Bargaining, Binding Contracts and Competitive Wages

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Abstract

In an analysis of a model where many workers bargain with a firm and sign binding contracts, we show existence of a stationary subgame perfect equilibrium. If the production function satisfies decreasing returns, each worker receives a share of his marginal product (treating all other workers as employed) in equilibrium. Thus, wages are competitive. This is in contrast to Stole and Zweibel (1996), who assume that contracts are non-binding and find that the payoff of a worker is a weighted average of the inframarginal contributions. Hence, binding contracts imply lower wages than non-binding contracts.

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1 Introduction

During the last two decades, noncooperative bargaining models have been studied extensively. Some of the earliest analyses are those of Ståhl (1972) and Rubinstein (1982), while an overview can be found in Osborne and Rubinstein (1990). In firm-worker bargaining models there is usually one worker who bargains with the firm. Horn and Wolinsky (1988) and Jun (1989) study a situation in which the firm bargains with two workers. However, with the exception of Stole and Zweibel (1996), there are few models describing a situation where a firm bargains with several workers.

Stole and Zweibel assume that labor contracts can be renegotiated at any time before production starts. This assumption leads to wages and profits corresponding to the Shapley value. Thus, a worker gets a weighted average of the inframarginal units. One objection to the approach in Stole and Zweibel is that it is normally not so simple to renegotiate contracts. For example, a contract is often legally binding and might stipulate that a specific period of time has to elapse before a contract can be terminated. In addition, breaking of contracts might lead to a legal conflict that is costly.

The purpose of this paper is to study a firm which bargains with several workers when contracts are binding. The theoretical model builds primarily on Gül (1989). The bargaining takes place as follows. One of the workers is chosen at random to bargain with the firm. The proposer is randomly chosen to be either the firm or the worker. The proposer makes a proposal and the respondent accepts or rejects the proposal. Then one of the workers without an agreement is chosen to bargain with the firm and so on.

The payoffs in equilibrium are computed. If the production function satisfies decreasing returns, each worker gets a share of his marginal contribution, treating all other workers as employed. Thus, workers do not get a share of the inframarginal units, but wages are “competitive”. To see this, consider an example with two workers. In subgames where one worker has already been hired, the wage for the remaining worker is competitive, since the surplus from hiring the last worker - the marginal product - is shared between the bargainers. In subgames where both workers are without an agreement, either both firm-worker meetings end in agreement or only one meeting ends in agreement. First, if both meetings end in agreement, a worker cannot get less (more) than the payoff in the subgame where the worker is the only one without an agreement, since otherwise the worker (firm)
could wait until an agreement is reached with the other worker. Thus, wages are competitive. Second, if only one meeting ends in agreement, it can be shown that the worker, say worker 1, and the firm share the (net) value of hiring both workers. If the production function satisfies decreasing returns, the value of hiring both workers is larger than the marginal product. Thus, the wage for worker 1 is higher than the competitive wage. Note also that worker 2 gets the competitive wage, since worker 2 always agrees with the firm when worker 1 has already been hired. Then the firm has a profitable deviation, by first agreeing with worker 2 and forcing worker 1 to wait. To agree with worker 2, the firm needs to offer slightly more than the equilibrium payoff for worker 2, i.e., the competitive wage. Since worker 1 is the last to agree with the firm, worker 1 also gets the competitive wage. The firm pays competitive wages to both workers and hence gains. Then, in equilibrium, both meetings end in agreement and wages are competitive. Thus, payoffs in equilibrium are in stark contrast to the payoffs in Stole and Zweibel.

The model is developed in the next section. Equilibrium is analyzed in section 3 and section 4 concludes.

2 The Model

The agents consist of a set of workers, denoted $W$, with generic element $i$, and a firm, denoted $F$. There are two goods, leisure and a consumption good. For simplicity, we normalize the utility of leisure to zero. A worker either works some fixed amount of time or does not work at all.

Let $W$ be the set of subsets of $W$, including the empty set. Let $f : W \to \mathbb{R}$ denote the production function. The production function satisfies the following requirement. For any $i \in W$ and $C \subseteq W$ such that $i \notin C$, the marginal contribution of a worker $i \notin C$, $f(C \cup i) - f(C)$, is positive. Also, for all $i \in W$, let $MP_i = f(W) - f(W \setminus i)$ be the marginal product for worker $i$.

2.1 The Bargaining Game

At the beginning of the game none of the workers are employed by the firm. The firm bargains with one worker at a time. A worker is randomly chosen from among the workers without an agreement (selection probabilities are described in more detail below). Next,
bargaining takes place between the chosen worker and the firm. The proposer is randomly chosen to be either the firm or the worker. The proposer then makes a proposal and the respondent answers yes or no. If no agreement is reached, the firm continues to bargain with the same set of workers. If an agreement is reached, the worker who signed the agreement leaves the game and bargaining continues with the remaining workers. The firm can use the labor from worker $i$ for the remainder of the bargaining game.

Let $\Gamma(S)$ denote the game when the workers in $S$ are without an agreement with the firm. The bargaining game is depicted in Figure 1.

Let $\delta \in [0, 1)$ be the common discount factor for all players. Consider an outcome of the bargaining game. Suppose an agreement between worker $i$ and the firm is reached at time $T$. Let $w_{iT} \in \mathbb{R}$ denote the wage payment to worker $i$ at time $T$. The payoff of worker $i$ is given by $\delta^{T-1}w_{iT}$. Since contracts are binding, it does not matter whether wages are paid out as a one-time payment as above or as a stream of payments each period.

**Figure 1.** The Bargaining Game.

Let $w_t \in \mathbb{R}$ be the wage payment by the firm at time $t$. Let $G_t$ denote the set of workers
with whom the firm has reached an agreement at time $t$. The payoff of the firm is given by
\[ \sum_{t=1}^{\infty} \delta^{t-1} \left( (1 - \delta) f(G_t) - w_t \right). \]

A strategy in the game for player $i$ is denoted $\sigma_i$. Let $\sigma = (\sigma_i)_{i \in W \cup F}$. In general, the strategies at any time are a function of all possible histories up to period $t-1$. As is commonly known, a subgame perfect equilibrium in a two-player Rubinstein/Ståhl bargaining game has the property that the players use stationary strategies. Bargaining with more than two players is more difficult to analyze. One reason is that there is a plethora of equilibria, as in Shaked and Suttons’s (1984) example with three players. It seems reasonable to focus on stationary strategies, because of their simplicity:

**Definition 1** A strategy $\sigma_i$ for player $i$ is stationary if, for any two histories such that the firm has agreed with the same set of workers, the strategy prescribes the same action.

A strategy profile $\sigma$ is stationary if the strategy for each player is stationary. Let $\Sigma^s$ denote the set of stationary strategy profiles.

Now consider the selection probabilities. Clearly, since selection probabilities must sum to one, they depend on the number of workers remaining. We assume that the probability that a worker is selected to bargain with the firm depends on the set the workers without an agreement. Thus, selection probabilities are not restricted to depend only on the number of workers who remain, but can also depend on which workers that remain. Formally, for any set $E$ of workers without an agreement, where $E \subseteq W$, let $p_i(E)$ denote the probability that worker $i \in E$ is selected to bargain with the firm. Let $p_i^p$ denote the probability that worker $i$ is selected as proposer.

Stationarity implies that the continuation payoff is the same for any two histories such that the firm has reached an agreement with the same set of workers. For all $\sigma \in \Sigma^s$, $E \subseteq W$ and $i \in F \cup E$, let $U_i(E, \sigma)$ denote the (expected) continuation payoff at the beginning of a subgame where $E$ workers are without an agreement.

**Comparison**

The model analyzed by Stole and Zweibel (1996) differs in two important respects from the model above. First, in their model, contracts can be renegotiated at any time before all workers are hired. Second, if a negotiation between a worker and the firm breaks down, the
worker leaves the firm forever. Thus, their model has the feature that players cannot commit credibly to a contract, but they can commit credibly to terminating the relationship forever. In our paper, a worker terminates the relationship only if it is profitable, by choosing actions that never lead to an agreement. Another difference is that the order of worker selection is predetermined in their model.

In the paper by Gül (1989) there is a finite number of agents who each owns some asset. In each period two agents are matched randomly. In our paper, only workers are chosen at random to bargain with the firm in each period. Thus, there is an asymmetry in how the players are selected. The motivation for always selecting the firm is that the owner of the firm owns some specific asset. Without this asset, two workers joined together cannot produce any surplus. Hence, it is not meaningful for two workers to bargain with each other.

3 Equilibrium

We begin by showing how payoffs are computed for a candidate stationary subgame perfect equilibrium (SPE) strategy profile. To find the payoffs, we first describe the payoffs when one worker remains, and then we proceed by induction to describe the payoffs when several workers remain.

Suppose only one worker remains. In any equilibrium, when only one worker remains, the firm and the worker share the surplus according to proposer selection probabilities. Moreover, the proposer offers the respondent his continuation payoff and the respondent accepts. Continuation payoffs are given by the following Lemma, which is a standard result in the bargaining literature.

**Lemma 1** Let $\sigma$ be a SPE strategy profile. For all $E \subseteq W$ such that $|E| = 1$, if $i = E$, then

$$U_i(i, \sigma) = p_i MP_i$$

and

$$U_F(i, \sigma) = f(W) - p_i MP_i.$$ 

We require the candidate equilibrium strategy profile to prescribe actions as above in subgames where only one worker remains.

Suppose that $E$ contains more than one worker and that the firm is without an agreement with the workers in $E$. Suppose the firm bargains with worker $i$ and is the proposer. Then,
if
\[\delta U_F(E \setminus i, \sigma) + (1 - \delta)f(W \setminus E \cup i) - \delta U_i(E, \sigma) \geq \delta U_F(E, \sigma) + (1 - \delta)f(W \setminus E), \]
(1)
it is profitable to make an acceptable offer and if (1) does not hold, the firm makes an unacceptable proposal. It is easily seen that if the worker is the proposer, the same condition determines whether an acceptable offer is made. If (1) holds strictly, then both proposers make acceptable offers. In the knife-edge case where (1) holds with equality for some worker \(i\), it might be the case that, when worker \(i\) is selected to bargain with the firm, proposers make acceptable offers with a probability less than one. Furthermore, if the proposer makes an acceptable offer, it must be equal to the continuation payoff of the respondent.

Now consider the continuation payoffs and suppose \(E\) workers remain. Let \(G_F\) denote the set of workers to whom the firm makes acceptable offers. Given some strategy profile \(\sigma\), let \(p_i^F(E)\) (and \(p_i^W(E)\)) denote the probability the firm agrees with worker \(i\), conditional on the firm (worker) being selected as proposer. Since we allow for mixed strategies, we have \(p_i^F(E) \leq p_i(E)(1 - p_i^F)\) and \(p_i^W(E) \leq p_i(E)p_i^W\). The continuation payoff for the firm is
\[
U_F(E, \sigma) = \sum_{i \in G_F} p_i^F(E) \{\delta U_F(E \setminus i, \sigma) + (1 - \delta)f(W \setminus E \cup i) - \delta U_i(E, \sigma)\} + (1 - \sum_{i \in G_F} p_i^F(E)) \{\delta U_F(E, \sigma) + (1 - \delta)f(W \setminus E)\}.
\]
(2)
The term after the first summation sign is the payoff for the firm if it is chosen as proposer when bargaining with worker \(i \in G_F\). The last term is the payoff when either some worker is the proposer and makes an acceptable offer or when unacceptable offers are made.

Analogously, the continuation payoff for worker \(i \in E\) is given by
\[
U_i(E, \sigma) = p_i^W(E) \{\delta U_F(E \setminus i, \sigma) + (1 - \delta)f(W \setminus E \cup i) - \delta U_F(E, \sigma) - (1 - \delta)f(W \setminus E)\} + p_i^F(E)\delta U_i(E, \sigma) + (1 - \sum_{j \in E} [p_j^F(E) + p_j^W(E)])\delta U_i(E, \sigma) + \sum_{j \in E \setminus i} [p_j^F(E) + p_j^W(E)]\delta U_i(E \setminus j, \sigma).
\]
(3)
The first term is the payoff when worker \(i\) is selected to bargain with the firm and the worker is the proposer. The second term is the payoff for worker \(i\) when selected to bargain and the firm is the proposer. The third term is the payoff when unacceptable proposals are made and the last term is the payoff when the firm agrees with some other worker.

Since marginal contributions are positive, (1) must hold for some worker. To see this, suppose (1) does not hold for any worker. Then, an equilibrium strategy profile prescribes
 unacceptable proposals to all workers. By using (3), the continuation payoffs for all workers are zero. Since the marginal contributions are positive, the firm can deviate and make a positive offer to some worker. If this offer is small enough, the firm gains.

3.1 Existence

We now show existence of equilibrium. To do this, we demonstrate that the following strategy profile is an equilibrium. First, respondents always accept proposals equal to their continuation payoff. Second, the firm and workers optimally choose whether to make acceptable offers or not, allowing for mixed strategies. Thus, the firm and worker, respectively, choose $p^F_i(E) \in [0, p_i(E)(1 - p^F_i)]$ and $p^W_i(E) \in [0, p_i(E)p^W_i]$. A probability $p^F_i(E) \in (0, p_i(E)(1 - p^F_i))$ or $p^W_i(E) \in (0, p_i(E)p^W_i)$ is interpreted to imply that the firm or worker makes an acceptable offer with some positive probability less than one.

Here, let $p^F_i(E) = (p^F_i(E))_{i \in E}$, $p^W_i(E) = (p^W_i(E))_{i \in E}$ and $p(E) = (p^F_i(E), p^W_i(E))$. Let $P^F = \{p^F_i(E) \in \mathbb{R}^{|E|}_{+} | p^F_i(E) \leq p_i(E)(1 - p^F_i)\}$ and $P^W = \{p^W_i(E) \in [0, p_i(E)p^W_i]\}$ denote the set of possible probabilities for the firm and worker $i$, respectively.

Now we define a mapping that enables us to find an equilibrium. Let $M^{|E|} = [0, f(W)]^{|E|+1}$ and $A^{|E|} = \times_{i \in E} ([0, p_i(E)(1 - p^F_i)] \times [0, p_i(E)p^W_i])$. Furthermore, let $B^{|E|} = M^{|E|} \times A^{|E|}$. Note that $B^{|E|}$ is compact and convex. For some $m \in M^{|E|}$ and $a \in A^{|E|}$ we define

$$
\mu^F_F(m, a) = \max_{p^F_i(E) \in P^F} \sum_{i \in E} p^F_i(E) \{\delta U_F(E \setminus i, \sigma) + (1 - \delta)f(W \setminus E \cup i) - \delta m_i\}
+ (1 - \sum_{i \in E} p^F_i(E)) \{\delta m_F + (1 - \delta)f(W \setminus E)\}
$$

(4)

and

$$
\mu^W_i(m, a) = \max_{p^W_i(E) \in P^W} p^W_i(E) \{\delta U_F(E \setminus i, \sigma) + (1 - \delta)f(W \setminus E \cup i) - \delta m_F - (1 - \delta)f(W \setminus E)\}
+ a^F_i \delta m_i + (1 - \sum_{j \in E \setminus i} [a^F_j + a^W_j]) \delta m_i - [a^F_i + p^W_i(E)] \delta m_i + \sum_{j \in E \setminus i} [a^F_j + a^W_j] \delta U_i(E \setminus j, \sigma).
$$

(5)

Also, let

$$
\alpha^{F,E}(m, a) = \arg \max_{p^F_i(E) \in P^F} \sum_{i \in E} p^F_i(E) \{\delta U_F(E \setminus i, \sigma) + (1 - \delta)f(W \setminus E \cup i) - \delta m_i\}
+ (1 - \sum_{i \in E} p^F_i(E)) \{\delta m_F + (1 - \delta)f(W \setminus E)\},
$$

(6)
let $a_i^{F,E}(m,a)$ be the $i$th element of $\alpha^{F,E}(m,a)$ and let

$$a_i^{W,E}(m,a) = \arg \max_{p_i^W(E) \in D_i^W} p_i^W(E) \left\{ \delta U_F(E \setminus i, \sigma) + (1 - \delta) f(W \setminus E \cup i) - (1 - \delta) f(W \setminus E) \right\}.$$

$$-\delta m_F \} + a_i^F \delta m_i + \left( 1 - \sum_{j \in E \setminus i} [a_j^F + a_j^W] \right) \delta m_i - [a_i^F + p_i^W(E)] \delta m_i + \sum_{j \in E \setminus i} [a_j^F + a_j^W] \delta U_i(E \setminus j, \sigma). \quad (7)$$

Finally, we let

$$\Phi^E(m,a) = \left( (\mu_i^E(m,a))_{i \in F \cup E}, \left( \alpha_i^{F,E}(m,a), \alpha_i^{W,E}(m,a) \right)_{i \in E} \right).$$

Thus, $\Phi^E$ is a mapping consisting of continuation payoffs and optimal choices of the players, given $m$ and $a$.

We claim that a fixed point of $\Phi^E$ gives equilibrium continuation payoffs and strategies in the subgame with $E$ workers remaining, given that an equilibrium has been found in subgames with $H \subseteq E$ workers remaining. To see that a fixed point of $\Phi^E$ gives equilibrium continuation payoffs and strategies, first note that, if $\mu^E_F(E, \sigma) = \mu^F_F(m,a) = m_F$, $U_i(E, \sigma) = \mu^F_i(m,a)$, $p_i^F(E) = \alpha_i^{F,E}(m,a) = a_i^F$ and $p_i^W(E) = \alpha_i^{W,E}(m,a) = a_i^W$ for all $i \in E$, then (4) and (5) are the same as (2) and (3). Second, (4), (5), (6) and (7), respectively, imply that the firm and workers optimally choose whether to make an acceptable offer or not. Moreover, by construction, respondents choose optimally.

**Theorem 1** *A stationary subgame perfect equilibrium exists.*

**Proof.** Fix $E \subseteq W$. By Lemma 1, an equilibrium exists in subgames with one worker remaining. Suppose that an equilibrium exists in subgames with $H \subseteq E$ workers remaining.

By the maximum theorem under convexity, $\alpha^{F,E}(m,a)$ and $\alpha_i^{W,E}(m,a)$ are upper-semicontinuous, convex-valued and compact-valued correspondences on $B^{[E]}$ and $\mu^E_i(m,a)$ is a continuous function on $B^{[E]}$ for all $i \in F \cup E$. Thus, $\Phi^E$ is an upper-semicontinuous, convex-valued and compact-valued correspondence. Then the Kakutani fixed-point theorem implies that a fixed point exists. Hence, if a stationary SPE exists in subgames where less than $E$ workers remain, then a stationary SPE exists in subgames where $E$ workers remain.

By induction, existence of a stationary SPE follows. $\blacksquare$

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1 *A related result is found in Gomes (1999).*
3.2 Uniqueness

Theorem 1 shows that an equilibrium exists. The following example demonstrates that uniqueness cannot be guaranteed, unless some constraints are imposed on the production function.

**Example 1** Assume that we have two identical workers. Then \( f(1) = f(2) \). Both workers are selected to bargain with the firm with probability \( \frac{1}{2} \) when no agreement is signed. Furthermore, for \( i \in W \), \( p_i^W = \frac{1}{2} \). If \( \sigma \) is a SPE, then, by Lemma 1, \( U_i(i, \sigma) = \frac{MP}{2} \) and \( U_F(i, \sigma) = f(W) - \frac{MP}{2} \).

Assume that both firm-worker meetings result in agreement when both workers are without an agreement. Denote this strategy profile \( \sigma^{12} \). Using (2) and (3) gives

\[
U_i(W, \sigma^{12}) = \frac{1}{4 - 3\delta} \left[ \delta f(W) + (1 - \delta) f(1) \right] + \frac{1 - \delta}{4 - 3\delta}
\]

for \( i = 1, 2 \) and

\[
U_F(W, \sigma^{12}) = \frac{2}{4 - 3\delta} \left\{ \left( 1 - \frac{\delta}{2} \right) [\delta f(W) + (1 - \delta) f(1)] - \delta \frac{MP}{2} \right\}.
\]

We also need to show that it is optimal to make acceptable offers when both workers are without an agreement. Some algebra shows that (1) holds, irrespectively of the value of \( \delta \) and \( f \). Thus, the strategy profile above is an equilibrium, for all \( \delta \) and \( f \).

Now suppose that, when both workers are without an agreement, only the meeting with worker 2 leads to an agreement. Denote this strategy profile \( \sigma^2 \). Then \( U_1(W, \sigma^2) = \frac{\delta}{2 - \delta} \frac{MP}{2}, \)

\[
U_2(W, \sigma^2) = \frac{1}{4 - 2\delta} \left\{ \delta \left[ f(W) - \frac{MP}{2} \right] + (1 - \delta) f(1) \right\}
\]

and

\[
U_F(W, \sigma^2) = \frac{1}{4 - 2\delta} \left\{ \delta \left[ f(W) - \frac{MP}{2} \right] + (1 - \delta) f(1) \right\}.
\]

It can be shown that it is always profitable to make acceptable offers to worker 2. Moreover, it must be optimal to make unacceptable offers when worker 1 is selected to bargain with the firm. Then the reverse inequality holds in (1). This is true if

\[
[5\delta^2 - 4\delta] f(W) + [10\delta - 8 - 5\delta^2] f(1) > 0.
\]

This condition depends on the production function and the value of \( \delta \). If \( \delta \to 1 \) then the expression above holds if \( f(W) > 3f(1) \).
Suppose \( \delta \) is close to one. Then, if the production function satisfies decreasing returns, \((f(W) < 3f(1))\), there is a unique stationary equilibrium where each worker gets half of his marginal product and the firm gets the rest. Wages are competitive. If \( f(W) > 3f(1) \) there are three equilibria. Besides the first equilibrium, there are two equilibria where one worker gets approximately \( \frac{1}{2} [f(W) - \frac{MP}{2}] \), the other \( \frac{MP}{2} \) and the firm the rest.

In the example above, there is always an equilibrium where each worker gets approximately half of the marginal product, for \( \delta \) close to one. This is not the case in general. To see this, suppose we have three identical workers. All workers are almost perfect complements, i.e., if one or two of the workers are hired, production is close to zero, but if all workers are hired, production is large. Then, if workers are paid approximately half their marginal product, the wage for each worker is almost half of total production. If all three workers are hired, the firm makes a loss. If the firm instead makes unacceptable offers to all workers, no worker is hired and the firm makes zero profits. Thus, there does not exist an equilibrium where all workers are paid half their marginal product.

In the example, wages are competitive if the production function satisfies decreasing returns. Below, we show that the intuition from the example generalizes. First, we define decreasing returns.

**Definition 2** The production function \( f \) satisfies decreasing returns if, for all \( i \in W \) and all \( D, E \subset W \) such that \( i \notin E \) and \( D \subset E \), we have

\[
f(D \cup i) - f(D) > f(E \cup i) - f(E).
\]

Thus, the increase in production from adding worker \( i \) is smaller, when the set of workers already employed is larger.

Consider subgames where at least two workers are without an agreement. It turns out that equilibrium continuation payoffs depend crucially on whether, in any subgame, at least two firm-worker meetings end in agreement or if there is a subgame where only one firm-worker meeting ends in agreement. To prove the main result, we begin by computing the expected utilities for an equilibrium strategy profile where, in any subgame where at least two workers remain, at least two firm-worker meetings always end in agreement. This is done in Lemma 2 and Theorem 2. Then we (partially) compute expected utilities for an equilibrium strategy profile where there is a subgame in which only one firm-worker meetings ends in agreement when at least two workers remain. This is done in Lemma 3. Theorem
3 then shows that, for all $E \subseteq W$ such that $E$ consists of at least two workers, at least two firm-worker meeting end in agreement when $E$ workers remain, if the production function satisfies decreasing returns. This implies that wages are competitive.

Any equilibrium strategy profile is constructed by using (2) and (3). Let $\sigma^*$ denote a strategy profile such that, for all $E \subseteq W$ ($|E| > 1$), at least two firm-worker meetings end in agreement when $E$ workers remain. Let $\hat{\sigma}^E$ be a strategy profile, such that at least two firm-worker meetings end in agreement when $E$ ($|E| > 1$) workers remain.

**Lemma 2** Let $\{\delta^k\}_{k=1}^\infty$ be a sequence such that, for all $k$, $\delta^k \in [0,1]$. Suppose $\hat{\sigma}^E(\delta^k)$ is a stationary SPE for all $k$. Suppose that $\lim_{k \to \infty} \delta^k = 1$. If for any $H \subseteq E \subseteq W$ such that $|H| = |E| - 1$,

$$\lim_{k \to \infty} U_F(H, \hat{\sigma}^E(\delta^k)) = f(W) - \sum_{j \in H} p_j M P_j$$

(8)

and, for all $i \in H$,

$$\lim_{k \to \infty} U_i(H, \hat{\sigma}^E(\delta^k)) = p_i M P_i$$

(9)

then,

$$\lim_{k \to \infty} U_F(E, \hat{\sigma}^E(\delta^k)) = f(W) - \sum_{j \in E} p_j M P_j$$

(10)

and, for all $i \in E$,

$$\lim_{k \to \infty} U_i(E, \hat{\sigma}^E(\delta^k)) = p_i M P_i$$

(11)

**Proof. Step 1.** $\lim_{k \to \infty} U_F(E, \hat{\sigma}^E(\delta^k)) \geq f(W) - \sum_{j \in E} p_j M P_j$.

Suppose that $\lim_{k \to \infty} U_F(E, \hat{\sigma}^E(\delta^k)) < f(W) - \sum_{j \in E} p_j M P_j$. For all $i$, let

$$K_i(\delta) = (1 - \delta) f(W \setminus E \cup i) + \delta U_F(E \setminus i, \hat{\sigma}^E(\delta)) - \delta U_i(E, \hat{\sigma}^E(\delta)).$$

Let $G = \{i \in E \mid p_i^F(E) + p_i^W(E) > 0\}$ be the set of workers with whom the firm agrees. From the hypothesis in the Lemma, $G$ has at least two elements. Choose $l \in G$ such that $K_l(\delta) \leq K_j(\delta)$ for all $j \in G$.

Consider $U_F(E, \hat{\sigma}^E(\delta))$. If $p_i^F(E) > 0$ for some $i \in G$ then, using (2),

$$U_F(E, \hat{\sigma}^E(\delta)) \geq \frac{\sum_{i \in G_F} p_i^F(E) K_i(\delta)}{1 - \delta + \sum_{i \in G_F} p_i^F(E) \delta} + \frac{(1 - \delta) f(W \setminus E)}{1 - \delta + \sum_{i \in G_F} p_i^F(E) \delta} \left(1 - \sum_{i \in G_F} p_i^F(E)\right) = B(\delta).$$

(12)
If \( p_i^F(E) = 0 \) for all \( i \in E \), then \( K_j(\delta) = \delta U_F(E, \hat{\sigma}^E(\delta)) + (1 - \delta) f(W \setminus E) \) for all \( j \in G \) since, if a worker makes acceptable and the firm unacceptable offers, \((1)\) must hold with equality. Then we have \( U_F(E, \hat{\sigma}^E(\delta)) = K_i(\delta) = B(\delta) \). Continuity of payoffs implies that

\[
\lim_{k \to \infty} B(\delta^k) = K_i(1). \tag{13}
\]

Now consider the payoff for worker \( l \). We have

\[
U_l(E, \hat{\sigma}^E(\delta)) 
\leq \frac{p_l^W(E)}{1 - \delta + \sum_{j \in E} [p_j^F(E) + p_j^W(E)] \delta U_l(E \setminus j, \hat{\sigma}^E(\delta))} \left( p_l^F(E) - p_l^W(E) \right) [K_i(\delta) - \delta B(\delta) - (1 - \delta) f(W \setminus E)] + \sum_{j \in E \setminus l} \left( p_j^F(E) + p_j^W(E) \right) \delta U_l(E \setminus j, \hat{\sigma}^E(\delta)) - \delta [p_l^F(E) - p_l^W(E)] \delta \] 

Then

\[
\lim_{k \to \infty} U_l(E, \hat{\sigma}^E(\delta^k)) \leq \frac{\sum_{j \in E \setminus l} \left( p_j^F(E) + p_j^W(E) \right) \delta U_l(E \setminus j, \hat{\sigma}^E(\delta))}{2p_l^W(E) + \sum_{j \in E \setminus l} \left( p_j^F(E) + p_j^W(E) \right)} p_l M P_l \leq p_l^W M P_l. \tag{14}
\]

Next, consider the following deviation by the firm. Make unacceptable offers to any \( j \in G \setminus l \) and reject all offers from any \( j \in G \). Worker \( l \) is offered \( \delta U_l(E, \hat{\sigma}^E(\delta^k)) + \epsilon \) where \( \epsilon > 0 \). Denote this strategy \( \sigma'_F(\delta) \) and let \( \sigma'(\delta) = (\sigma'_F(\delta), \hat{\sigma}_1^E(\delta), ..., \hat{\sigma}_{|E|}^E(\delta)) \). The payoff for the firm is

\[
U_F(E, \sigma'(\delta)) = \frac{(1 - p_l(E))(1 - p_l^F)}{1 - (1 - p_l(E))(1 - p_l^F)} \left( 1 - \delta \right) f(W \setminus E) + \frac{p_l(E)(1 - p_l^F)}{1 - (1 - p_l(E))(1 - p_l^F)} \left( \delta U_F(E \setminus l, \hat{\sigma}^E(\delta^k)) + (1 - \delta) f(W \setminus E \cup l) - \delta U_l(E, \hat{\sigma}^E(\delta^k)) - \epsilon \right).
\]

Then \((9)\) and \((14)\) imply that

\[
\lim_{k \to \infty} U_F(E, \sigma'_F(\delta^k)) \geq f(W) - \sum_{j \in E} p_j^F M P_j - \frac{p_l(E)(1 - p_l^F)}{1 - (1 - p_l(E))(1 - p_l^F)} \epsilon,
\]

contradicting the optimality of \( \hat{\sigma}^E(\delta^k) \) for the firm for \( \epsilon \) sufficiently small.

**Step 2.** \( \lim_{k \to \infty} U_i(E, \hat{\sigma}^E(\delta^k)) \geq p_i^W M P_i \) for all \( i \in E \).

The result easily follows for workers \( i \in E \setminus G \) by using \((3)\) and \((9)\). Now, suppose that \( \lim_{k \to \infty} U_i(E, \hat{\sigma}^E(\delta^k)) < p_i^W M P_i \) for some \( i \in G \). Consider the strategy for the worker when he rejects any offer by the firm and makes unacceptable proposals in subgames where \( E \) workers remain. Otherwise, the strategy profile is identical to \( \hat{\sigma}_i^E(\delta) \). Denote this strategy \( \sigma'_1(\delta) \) and let \( \sigma' = (\sigma'_F(\delta), \hat{\sigma}_1^E(\delta), ..., \hat{\sigma}_{|E|}^E(\delta)) \). The payoff of the worker is

\[
U_i(E, \sigma'(\delta)) = \frac{1}{1 - \delta + \sum_{j \in E \setminus i} \left( p_j^F(E) + p_j^W(E) \right) \delta U_i(E \setminus j, \hat{\sigma}^E(\delta))} \sum_{j \in E \setminus i} \left( p_j^F(E) + p_j^W(E) \right) \delta U_i(E \setminus j, \hat{\sigma}^E(\delta)).
\]
Then (9) implies that
\[
\lim_{k \to \infty} U_i(E, \sigma^*(\delta^k)) = \frac{\sum_{j \in E \setminus i} \left[p_j^F(E) + p_j^W(E)\right]}{\sum_{j \in E \setminus i} \left[p_j^F(E) + p_j^W(E)\right]} p_i^P MP_i = p_i^P MP_i,
\]
contradicting the optimality of \(\hat{\sigma}_i^E(\delta)\) for \(\delta\) close to one.

**Step 3.** From step 1 we have \(\lim_{k \to \infty} U_F(E, \hat{\sigma}^E(\delta^k)) \geq f(W) - \sum_{j \in E} p_j^P MP_j\) and from step 2, for all \(i \in G\), \(\lim_{k \to \infty} U_i(E, \hat{\sigma}^E(\delta^k)) \geq p_i^P MP_i\). Since, for all \(\delta^k \leq 1\), we have
\[
U_F(E, \hat{\sigma}^E(\delta^k)) + \sum_{j \in E} U_i(E, \hat{\sigma}^E(\delta^k)) \leq f(W),
\]
the conclusion follows. ■

The intuition is as follows. If worker \(i\) gets less than \(p_i^P MP_i\) in the limit, then he can wait until the firm has agreed with one of the other workers. Then worker \(i\) gets at least \(p_i^P MP_i\) in the limit. Now consider the firm. It can be shown that one of the workers with whom the firm agrees (say worker \(l\)) gets at most \(p_l^P MP_l\) in the limit. If the firm gets less than \(f(W) - \sum_{j \in E} p_j^P MP_j\) in the limit, the firm can deviate and make acceptable offers only to worker \(l\). Since the firm gets the payoff in (8) when it has agreed with \(l\), the firm gains by this deviation. Thus, the firm must get at least \(f(W) - \sum_{j \in E} p_j^P MP_j\) in the limit. Since workers get at least \(p_i^P MP_i\), the firm gets at least \(f(W) - \sum_{j \in E} p_j^P MP_j\) and the sum of payoffs is at most \(f(W)\), the conclusion follows.

By using Lemmas 1, 2 and induction we can show the following.

**Theorem 2** Let \(\{\delta^k\}_{k=1}^\infty\) be a sequence such that \(\delta^k \in [0, 1)\) for all \(k\). Suppose \(\sigma^*(\delta^k)\) is a stationary SPE for all \(k\). If \(\lim_{k \to \infty} \delta^k = 1\) then,
\[
\lim_{k \to \infty} U_F(W, \sigma^*(\delta^k)) = f(W) - \sum_{j \in W} p_j^P MP_j
\]
and, for all \(i \in W\),
\[
\lim_{k \to \infty} U_i(W, \sigma^*(\delta^k)) = p_i^P MP_i. \tag{15}
\]

Now consider the case where, for some \(E \subseteq W\ (|E| > 1)\), the firm agrees with only one worker in equilibrium. Let \(\hat{\sigma}^{E,i}(\delta)\) denote a strategy profile defined as follows. In the subgame with \(E\) workers remaining, only the meeting with worker \(i\) results in agreement. In any subgame of \(E\) with more than two workers remaining, at least two firm-worker meetings result in agreement. The following Lemma gives continuation payoffs for \(\hat{\sigma}^{E,i}(\delta)\).
Lemma 3. Let \( \{\delta^k\}_{k=1}^{\infty} \) be a sequence such that \( \delta^k \in [0, 1) \) for all \( k \). Suppose \( \bar{\sigma}^{E,i}(\delta^k) \) is a stationary SPE for all \( k \). If \( \lim_{k \to \infty} \delta^k = 1 \) then,

\[
\lim_{k \to \infty} U_F(E, \bar{\sigma}^{E,i}(\delta^k)) = (1 - p_i^p) \left[ f(W) - f(W \setminus E) - \sum_{j \in E \setminus i} p_j^p MP_j \right] + f(W \setminus E), \tag{16}
\]

\[
\lim_{k \to \infty} U_i(E, \bar{\sigma}^{E,i}(\delta^k)) = p_i^p \left[ f(W) - f(W \setminus E) - \sum_{j \in E \setminus i} p_j^p MP_j \right] \tag{17}
\]

and, for \( j \in E \setminus i \),

\[
\lim_{k \to \infty} U_j(E, \bar{\sigma}^{E,i}(\delta^k)) = p_j^p MP_j.
\]

Proof. Let \( H(\delta^k) = \delta^k U_F(E \setminus i, \bar{\sigma}^{E,i}(\delta^k)) + (1 - \delta^k) f(W \setminus E \cup i) \).

Solving for \( U_F(E, \bar{\sigma}^{E,i}(\delta^k)) \) and \( U_i(E, \bar{\sigma}^{E,i}(\delta^k)) \) in (2) and (3), respectively, and substituting the solution for \( U_F(E, \bar{\sigma}^{E,i}(\delta^k)) \) into \( U_i(E, \bar{\sigma}^{E,i}(\delta^k)) \) gives

\[
U_i(E, \bar{\sigma}^{E,i}(\delta^k)) \left[(1 - \delta^k) + (p_i^W(E) + p_i^F(E)) \delta^k\right] = p_i^W(E) H(\delta^k) - p_i^W(E)f(W \setminus E).
\]

**Case 1.** Condition (1) holds strictly. Then, \( p_i^W(E) + p_i^F(E) = p_i(E) \) and \( p_i^W(E) = p_i(E)p_i^p \) and we have

\[
\lim_{k \to \infty} U_i(E, \bar{\sigma}^{E,i}(\delta^k)) = p_i^p \left\{ \lim_{k \to \infty} U_F(E \setminus i, \bar{\sigma}^{E,i}(\delta^k)) - f(W \setminus E) \right\}. \tag{18}
\]

Also, from rearranging the payoffs in (2) we have

\[
\lim_{k \to \infty} U_F(E, \bar{\sigma}^{E,i}(\delta^k)) = \lim_{k \to \infty} \left\{ U_F(E \setminus i, \bar{\sigma}^{E,i}(\delta^k)) - U_i(E, \bar{\sigma}^{E,i}(\delta^k)) \right\}.
\]

Then

\[
\lim_{k \to \infty} U_F(E, \bar{\sigma}^{E,i}(\delta^k)) = \lim_{k \to \infty} \left\{ U_F(E \setminus i, \bar{\sigma}^{E,i}(\delta^k)) - p_i^p \left[ U_F(E \setminus i, \bar{\sigma}^{E,i}(\delta^k)) - f(W \setminus E) \right] \right\}. \tag{19}
\]

By induction, \( \lim_{k \to \infty} U_F(E \setminus i, \bar{\sigma}(\delta^k)) \) is given by (10). Substituting into (18) and (19) gives the result. The result for workers \( j \in E \setminus i \) follows from (3).

**Case 2.** Condition (1) holds with equality. Then the payoff when acting as proposer is the same as when acting as respondent. Using (2) gives \( U_F(E, \bar{\sigma}^{E,i}(\delta^k)) = f(W \setminus E) \) and using (3) gives \( U_i(E, \bar{\sigma}^{E,i}(\delta^k)) = 0 \). Since (1) holds with equality we have \( f(W) - \sum_{j \in E \setminus i} p_j^p MP_j = f(W \setminus E) \). Then (16) and (17) follow. The result for workers \( j \in E \setminus i \) follows from (3).
By construction of the strategy profile, the firm has to agree with worker $i$ before it hires the other workers. Thus, the firm has to agree with worker $i$ before it can reap the surplus from negotiating with the remaining workers. This is similar to a situation where the firm only bargains with one worker and the surplus consists of the gain from hiring the remaining workers. This gain is $f(W) - f(W \setminus E) - \sum_{j \in E \setminus i} p_j f_j M P_j$. Lemma 3 shows that the gain is shared according to proposer probabilities. Theorem 2 and Lemma 3 imply the following result.

**Theorem 3** Suppose that the production function satisfies decreasing returns. There exists $\delta < 1$ such that, for all $\delta \in [\delta, 1]$, in any stationary SPE strategy profile, for all $E \subseteq W$ ($|E| > 1$), at least two firm-worker meetings end in agreement.

**Proof.** Let $\tilde{\sigma}(\delta)$ denote the equilibrium strategy profile. Suppose the theorem is false. Then for any $\delta' < 1$ there exists a $\delta \in (\delta', 1)$ such that, for some $E$ ($|E| > 1$) only one of the firm-worker meetings results in an agreement in equilibrium. Then we can find $E$ such that only one firm-worker meeting results in agreement and, for all $H \subset E$ ($|H| > 1$), we have either two firm-worker meetings resulting in agreement or $|E| = 2$. The continuation payoffs for $H \subset E$ can be computed by using Lemmas 1 and 3.

Consider the following deviation by the firm, whose actions change only when $E$ workers remain. To workers $j \in E \setminus i$, offer the continuation payoff plus $\epsilon > 0$. Reject all offers. Make an unacceptable offer to worker $i$. Denote this strategy $\sigma'_F(\delta)$ and let $\sigma'(\delta) = (\sigma'_F(\delta), \tilde{\sigma}_1(\delta), ..., \tilde{\sigma}_{|E|-1}(\delta))$. Note that, by Lemma 3, we have

$$\lim_{k \to \infty} U_j(E, \tilde{\sigma}(\delta)) = p_j f_j M P_j.$$ 

The payoff for the firm is

$$U_F(E, \sigma'(\delta)) = \frac{\sum_{j \in E \setminus i} p_j f_j(E)}{1 - \delta + \sum_{j \in E \setminus i} p_j f_j(E) \delta} \{\delta U_F(E \setminus j, \tilde{\sigma}(\delta)) + (1 - \delta) f(W \setminus E \cup j) - \delta U_j(E, \tilde{\sigma}(\delta)) - \epsilon\} + \frac{1 - \sum_{j \in E \setminus i} p_j f_j(E)}{1 - \delta + \sum_{j \in E \setminus i} p_j f_j(E) \delta} \{(1 - \delta) f(W \setminus E)\}.$$ 

Using Lemmas 2 and 3 gives

$$\lim_{k \to \infty} U_F(E, \sigma'(\delta^k)) = f(W) - \sum_{j \in E} p_j f_j M P_j - \epsilon.$$ 

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Using (16) in Lemma 3, the change in payoff from deviating is

$$\lim_{k \to \infty} \left[ U_F(E, \sigma'(\delta^k)) - U_F(E, \tilde{\sigma}(\delta^k)) \right] = p_i^p \left[ f(W) - f(W \setminus E) - \sum_{j \in E \setminus i} p_j^p MP_j - MP_i \right] - \epsilon. \tag{20}$$

Note that decreasing returns imply that \(^2\)

$$f(W) - f(W \setminus E) > \sum_{i \in E} MP_i.$$ 

Thus, decreasing returns imply that (20) is positive when \(\epsilon\) is small. By continuity of payoffs there exists a profitable deviation when \(\delta\) is close to one. Existence of \(\delta\) follows.

The intuition is the following. First note that decreasing returns imply that the wage for worker \(i\), as given by (17), is larger than \(p_i^p MP_i\). Suppose that, at some point, the firm only agrees with one worker \(i\). Then, the firm can choose some other worker \(j\) instead of worker \(i\) and offer this worker \(p_j^p MP_j + \epsilon\). Since the continuation payoff for worker \(i\) is approximately \(p_i^p MP_i\) in any subgame where less than \(E\) workers remain, the firm pays a lower wage to worker \(i\) and approximately the same to the other workers. Thus, the firm gains by the above deviation. Hence, the firm must agree with at least two workers in any subgame where more than one worker remains.

Theorem 3 implies that, as \(\delta \to 1\), equilibrium payoffs converge to the payoffs in Theorem 2, if an equilibrium exists. Existence is guaranteed by Theorem 1. Workers get a share of the marginal product and wages are competitive. In particular, when \(p_i^p = \frac{1}{2}\), worker \(i\) gets

$$\frac{1}{2} \left[ f(W) - f(W \setminus i) \right].$$

In this model, the payoff for a worker depends only on the marginal product of the worker. In the model of Stole and Zweibel, worker \(i\) gets:

$$\frac{1}{2} \sum_{C \subseteq W \setminus i} \frac{|C|! (|W| - |C| - 1)!}{|W|!} \left[ f(C \cup i) - f(C) \right]. \tag{21}$$

\(^2\) Too see this, number workers in \(E\) from 1 to \(|E|\). Let \(D_k\) be the workers with labels up to \(k\). Then

$$f(W) - f(W \setminus E) = \sum_{k=1}^{[E]} f(W \setminus D_{k-1}) - f(W \setminus D_k) > \sum_{i \in E} MP_i,$$

where the last inequality follows from decreasing returns.
Thus, each worker/agent gets a weighted average of the inframarginal contributions of worker $i$. Since the weights sum to one, decreasing returns imply that wages are lower with binding contracts. In Gül’s model, payoffs are almost the same as in (21). The summation is multiplied by one instead of $\frac{1}{2}$.

4 Conclusions

In a study of stationary subgame perfect equilibria in a bargaining game, we have shown that a stationary subgame perfect equilibrium exists. In equilibrium, it is found that the payoff for a worker is given by a share of the marginal product, if the production function satisfies decreasing returns to scale. Thus, wages are “competitive”. If a worker asks for more than this wage, the firm can wait and agree with other workers instead, leaving the worker without an agreement. Since a worker gets the competitive wage when he is the only one without an agreement, this is profitable for the firm. A similar argument holds if the firm asks for too much.

This result is in contrast to that of Stole and Zweibel (1996) where the workers get the Shapley value and thus a share of the inframarginal units. The intuition behind their result is that if a worker disagrees with the firm, this leads to renegotiation with the other workers. A worker who disagrees with the firm not only withdraws his own labor, but also inflicts the cost of paying renegotiated and higher wages to the other workers hired by the firm. In the model analyzed here, this is not the case.
References


