A Resonant Parallel Elastic Actuator for Biorobotic Applications

Angelo Sudano*, Nevio Luigi Tagliamonte, Dino Accoto and Eugenio Guglielmelli

Abstract—In cyclic motions, e.g. during locomotion, part of total kinetic energy, which otherwise would be wasted during direction inversion, can be conveniently stored in elastic elements in order to be later reused. Such energy recovery strategy, which can be adopted in robotic systems to improve energetic autonomy, mimics biological solutions, where muscles, tendons and ligaments are exploited also as elastic energy buffers.

The addition of compliant elements, in parallel or in series to conventional actuators, may result in complex assemblies. Conversely, compact solutions can be obtained if elasticity is embedded in the structure of the actuator.

In this paper an electric rotary compliant actuator, integrating a magnetic non-linear torsion spring, is presented. The system is a parallel elastic actuator and it allows to efficiently produce oscillating motions, as needed in several biorobotic applications. The stator comprises mono-phase windings while the rotor includes permanent magnets in axial Halbach configuration. Additional magnets are advantageously located in the stator to embody the intrinsic parallel elasticity.

A proof-of-concept prototype was developed, whose winding coils were designed based on magneto-static FEM simulations. The actuator was experimentally characterized and the present dynamic behavior was compared to the theoretical one calculated by considering a non-linear electro-mechanical model.

I. INTRODUCTION

In the biological domain there exist many examples of locomotion strategies which are optimized by cyclically storing/releasing potential elastic energy. In this regard, several studies describe how tendons, muscles and ligaments can be exploited also as energy storage elements [1], [2]. Such strategy has been demonstrated in a wide variety of animals that differ functionally, structurally and dimensionally [3]. The cyclic energetic buffering can be conveniently exploited also in walking/running [4], [5], swimming [6] or flapping [7] robots. To this aim elasticity is purposively introduced in the actuation system.

Actuators with intrinsic compliance can be grouped in two classes: Series Elastic Actuators (SEAs) [8], [9] and Parallel Elastic Actuators (PEAs) [10], [11]. A comparison between these and other possible compliant actuation schemes, in terms of energy consumption, is reported in [11]. Parallel elastic elements are capable of reducing power/torque requirements for actuators driving cyclic movements [12]. Many examples of this concept are present in literature, and they are also related to exoskeletons [13] and prostheses [14]. A new actuation concept, Series-Parallel Elastic Actuator (SPEA), was described in [15]. This actuator consists of multiple parallel springs connected to a system that enables their variable recruitment. Further actuation architectures have been developed, based on more complex combinations of springs and on mechanisms able to vary the output elasticity [16].

Compliant rotary actuators are typically designed by considering the motor and the elastic element as two distinct components [17]–[19]. This approach facilitates the design but it possibly results in heavy and bulky assemblies.

The present work aims at integrating a magnetic spring in the same design of an electric rotary motor, thus simplifying the overall device structure. In particular, a novel compact PEA, capable of producing resonant oscillations, is presented. The system is suitable for applications where efficient reciprocating motion is required, such as autonomous robots with bioinspired locomotion capabilities [20]. In the presented design the magnetic field generated by motor permanent magnets is used not only to produce current-induced actuation torques, but also to simultaneously deliver parallel elastic torques. Indeed, additional permanent magnets were purposively introduced in the rotor to embed a magnetic non-linear torsion spring.

The design of a proof-of-concept prototype is described in Sec. II. The experimental characterization of the actuator parameters, based on an electro-mechanical model of the system, is reported in Sec. IV. The dynamic behavior of the system is analyzed in Sec. V.

II. DESIGN

As shown in Fig. 1-(a), the actuator includes a pancaketype motor and consists of three layers: two rotor discs and one stator disc sandwiched in between. The motor is an Axial Flux Permanent Magnet (AFPM) double-sided machine with NS magnets configuration. The two external rotor discs are mechanically coupled to implement a single rotor component and are separated from the stator by two air-gaps \( d_g = 4 \) mm. Rotor discs, whose distance is \( d_h = 10 \) mm, are coupled to a central rod, fixed to the stator, by means of two ball bearings, as shown in Fig. 1-(b). Both for the rotor and for the stator, axially magnetized neodymium-iron-boron (NdFeB) cylindrical magnets are used, with diameter and height of 2 mm, remanence \( B_r = 1.2 \) T and coercive force \( H_c = 868 \) kA/m. In each rotor disc, 32 magnets are arranged in a double Halbach array configuration with 4 poles; in particular, magnets are placed along two concentric circumferences with radius \( r = 7.5 \) mm and \( r + d_m = 10.5 \) mm (\( d_m = 3.0 \) mm, Fig. 1-(d)). In the stator, 8 magnets are arranged in 4 couples spaced at 90 deg and axially oriented with alternate magnetization. Rotor consists of 3D-printed
acrylic resin discs, each one with housings for a ball bearing and for the circularly arranged magnets, as shown in Fig. 1-(b) and in Fig. 1-(c). Rotor discs are assembled so that the magnet poles on one disc are aligned with the opposite polarity of the magnet poles located on the other one. The stator disc is made of 3D-printed acrylic resin and it has a cross-shaped frame where the housings for the spring permanent magnets are obtained. The stator includes 4 coils, spaced at 90 deg, to wind mono-phase coils, as depicted in Fig. 1-(d). For characterization purpose, the central rod was framed to a fixed plate (M-MRP4-1, Newport). The actuator was also equipped with an optical incremental encoder with 500 cpt (AEDB-9140-A12, Avago Technologies), whose disc is fixed to the rotor and whose reading head is supported by a 3D-printed acrylic scaffold, as shown in Fig. 1-(a). The actuator, without encoder, weights 12.1 g, is $h = 20\text{mm}$ high and has a diameter of $\phi = 32\text{mm}$.

A. Motor

A brushless motor type was preferred to avoid electric and mechanical noise due to sparks occurring during brushes commutations. Despite its low air-gap flux density, the air-cored architecture was selected because of its simpler design and ease of manufacture. Air-cored motors have no core losses; however, they can operate with higher efficiency only if magnetic flux density is comparable to the one of cored motors, e.g. if the amount of magnetic material is increased or the air paths are reduced. For this reason, an internal stator with axial coils and permanent magnets (PMs) in NS configuration was used. Furthermore, to better concentrate the magnetic field without the aid of ferromagnetic cores, a Halbach array configuration was used for the permanent magnets in the rotor. In [21] the main advantages of Halbach arrangement are reported: i) the magnetic field is about 40% higher with respect to conventional PM arrays and efficiency is doubled; ii) the introduction of backing steel magnetic circuits is not required and PMs can be bonded directly to a non-ferromagnetic support; iii) the sinusoidal magnetic field is less distorted than in conventional PM configurations; iv) back-side magnetic fields are negligible.

With reference to Fig. 2-(a), considering a Cartesian coordinate system ($x, y, z$) and a circular Halbach array in the $x$-$y$ plane, the magnetization orientation $\mathbf{M}_i$ for all the array magnets, indexed with the $i$ counter ($i = 0, \ldots, 15$), is expressed as:

$$
\mathbf{M}_i = \begin{bmatrix}
\sin(i\pi/8 + j\pi) \sin(i\pi/4) \\
-\cos(i\pi/8 + j\pi) \sin(i\pi/4) \\
\cos(i\pi/4)
\end{bmatrix}
$$

where $j$ is 0 for the upper rotor disc and 1 for the lower one. Counterclockwise direction is assumed positive and magnets with index $i = 0$ are considered to lie on the $x$ axis.

For this configuration a magneto-static FEM simulation, in a current-free space, was performed to evaluate the $z$-axis magnetic flux density ($B_z$ [T]) evaluated through FEM simulations in the middle plane between rotor discs.

![Fig. 2: (a) Magnetization orientation of the magnets for the Halbach circular arrays included in the upper ($j = 0$) actuator rotor. The orientation vector is mathematically described by (1). (b) $z$-axis component of the magnetic flux density ($B_z$ [T]) evaluated through FEM simulations in the middle plane between rotor discs.](image)
two rotor discs. Fig. 2-(b) shows the flux density, whose distribution varies sinusoidally with the angular variable \( \alpha \) indicated in Fig. 2-(a). Based on simulation results, windings were purposely dimensioned to maximize the flux linkage (\( \lambda \)), that can be approximated as:

\[
\lambda = \xi \sum_{n=1}^{N} B_{z} d \Omega
\]

where \( \Omega_{n} \) is the surface of the \( n \)-th coil turn, \( N \) is the number of turns per coil and \( \xi \) is the number of coils. The four coils were designed with the shape dictated by the contour lines of \( B_{z} \) in Fig. 2-(b). They consist of sixty turns of a polyurethane-coated copper wire with a conductor diameter of 0.2 mm and a coating of 33 \( \mu \)m. Winding coils were wounded alternatively in clockwise and counterclockwise directions and were electrically connected in series (monophase motor). Each coil has a mean area of about 45 mm\(^2\), while the maximum magnetic flux density \( B_{z,max} \) averaged over the mean coil area is 28 mT.

The flux linkage is \( \lambda (\theta) = \lambda \sin(2\theta) \) with \( \lambda = 0.302 \) mWb and \( \theta \) the rotor angular position. The back-emf can be estimated as \( V_{c} = \frac{d \lambda}{d \theta} = 0.604 \dot{\theta} \cos(2\theta) \) mV. Torque characteristic can be derived from mechanical transferred power \( P_{m} = V_{c} I = \tau_{m} \dot{\theta} \) and it results in \( \tau_{m} = P_{m} = 0.604 \dot{\theta} \cos(2\theta) \) mNm.

B. Magnetic torsion spring

A torsion spring is connected in parallel to the actuator output shaft thus embodying a PEA architecture and improving efficiency of oscillating motion. Four couples of magnets, identical to those used in the rotor, were axially oriented and mounted on the stator in the same way as the rotor permanent magnets in the positions \( i = 0, 4, 8, 12 \). In addition, they were shifted by 45 deg with respect to the center of the stator coils (Fig. 1-(d)). In order to preliminarily evaluate the magnetic spring torque-angle characteristic, a set of magneto-static FEM simulations were computed (Fig. 3). Simulated torque-angle characteristic was fitted with a sinusoidal curve in the form \( \tau_{s}(\theta) = \tilde{k}_{s,FEM} \sin(2\theta) \). A value \( \tilde{k}_{s,FEM} = 4.10 \) mNm was found, with a correlation coefficient \( R^2 = 0.999 \).

\[
\text{Fig. 3: Magnetic spring torque-angle characteristic. Green dots: FEM simulations; black dots: experimental data (standard error bars represent the 95% confidence interval); solid line: experimental data fitted according to (5) (correlation coefficient } R^2 = 0.997).\]

C. Motor drive and sensors

Due to the low inductance, the actuator can be efficiently driven with a full MOSFET H-bridge. Two dual N and P channel MOSFETs (International Rectifier, IRF5851), with low drain-to-source resistance in ON-state, were used. The gates of these MOSFET are driven by 4 High-Speed Power MOSFET Drivers (Microchip, TC4428A). For the actuator characterization, an optical encoder, as described in Sec. II, was added (Fig. 4).

III. MODEL

Let \( I \) be the current in the coils, \( \theta \) the angular position of the rotor, \( \tau \) the output torque, \( J \) the moment of inertia of the rotor, \( c \) the viscous friction coefficient, \( \tau_{s}(\theta) \) the torque-angle characteristic of the magnetic spring, \( L \) the electric inductance of the armature, \( R \) the electric resistance, \( \tau_{m}(l, \theta) \) the current-induced motor torque and \( V_{c}(\theta, \dot{\theta}) \) the back-emf. The non-linear model of the actuator can be described as:

\[
\begin{align*}
\tau &= J \ddot{\theta} + c \dot{\theta} + \tau_{s}(\theta) + \tau_{m}(l, \theta) \\
V &= LI + RI - V_{c}(\theta, \dot{\theta})
\end{align*}
\]

(3)

Fig. 4: Actuator electronic driver and sensor. \( V \) is the voltage supply, while a, b, c, d are the H-bridge commands provided to the four MOSFET gates.

Considering a sinusoidal magnetic flux density produced by rotor magnets, as obtained from FEM simulations, and the winding location, the motor current-induced torque and the back-emf can be written as:

\[
\begin{align*}
\tau_{m}(l, \theta) &= lk_{t} \cos(2\theta) \\
V_{c}(\theta, \dot{\theta}) &= \dot{\theta} kk_{v} \cos(2\theta)
\end{align*}
\]

(4)

where \( k_{t} \) and \( k_{v} \) represent the related constant coefficients. The sinusoidal torque-angle characteristic of the spring, as also results from FEM simulations, can be modeled as:

\[
\tau_{s}(\theta) = k_{s} \sin(2\theta)
\]

(5)

where \( k_{s} \) is the maximum elastic torque deliverable by the magnetic spring.

IV. EXPERIMENTAL CHARACTERIZATION

A. Magnetic torsion spring

In order to experimentally evaluate the spring torque-angle characteristic, the torque \( \tau \) delivered by the actuator to maintain an angular position \( \theta \) was measured. This measurement was carried out in static condition (\( \dot{\theta} = 0 \)) and without exciting actuator windings (\( l = 0 \)), by employing the custom test-bed shown in Fig. 5.
Starting from the rest configuration of the actuator, the torques associated to rotations ranging from 0 to 90 deg (step: 1 deg), were measured, by considering five repetitions for each trial. Torque values were averaged over the repetitions in order to regress the data with a sinusoidal curve as in (5). Fig. 3 shows the average data with standard error bars (confidence interval: 95%) and the fitting curve (correlation coefficient $R^2 = 0.997$). The value for the spring parameter related to model (5), resulting from the regression of the experimental data, is $k_{s,exp} = 2.97$ mN m.

B. Electric resistance and inductance

To characterize the electric resistance and inductance, a set of measurements which consisted in probing the motor armature terminals through an LCR-meter (ISO-TECH 9053) in static conditions ($\dot{\theta} = 0$) was carried out. Values obtained by averaging five different measurements are $L = 49 \pm 1 \mu$H and $R = 3.45 \pm 0.10 \Omega$.

C. Back electromotive force

The setup used to characterize the back-emf includes the incremental optical encoder, two low-noise operational amplifiers (Texas Instruments, LMV722IDR), a 8-bit microcontroller (PIC18F4331, Microchip), a UART to RS232 interface and a PC. One operational amplifier is used as voltage reference, while the other one is employed as inverting amplifier. A potentiometer allows regulating the gain to avoid saturation. The analog voltage is converted by an ADC module of the microcontroller, which simultaneously reads the rotor position from the encoder through a dedicated module. Both position and voltage are collected by PC at 200Hz.

The characterization procedure consisted in measuring the actuator free oscillations (windings not powered) when the rotor started moving from a position different from the equilibrium point. During oscillations also armature voltage was measured. Actuator velocity $\dot{\theta}$ was computed by numerically differentiating the position and it was fitted with voltage data according to the model $V_{e}(\theta, \dot{\theta}) = \dot{\theta} \hat{k}_{e} \cos(2\theta)$. Results obtained from the fitting provided a back-emf coefficient $\hat{k}_{e} = 0.58 \text{ mV rad}^{-1}$ (correlation coefficient $R^2 = 0.990$). The measured angle, the measured back-emf and the curve resulting by regressing it with the calculated angular velocity are reported in Fig. 6.

D. Current-induced torque

Torques $\tau_{m}$, modeled by (4), were measured with the setup presented in IV-A. In static conditions ($\dot{\theta} = 0$) a power supply with adjustable current amplitude and a multimeter electrically connected in series to the windings to directly measure current were used. For rotations in the ranges 0–45 deg (step: 5 deg) and current values in the range 0–500 mA (step: 100 mA), torque $\tau_{m}$ was measured with the load cell. The surface which fits measured data according to the model (4) provided a current-induced torque coefficient $\hat{k}_{i} = 0.52 \text{ mN m A}^{-1}$ (correlation coefficient $R^2 = 0.872$). Regression results are reported in Fig. 7.

E. Inertia and damping

For the identification of inertia and viscous friction coefficient, data collected during free oscillations, and previously used for the back-emf characterization (Sec. IV-C), were analyzed. Inertia $J$ and viscous friction $\dot{c}$ were estimated by
means of logarithmic decrement method. In this regard, only rotations with amplitude smaller than 10 deg were taken into account, in order to consider small oscillations and hence a linear torque-angle characteristic for the spring. The stiffness function was calculated as $\kappa(\theta) = \frac{dC}{d\theta} = 2k_s \cos(2\theta)$. In the linear regime the stiffness is constant and can be approximated as $R \approx 2k_s = 5.94$ Nm rad$^{-1}$. Considering the actuator as a simple linear mass-spring-damper system with known spring stiffness, the logarithmic decrement method provided the values $\hat{F} = 1.50 \cdot 10^{-6}$ Nms$^2$ rad$^{-1}$ and $\hat{\varphi} = 2.87 \cdot 10^{-6}$ Nms rad$^{-1}$ (correlation coefficient $R^2 = 0.999$).

V. DYNAMIC BEHAVIOR

To evaluate the actuator dynamic behavior, a set of tests was carried out. Absorbed current was limited by means of a limiter on the power supply. Experimental trials consisted in exciting the actuator through current square waves with duty cycle 50% while measuring rotation through the encoder (sampling frequency: 200 Hz). Measurements were performed for frequencies in the range 1–20 Hz (step: 0.5 Hz) and currents in the range 100–500 mA (step: 100 mA). Each trial had a duration of 20 s. A representative test with current amplitude of 500 mA and excitation frequency of 8.5 Hz is reported in Fig. 8. The same working conditions were replicated in simulation by using the model (3)–(5), with parameters experimentally identified as explained in Sec. IV. Both for simulated and experimental data, the RMS value for the oscillation amplitude is reported as a function of the oscillation frequency in Fig. 9. It can be noticed that the amplitude increases with the magnitude of the forcing input but the fundamental frequency decreases with it. This behavior is typical of some non-linear oscillators [22].

VI. DISCUSSION AND CONCLUSIONS

This paper presents a novel compact PEA design, which advantageously embeds a magnetic torsion spring in a custom axial flux electric rotary motor. The working principle of the actuator was presented and a proof-of-concept prototype was described. The experimental characterization of the system was performed based on a non-linear electro-mechanical model. The capability of the actuator of producing resonant oscillatory motion was experimentally proven and the actual prototype behavior was compared to the theoretical one based on the estimated values of the model parameters.

The adopted design presents the combined advantages of magnetic springs and Halbach AFPM motors. In particular, the actuator does not exhibit fatigue, overloading and friction issues due to the magnetic nature of the spring (the rotor and the stator remain mechanically isolated despite the introduction of elasticity connecting them). The actuator is intrinsically resonant and can be used in many robotic applications where a cyclic motion has to be produced (e.g. bioinspired robotic locomotion such as swinging, walking or flapping). The system presents very low back-side fields thus not requiring any backing steel magnetic circuit. This results in a low rotor inertia and in a sinusoidal magnetic flux density. The use of mono-phase windings only requires simple driving electronics. The air-cored architecture does not imply complex cores manufacturing and it results in a lightweight design. Nonetheless, the absence of cores reduces cogging effects. Finally, due to AFPM motor topology, the actuator shows a high level of modularity that allows possibly stacking several actuators or simply functional elements in series or in parallel.

The presented prototype is a proof-of-concept of the actuator working principle and therefore is open to future fabrication improvements to increase the magnetic flux linkage and thus spring and motor torques. Such improvements include the reduction of the air-gap and the use of magnets...
with higher energy product and/or with different geometry. Simulation results were employed to dimension the stator winding coils. The actuator model was reported and the values of its parameters were experimentally determined through different characterization tests. The spring torque-angle characteristic was found to be sinusoidal, as expected based on the theoretical model. The regression of the experimental data provided a correlation coefficient $R^2 = 0.997$. The shape of the curve was also coherent with FEM simulations. Anyhow, the maximum deliverable simulated elastic torque ($\hat{\tau}_{\text{FEM}}$) was found to be about 28% higher than the experimental one ($\hat{\tau}_{\text{exp}}$). This discrepancy is likely due to the uncertainty on the nominal values of the magnets parameters and to mounting errors due to the manual assembly (position and orientation) of the magnets.

The dynamic behavior was experimentally tested by exciting the actuator without additional loads through current square waves at different amplitudes and frequencies. These experimental conditions were also replicated in simulation. In both cases RMS values of the rotation amplitude as a function of the oscillation frequency highlighted the nonlinear behavior of the actuator. The system fundamental frequency depends on the oscillation amplitude and the quality factor of the resonance is rather high. In the actual prototype maximum oscillations for the different current values were in the range 8–10 Hz as shown by peaks A in Fig. 9. It is worth adding that the peaks B and C (at 1.5 and 3 Hz) are generated by frequency components in the excitation square wave higher than the fundamental one. Minor discrepancies were found between actual and simulated behaviors. In particular, simulations slightly overestimated (about 0.5 Hz) the oscillation frequency for the amplitude peaks A. This result demonstrates the reliability of experimental results and of the adopted model, which can be used also to simulate actuator performance with different loads and control algorithms.

In the present prototype, the current-induced torque is higher than the (parallel) elastic torque only for a limited angular range. This condition precludes the effective use of a pure position control scheme outside this angular window. Despite this limitation, high-amplitude oscillations are possible, without employing any position controller, due to the PEA resonant nature. Future research will be devoted to implement a controller able to induce self-oscillation, thus further improving actuator efficiency through auto-resonance.

**References**


