On the behaviour of ancient masonry structures subjected to static loads: case studies

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Abstract—The paper deals with the mechanical behaviour of ancient masonry constructions subjected to static loads. The cases considered concern structures designed to cover a given span. Masonry is modelled as a no-tension orthotropic material: the two-dimensional adopted continuum is characterised by infinite resistance to compression and no tension strength. Three cases are analysed: pseudo-arches, pseudo-domes and walls with windows. The considered applied load at this stage is only own weight and the purpose of the study is to recognise for each analysed case the bearing structure. As a consequence of the modelling the masonry is divided into macro-elements separated by possible fracture lines which define the starting of probable regions where damage can be localised.

Keywords—Historic constructions, ancient masonry, no tension material, resistant sections.

I. INTRODUCTION

The structural safety assessment of historical buildings is one of the central issues in the maintenance of the national and worldwide architectural heritage. Historical buildings, mostly due to the age, are placed at significant risk for the effects of thermal variations and, especially, of dynamic actions like those produced by earthquakes, traffic and wind induced vibrations. They are sensible to the decay of the resistance characteristics of the masonry, as well as to the condition of the soil and foundation, and vulnerable to cycling loads. These problems are mainly caused by atmospheric agents, settlement of foundation with consequent leaning, and as already said by wind and seismic actions.

Most of the historic constructions present in Italy, part of the national cultural and artistic heritage, are made of masonry, with structural elements often built in stone blocks or bricks with mortar. Such construction technique, inherit hand down through the centuries, has been consolidated during the ages. The intrinsic frailty of masonry structures is related to the scarce capacity to support tension, due to the mortar utilized, often degraded or lacking at all. The effect of applied loads, especially when horizontal and with high intensity can increase the vulnerability of such structures producing damage and possible collapse.

Several models are present in literature to represent the behaviour of masonry structures both in the linear elastic and non linear field, with discrete or continuum approaches; the applied loads can be either static or dynamic. Masonry is generally considered as a composite material with periodic structure so that difficulties arise in mathematical modelling its behaviour as that of a continuum. In the last years, continuum micro-mechanical models became increasingly popular among the masonry community: they basically make use of homogenisation techniques, e.g. [1]-[4]. The advantage of such models is that they can be implemented through the finite element method, therefore developing numerical procedures. The main idea is that masonry can be considered as a periodic structure obtained by regular repetition of elementary inhomogeneous cells of small size with respect to that of the whole structure [5]. Alternative procedures, still referring to a continuum approach, have been also proposed, basically based on phenomenological and then macro-mechanical models; among the numerous ones, the most used corresponds to the assumption of no tension behaviour. In this line, the first pioneering work is due to Signorini [6], who modelled masonry as a homogeneous elastic continuum with infinite to compression and no tensile strength, a prototype of unilateral constitutive relationship; further on, others followed this school of thought e.g [7]-[8].

With the same assumption made by Signorini, but with different purposes firstly proposed by Milankovitch [9], moves this paper, which belongs to a wide research program aimed at the protection and safety of ancient masonry structures subjected to applied loads. The present study - the first step of the program - investigates the best shape of masonry when the structure is designed to cover a given span: we refer to the typology of arched or vaulted masonry structures, under the assumption that the construction technique is in any case characterised by horizontal layered strata of material. This feature is common to structures of different historical periods and places, whose construction technique is closely connected to the tradition of *tholoi* of the Mediterranean proto-history. The use of roughly hewn stone carefully arranged in slightly projecting horizontal layers is in fact a characteristic of so-called false-domes, as is the case of the *nuraghe* (Fig. 1), built in Sardinia between the seventeenth and fifth centuries BC, the *trulli* (Fig. 2), traditional conical constructions to be found in Puglia, and *casididi* and *caselle*, simple rural buildings whose tradition is still present in Italy and in Istria.

By assuming that masonry can be modelled as a continuum, stratified across horizontal courses, with infinite compressive strength and no tension behaviour, a semi-analytical approach is proposed to identify the best shape to give to these special structures; a constitutive friction relationship is anyway
assumed along horizontal joints to guarantee equilibrium in presence of horizontal actions.

Figure 1. Example of Sardinian Nuraghe

The goal is to identify inside the masonry the supporting structure of minimum horizontal thickness, which defines at various vertical levels the corresponding resisting transversal sections. By this approach, the construction can be divided into macro-elements, which may vary depending on the position and intensity of applied loads and structure dimensions. Macro-elements are bounded at one lateral side by a defined geometrical profile and at the other by fracture lines, which can cause the pull-out of unused for resistance parts; fracture lines therefore define the starting of possible regions where damage can be localised. Along these directions sensors measuring displacements can be conveniently placed in order to monitor the thickness of openings. In such way it is possible monitoring in real time the health of historic constructions, by observing how they respond to external loads.

Three cases will be analysed and discussed: pseudo-arches, pseudo-domes and walls with windows. Considered applied loads at this stage are only due to own weight.

II. CASE STUDIES

Three cases, some of them typical examples of Italian historical constructions, are analysed and discussed: pseudo-arches, pseudo-domes and walls with windows subjected to static self-weight. Pseudo-arches are arches built by horizontal courses which do not need horizontal trust to guarantee equilibrium (Fig. 3). In this context, the problem analysed can be formulated as follows: having fixed the intrados line and the projection, that is the span to be covered, what is the minimum thickness required for the pseudo-arch to ensure stability at various heights in the absence of thrust?

Figure 2. Example of Apulian Trulli

Figure 3. Treasure of Atreus

Pseudo-domes or false-domes, here investigated, are axial-symmetric structures built by horizontal disposition of stones placed along circular rings; the difference between these kind of structures and stone domes built with radial joints along the meridians concern their structural behaviour – absence of interactions between meridian and parallels - and the law of thickness variation (Figs. 1-2).

The last example refers to walls with openings: in this case the problem is to understand how the masonry behaves to bypass the span, by means of appropriate pressure lines which allow to discharge the upper weight at sides of the window. It is moreover expected that the effect of the presence of an opening is a phenomenon with a local character, which
influences the masonry wall behaviour only inside a small region surrounding the opening itself.

In all the three study cases the aim is to recognize the best shape defining the bearing structure of minimum thickness, with the consequence that the masonry is divided into macro-elements separated by possible fracture lines.

III. MECHANICAL MODEL

Masonry is modelled as a no-tension orthotropic material. The two-dimensional adopted continuum assumes infinite resistance to compression and no tension behaviour concentrated along the horizontal planes, so that masonry is treated as a horizontally layered no-tension continuum. In any case, it is assumed that along horizontal joints a constitutive friction relationship is guaranteed in the presence of horizontal actions.

The considered applied loads are only due to own weight. The starting point of each analysis is the equation of equilibrium, in terms of resultant reaction acting at the generic horizontal level: the search criterion used to find the optimal solution is based on the best shape of a no-tension structure and therefore on the minimum thickness of the structure with wholly resistant transversal sections. Since the position of the centre of pressure on the joint depends on the distribution of normal pressure on the joint itself, having assumed no-tension behaviour, care has to be taken to avoid tensile stress being generated at each joint with consequent reduction of the resistant section: the centre of pressure must then fall within the cross section core, the limits of which represent the boundary defining the minimum thickness, that is the best shape. By knowing the best shape of the bearing structure, the whole masonry appears to be divided into sub-structures, that are macro-elements.

In the following, for each case, the equilibrium equations will be given, recalling specific references for details on how to obtain them. The equilibrium equations correspond to differential relationships between the thrust centre at generic level and the weight to be supported; due to the no-tension behaviour the best shape is obtained as that which guarantees the pressure distribution in accordance with the middle-third law.

A. Pseudo-arch

The differential equation governing the thrust centre position, indicated by variable $x$ (see for details [10]) as a function of the $y$ level, is:

$$\frac{d^2 y}{d x^2} - \frac{2}{\tan \beta} \left( \frac{dy}{dx} \right)^2 = 0$$  \hspace{1cm} (1)

which must satisfy the boundary conditions:

$$y\Big|_{1/3} = 0$$

$$\frac{dy}{dx}\Big|_{1/3} = \frac{6P}{g}$$  \hspace{1cm} (2)

Equation (1) is a non-linear differential equation of the second order, with variable coefficients, where $\beta$ is the constant slope with respect to $x$ axis of the predefined intrados line, $P$ is the assigned weight acting at the top and $g$ the specific weight of the material. The solution of (1) has been determined numerically; it should be noted that it does not depend on the specific weight of the material, resistance being guaranteed by the geometric shape.

B. Pseudo-dome

In the case of pseudo-domes of conical shape (Fig. 2), the differential equation governing the thrust centre position, indicated by variable $x$ (see for details [10]) as a function of the $y$ level, is:

$$F(x,y) \frac{d^2 y}{d x^2} + S(x,y) \left( \frac{dy}{dx} \right)^2 + R(x,y) \frac{dy}{dx} = 0$$  \hspace{1cm} (3)

Equation (3) must satisfy the boundary conditions:

$$y\Big|_{1/2} = 0$$

$$\frac{dy}{dx}\Big|_{1/2} = \frac{12P}{g}$$  \hspace{1cm} (4)

where $P$ and $g$ have the same meaning of the quantities defined in case A, and $F(x,y)$, $S(x,y)$ and $R(x,y)$ are functions of the pseudo-dome geometry reported in [10]. Equation (3) is a non-linear differential equation of the second order, with variable coefficients, whose solution has been determined numerically. It should be noted that by extending the problem of the best shape from the two to three-dimensional structure, the analytical expressions and the corresponding calculation became extremely complicated. As in the case of the pseudo-arch, the equation governing the phenomenon does not depend on the specific weight of the material, resistance being purely guaranteed by the geometric shape.

C. Wall with window

In the case of masonry surrounding laterally and above a window, only half part is considered in the analysis due to the symmetry of the problem. Two main behaviours can be distinguished depending on the circumstance that the two half parts surrounding the window either interact or not with each other, so that behaviour is that of the arch or the pseudo-arch, respectively. With reference to the pseudo-arch behaviour, let assume that no interaction exists; thus, having fixed the span $L$ to be covered and the extension $d$ of masonry at sides of the window, it is possible to evaluate the required height of masonry – in the following denoted as $y_{max}$ – in order that the pseudo-arch can cover in projection the whole fixed span. The equation governing the thrust centre, indicated by variable $x$ as a function of the $y$ level, is (see for details [11]):

$$\frac{d^2 y}{d x^2} - \frac{18}{(L + d) - 3x} \frac{dy}{dx} = 0$$  \hspace{1cm} (5)

Equation (5) must satisfy the boundary conditions:

$$y\Big|_{0} = 0$$

$$\frac{dy}{dx}\Big|_{0} = \frac{24W_0}{p(L + d)^2}$$  \hspace{1cm} (6)

The solution of Equation (5) allows for estimating the thrust line and intrados best shape of the pseudo-arch; both are related to the height $y_{max}$ required to guarantee that the
pseudo-arch covers the whole span so that the weight can be discharged at sides of the window. The value of $y_{\text{max}}$ obviously depends on the geometrical and mechanical parameters of the problem, namely: the window width $L$, the extension $d$ of the masonry portion adjacent the opening, the weight $V_0$ applied at the top and the weight $p$ for unitary volume of material (Fig. 3). Having fixed the geometrical parameters, the value of $y_{\text{max}}$ increases with the applied load $V_0$, which represents the upper masonry weighting on the pseudo-arch; however, although in our approach the masonry deformability is disregarded, it seems reasonable to assume that, by increasing $V_0$ and consequently the height of masonry weighting on the pseudo-arch, the pseudo-arch behaviour extinguishes itself due to the starting of the natural arch behaviour generated by the bending deformability of masonry and the consequent interaction between the two symmetric half parts of the structure. The identification of the vertical extension $y_{\text{max}}$ for a pseudo-arch behaviour is therefore relevant in our investigation; it corresponds to its minimum value guaranteeing the covering of the span and therefore to the smallest value of $V_0$ (Fig. 4).

![Figure 4. Scheme of the masonry element without interaction](image1)

For a masonry height over the window greater than $y_{\text{max}}$, the behaviour transforms from that of the pseudo-arch into that of the natural arch through the generation of thrust due to the interaction of the two half parts. If $h$ represents the height of masonry over the value of $y_{\text{max}}$ defined by the pseudo-arch behaviour, the problem of the best shape of the natural arch consists in determining the intrados profile which vertically extends along $y_{\text{max}}$ from the key intrados (Fig. 5) to the upper level of the window, so that the whole opening is covered by the natural arch; by this way pseudo-arch and natural arch behaviours are matched at the level where the behaviour varies.

For the natural arch, the equation governing the thrust centre position, indicated by variable $x$ as a function of the $y$ level, is (see for details [11]):

$$
H = \frac{3}{2} \left[ \frac{L + d}{6} - x \right]^2 \frac{d^2y}{dx^2} = 0
$$

Equation (7) must satisfy the boundary conditions:

$$
y_{\text{b}} = 0
$$

$$
\frac{dy}{dx} = \frac{24V_0}{24H - p(L + d)^2}
$$

The solution of equation (7) depends on the values of the applied weight $V_0$ and thrust $H$. The value of $H$ is defined as that guaranteeing the crossing of the thrust line at $x=0$ for $y=0$ (see Fig. 5), that is: $H = p(L + d)^2 / 16$. It is obvious that the matching of pseudo-arch and arch behaviours requires the same value of $y_{\text{max}}$ obtained in accordance to Equations (5) and (7).

![Figure 5. Scheme of the masonry element with interaction](image2)

### IV. RESULTS

#### A. Pseudo-arch

A typical result of the analysis performed by applying the procedure of Section III and integrating Equation (1) is shown in Fig. 6. The illustrated result has been obtained for $\beta=70^\circ$, $P=0.5$ t and specific weight of the stone: $g = 2.5$ t/m$. As can be observed, having fixed the intrados outline, the ideal lateral profile obtained by imposing the middle-third rule at each horizontal section is shaped as an arch, despite the vertical applied loads. The region extending from the intrados to the lateral profile can be defined as the “strength domain” of the pseudo-arch; it identifies the bearing part of the structure. Moreover, it is worthwhile to note that, starting from the top, the horizontal extension of the strength domain first increases and then decreases until, at a given vertical level, it reduces to zero; this level defines the end of the pseudo-arch behaviour, so that for greater heights an interaction between the two half parts of the structure is expected. Greater stability is therefore guaranteed by pseudo-arches of heights extending from the top to the level where the strength domain is of maximal horizontal extension.

Having fixed the position of $P$ - at the middle-third closest to the intrados of the unitary joint at the top - and its intensity, Fig. 7 shows how the slope intrados influences both the level of the section of maximal extension - in the following indicated as $y^*$ - and that at which the strength domain reduces to zero - in the following indicated as $y_{\text{max}}$; it can be...
also noticed that both levels increase by increasing the intrados slope.

![Figure 6. Typical strength domain for P=0.5 t, \(\beta=70^\circ\)](image)

**Figure 6.** Typical strength domain for \(P=0.5\) t, \(\beta=70^\circ\)

For a certain value of \(\beta\) (in this example for \(\beta=85^\circ\)) the slope of the thrust line and that of the extrados line are so high that the strength domain never reduces to a point (Fig. 8), since the strength domain is always increasing in extension; for this reason, for \(\beta=85^\circ\), \(y^*\) and \(y_{\text{max}}\) coincide.

In conclusion, fracture lines in the pseudo-arch can be located along the ideal external profile defining the boundary of the bearing structure.

**Figure 7.** Influence of the slope intrados \(\beta\) on \(y^*\) and \(y_{\text{max}}\)

![Figure 7. Influence of the slope intrados \(\beta\) on \(y^*\) and \(y_{\text{max}}\)](image)

**Figure 7.** Influence of the slope intrados \(\beta\) on \(y^*\) and \(y_{\text{max}}\)

**B. Pseudo-dome**

A typical result of the analysis performed by applying the procedure of Section III and integrating Equation (3) is shown in Fig. 9.

The result has been obtained for \(\beta=70^\circ\), \(P=0.5\) t and \(g=2.5\) t/m. As in the case of the pseudo-arch, the extension of the horizontal section at the top - in accordance with the middle-third rule - is defined by the point of application of the weight \(P\); at this point the neutral axis of the bearing structure starts.

The region extending from the intrados to the lateral profile can be defined as the “strength domain” of the pseudo-dome; it identifies the bearing part of the structure.

**Figure 8.** Typical strength domain for \(P=0.5\) t, \(\beta=85^\circ\)

![Figure 8. Typical strength domain for \(P=0.5\) t, \(\beta=85^\circ\)](image)

**Figure 9.** Typical strength domain for \(P=0.5\) t, \(\beta=70^\circ\)

![Figure 9. Typical strength domain for \(P=0.5\) t, \(\beta=70^\circ\)](image)
The optimal lateral profile for the pseudo-dome results to be a bottlenecked shape. Moreover, it can be noticed that when the problem is transferred from the two to three-dimensional structure (from pseudo-arch to pseudo-dome) the extension of the strength domain is considerably increased.

As for the pseudo-arch, the dependence of the strength domain extension on the slope of the intrados generatrix has been investigated. Having fixed the position and intensity of $P$, Fig. 10 shows how the slope intrados influences both the level of the maximal extension – indicated as $y^*$ - and that at which the strength domain reduces to zero – indicated as $y_{\text{max}}$.

Even in this case, the level at which the maximal extension of the horizontal section is reached and the level at which the strength domain reduces to zero increase more than linearly by increasing the intrados slope $\beta$. By comparing at equal conditions (same values of $P$ and $\beta$) the results here obtained with those of the pseudo-arch (Fig. 7), it can be noticed that increase of stability of the whole construction is reached mainly due to the axial-symmetric structure of the pseudo-dome. See for example the case of $\beta=75^\circ$; for the pseudo-dome $y^*$ is equal to 2.16 m and $y_{\text{max}}$ to 7.8 m, whereas for the pseudo-arch $y^*=0.97$ m and $y_{\text{max}}=3.9$ m. The level at which stability is guaranteed (known by observing $y^*$) is therefore increased more than two times.

As well as the weight applied at the top increases, the pseudo-arch behaviour requires a greater height $y_{\text{max}}$ necessary to cover the span; in fact, $y_{\text{max}}$ varies from 0.53 L for $h^*=0.1$ L, and different values of $V_0$ in order to test its influence on $y_{\text{max}}$.

In conclusion, fracture lines in the pseudo-dome can be located along the ideal external profile defining the boundary of the bearing structure.

**C. Wall with window**

In this case, the first unknown to be determined is the shape assumed by the ideal intrados outline corresponding to a pseudo-arch behaviour to cover the span over the window (over the architrave if there is one), having fixed due to symmetry the lateral profile as vertical. It is seen from Equation (5) that the best shape depends on the masonry dimensions and the weight $V_0$ applied at the top. According to actual dimensions of a wall with windows, we will assume: $L=1$ m, $d=2$ m, the specific weight of the stone $g = 2.5$ t/m$^3$ and $V_0$ is simulated as corresponding to a fictitious value of height $h^*$ of masonry. Results are reported in Fig. 11 where the fixed lateral profile is vertical; the thrust line (solution of the differential equation) is the dashed curve, while the continue curve corresponds to the ideal intrados line. The region extending from the ‘ideal’ intrados to the vertical lateral profile is the ‘compression strength domain’.

![Figure 10](image1.png)

**Figure 10.** Influence of the slope intrados $\beta$ on $y^*$ and $y_{\text{max}}$.

![Figure 11](image2.png)

**Figure 11.** Pseudo-arch behaviour over the window. Top: $h^*=0.1L$, Middle: $h^*=0.5L$, Bottom: $h^*=L$. 

As well as the weight applied at the top increases, the pseudo-arch behaviour requires a greater height $y_{\text{max}}$ necessary to cover the span; in fact, $y_{\text{max}}$ varies from 0.53 L for $h^*=0.1$ L,
to 2.53 L for \( h^* = 0.5 L \), until it reaches the value of 5.53 L for \( h^* = L \). Note that under the strict assumption of pseudo-arch behaviour the portion of masonry underneath the intrados line would weight the architrave, so that over a certain height \( y_{\text{max}} \) this weight could exceed acceptable values; however, the pseudo-arch behaviour is possible and compatible with the load weighting the architrave (if there is one) as soon as \( y_{\text{max}} \) covers the given span, that is for the minimum value of \( h^* \); as a consequence \( h^* \) has been therefore chosen as equal to 0.1 L corresponding to the height of a layer of bricks (Fig. 4). As already observed, for a masonry height over the window greater than \( y_{\text{max}} \), the masonry behaviour transforms into that of natural arch, which generates the thrust corresponding to the interaction of the two half parts. Starting from \( y = 0 \), the solution of Equation (7) is a straight line, whose slope depends on the geometry of masonry surrounding the window and the masonry height \( h \) over the level at \( y = 0 \) (Fig. 5). The natural arch fully covers the given span if \( y_{\text{max}} = 4hL/(L + d) \), so that \( y_{\text{max}} \) increases linearly with \( h \); thus, since the value of \( y_{\text{max}} \) of pseudo and natural arch must coincide to match the two behaviours, the best value of the height \( h \) can be therefore determined: it defines the optimal shape of the natural arch surrounding a window. The numerical investigation gives the value of \( y_{\text{max}} = 0.53 L \), corresponding to the height of masonry over \( y_{\text{max}} \) equal to \( h = 0.4 L \).

In Fig. 12, the vertical line represents the fixed lateral profile, the dashed line is the thrust line (solution of the differential equation) and the continuous straight one is the corresponding intrados ideal shape. The region horizontally extending from the ‘ideal’ intrados to the vertical lateral profile is the ‘compression strength domain’. It can be noticed that the arch is characterised by straight thrust and intrados lines. The intrados lines of pseudo and natural arch for the same value of \( y_{\text{max}} = 0.53 L \) are compared in Fig. 13.

In conclusion, having fixed the span of the window to be covered and the horizontal extension of the masonry at sides of the window, the height of masonry for the pseudo-arch behaviour is univocally determined, as well as the height \( h \) of masonry defining the thickness of the natural arch at the key.

Thus, the selected value of \( d/L \) decides the amount of masonry weighting on the architrave placed over the window; in our investigation it corresponds to about the weight of a triangle of height equal to half of the base \( L \). By varying the value of \( d/L \) (in general, for too small values of \( d \) in comparison with \( L \)), the size of the triangle can become too large to be supported by the architrave (if there is one): this is the case to build an artificial arch for protect the integrity of the architrave. Fracture lines over the window are located along the ideal intrados line generated by either the pseudo or the natural arch. Such regions represent places where damage of the structure can be localized. It assumes therefore importance to monitor the regionjust above the window at a level more or less equal to half height of the opening.

The results reported refer to typical assumed dispositions of the openings in masonry walls; results can be different if the masonry wall geometry drastically changes.

V. CONCLUSIONS AND FURTHER REMARKS

The paper dealt with the protection and safety of ancient masonry structures, numerous in Italy and part of the national cultural and artistic heritage.

Three common typologies of structures - the pseudo-arch, pseudo-dome and finally wall with windows - subjected to static loads due to self-weight have been analysed. The objective has been to outline the structural behaviour and possible damages, in relation to the structure geometry and the kind of applied loads. A semi analytical approach has been proposed for the analysis in order to evaluate the stress path; the goal has been to identify inside the masonry the bearing structure, by defining the resisting transversal sections. As a consequence the construction can be divided into macro-elements, which depend and may vary on the position of applied loads and the structure geometry. Adjacent resisting transversal sections are separated by fracture lines, opening along the vertical joints. Along the direction of fracture lines, the parts unhelpful for resistance are pointed out as possible regions where damage can be localised: thus, sensors (with new technologies e.g. wireless or optical fiber) measuring displacements can be conveniently placed in order to monitor the thickness of the openings. In such way it is possible monitoring in real time the health of an historic construction, observing how it responds to external loads, which may be not only static but also dynamic, such as those due to traffic vibrations, seismic or wind actions. In case of danger, quick measures for safety of the construction can be therefore taken.
VI. REFERENCES


