})^T(y(k) - \phi(k)\hat{\vartheta})$$

(1)

The vector parameters minimizing the criterion $J(\hat{\vartheta})$, denoted $\hat{\vartheta}(k)$, is called the estimated vector parameters defined by ([10]):

$$\hat{\vartheta}(k) = (\phi^T(k)\phi(k))^{-1}\phi^T(k)y(k)$$

(2)

where $\phi(k)$ is the observation matrix $\phi(k)$ of $p$-order. In order to identify the wide dynamics bandwidth of the servonanopositioner, we chose a Pseudo-Random Pattern Generator signal (PRPG). It has been already proved reliable for a good identification in many dynamic systems. The scheme of Fig.2 depicts the principle of the numerical identification. $U(Z)$ represents the numerical input signal (PRPG), $Y(Z)$ the numerical output signal of the system.

In Fig.2, the terms $G(S)$ and $H(z)$ represent the transfer functions of the continuous system and the discrete system, respectively. The parameters provided by the identification represent the $a_i$ and the $b_j$ of the discrete transfer function which is given by the following expression:

$$\frac{b_0 z^2 + b_1 z + b_2}{z^3 + a_1 z^2 + a_2 z + a_3}$$

(3)

The identification was carried out for several operating points, ranging from 0 to 60μm with elementary steps of
10μm order and driving frequencies lower than the resonance frequencies. The discrete models have been identified with averaged parameter values for x, y and z axes, respectively.

\[
\begin{align*}
H_x(z) &= \frac{0.00535z^2 + 0.0271z + 0.0507}{z^3 - 0.00557z^2 + 0.0196z - 0.93} \\
H_y(z) &= \frac{0.003z^2 + 0.00741z + 0.0117}{z^3 - 0.00579z^2 + 0.025z - 0.997} \\
H_z(z) &= \frac{0.00056z^2 + 0.00535z + 0.00761}{z^3 - 0.00566z^2 + 0.0222z - 1}
\end{align*}
\]

The results presented in Fig.3 show the experimental frequency responses. These results show good tracking performances with negligible error. However, it should be noticed that the identified models vary strongly due to uncertain dynamics and are subject to modeling uncertainties.

IV. PLUG- IN ROBUST CONTROLLER DESIGN

The existing controller is a proportional-integral (PI) and for the robustification of the existing controller we use the \(H_{\infty}\) loop shaping technique and how it can be integrated in a robust plug-in controller.

A. Coprime Factorization Approach

An approach was developed by ([11]) and ([12]) starting from the concept of the coprime factorization of transfer matrix. This approach presents interesting properties and its implementation calls upon traditional notions of control engineering.

B. Robust Controller Design using Normalized Coprime Factor

We define the nominal model of the system to be controlled starting from the coprime factors on the left: \(G(s) = \tilde{M}(s)^{-1}\tilde{N}(s)\). Then a perturbed model is written (see Fig.4).

\[
\tilde{G} = (\tilde{M} + \Delta_m)^{-1}(\tilde{N} + \Delta_n)
\]

where \(\tilde{G}\) is a left coprime factorization (LCF) of \(G\), and \(\Delta_m, \Delta_n\) are unknown and stable transfer functions representing the uncertainty. We can then define a family of models with the following expression:

\[
\xi_{\varepsilon} = \{ \tilde{G} = (\tilde{M} + \Delta_m)^{-1}(\tilde{N} + \Delta_n) : \| (\Delta_m, \Delta_n) \|_{\infty} < \varepsilon \}
\]

where \(\varepsilon_{\text{max}}\) represent the margin of maximum stability. The robust problem of stability is thus to find the greatest value of \(\varepsilon = \varepsilon_{\text{max}}\), such as all the models belonging to \(\xi_{\varepsilon}\) can be stabilized by the same corrector \(K\). The problem of robust stability \(H_{\infty}\) amounts finding \(\gamma_{\text{min}}\) and \(K(s)\) stabilizing \(G(s)\) such as:

\[
\left\| \begin{pmatrix} I & K \end{pmatrix} (I - G)K^{-1}(I - W_2GW_1) \right\|_{\infty} = \gamma_{\text{min}}^{-1} = \varepsilon_{\text{max}}^{-1}
\]

However, [13] showed that the minimal value \(\gamma\) of is given by:

\[
\gamma_{\text{min}} = \varepsilon_{\text{max}}^{-1} = \sqrt{1 + \lambda_{\text{sup}}(XY)}
\]

where \(\lambda_{\text{sup}}\) indicates the greatest eigenvalue of \(XY\). Moreover, for any value \(\varepsilon < \varepsilon_{\text{max}}\) a corrector stabilizing all
the models belonging to $\xi_e$ is given by:

$$K(s) = B^T X(sI - A + B^T X - \gamma^2 Z Y C^T C)^{-1} \gamma^2 Z Y C^T$$

$$Z = (I + Y X - \gamma^2 I)^{-1}$$

$$\gamma = \varepsilon^{-1}$$

where $A$, $B$ and $C$ are state matrices of the system defined by the function $G$ and $X$, $Y$ are the positive definite matrices and solution of the Ricatti equation.

V. CHARACTERIZATION OF PIEZOELECTRIC-STACK ACTUATED NANOMANIPULATOR

A. Implementation of the controllers

The controller $K$ is obtained by combining the pre-filter and the post-filter. The pre-filter and post-filter are used to shape the open-loop plant to achieve a desired frequency response according to some well-defined design specifications such as bandwidth and steady-state error ([15]). To obtain a high gain at low frequency, a PI controller is synthesized for the x-axis like pre-filter $W_1$ and in order to obtain a small gain at high frequency, a low-pass filter is synthesized like pre-filter $W_2$. In order to obtain a high performance and a good robustness, we add the following weight functions:

$$W_1 = 50 \times \frac{10s + 35.2}{10s}, W_2 = \frac{1}{s + 11.3}.$$  

Using this weight functions we obtain six-order the $H_\infty$ controller. In order to implement this controller we use the function of Matlab balmr to reduce the order and obtain a third-order approximation of the plug-in robust controller given by:

$$K_r(s) = \frac{-72.69s^2 - 1824s - 6706}{s^3 + 66.88s^2 + 625.7s - 5.07 \times 10^{-11}}.$$  

and the discrete controller:

$$K_{rd}(z) = \frac{0.1726z^2 - 0.3347z + 0.1622}{z^3 - 2.842z^2 + 2.688z - 0.846}.$$  

The Fig. 5 and Fig. 7 show the frequency responses of the weight functions $W_1$, $W_2$, of the open-loop system $W_1 P W_2$ and of the controller $K$. The results show that the open-loop remains close to the step response obtained after the choice of the shaping functions and $K_r$ ensures correct margins of stability. The Fig. 8 show the identified input signal (PRPG signal) with PI output Fig.8.(a) and with robustified PI controller Fig.8.(b).

B. Nanomanipulator Characterization

In this section, the nanopositioning is characterized in terms of range, sensitivity and resolution in the open- and closed-loop configurations. The calibration data showed some hysteresis in open-loop. Hysteresis is primarily due to the nonlinear relationship between applied voltage and displacement which are important for large deflections. To present the effectiveness of the $H_\infty$ closed-loop design, the hysteresis curves obtained in open-loop are compared with the closed-loop design using PI controller and the $H_\infty$ controller. We can
see clearly that for a nanopositioning displacement of 40μm, a maximum output hysteresis of 10μm (25%) was observed. The same experiment with the closed-loop controller showed that the effects were practically eliminated Fig.9. An important observation comes from the fact that the operating applied voltage is settled to 100V while the system was identified for a low value of input signal (PRPG signal). It shows clearly the linearity relationship between the input-output signals. Similar linearity results have been measured when considering creep effects. It should be noticed that important differences were observed for different displacement with PI and $H_\infty$ controller. High precision is obtained with robustified PI controller.

Fig. 9. PI and $H_\infty$ controller to remove the hysteresis (the zoom of the response).

C. Robust Experimentation in Microsphere pushing

To illustrate the behavior of the piezoelectric-stack actuated nanomanipulator system and to confirm the findings of the previous section, we carried out different experiments. The working space of the nanomanipulator is visualized through two orthogonal optical microscopes (top-view and side-view) and a high magnification microscope for a zooming view (see Fig.10). The micromanipulation task is to realize micro pushing tasks of microparticles of 5 μm of diameter on a glass substrate. During the micro pushing tasks, we made a comparison between the responses with PI and $H_\infty$ controllers when pushing a microparticle. We give different responses of the system with plug-in robust controller in the presence of the disturbance and position sensor noise. The set of experiments of Fig.11 shows the position with the plug-in robust controller. The experimental results demonstrate the excellent position tracking with the proposed plug-in robust controller. We can noticed better tracking performances of the
robustified PI controller Fig.11.(b). The experimental response with a step perturbation demonstrate the excellent robustness of the controller Fig.12.(a). To emphasize the noise problem of the closed loop system with the plug-in robust, Fig.12.(b) demonstrate the influence of this parameter on the performance of the robust controller. To emphasize the noise problem of the closed-loop system, Fig.12.(b) demonstrate its weak influence on the robustness performances.

VI. CONCLUSION

The Loop Shaping Design Procedure using $H_{\infty}$ synthesis has been applied for robustified controller design of piezoelectric-stack actuated nanomanipulators. In the proposed $H_{\infty}$ loop shaping design procedure, the model uncertainties are included as perturbations to the nominal model, and robustness is guaranteed by ensuring that the stability specifications (nanometer resolution) are satisfied in the worst-case uncertainty. As conclusion, a very good point of the design is the remarkable features that achieves robustness with marginal reduction of performance. Our future work will consist of using the piezoelectric-stack actuated nanomanipulators in robust nanoteleoperation of mouse embryonic stems cells (mESCs) for regenerative cell construction.

REFERENCES