Performance of Coded Uplink MC-CDMA with Combined-Equalization in Fading Channels

Ivan Cosovic, Stefan Kaiser, Michael Schnell  
German Aerospace Center (DLR), Institute of Communications and Navigation  
Oberpfaffenhofen, D-82234 Wessling, Germany  
Emails: ivan.cosovic@dlr.de stefan.kaiser@dlr.de michael.schnell@dlr.de  
www.dlr.de/kn/kn-s/cosovic  

Andreas Springer  
Inst. for Commun. and Information Engineering, University of Linz  
A-4040 Linz, Austria  
Email: a.springer@icie.jku.at

Abstract—In this paper, the novel concept of combined-equalization is investigated for coded uplink MC-CDMA. For combined-equalization it is assumed that channel state information (CSI) is available at the transmitter (Tx) as well as at the receiver (Rx). As a result, combined-equalization achieves considerable performance improvements over pre- or post-equalization applied alone if the corresponding pre- and post-equalization techniques within combined-equalization are chosen in a preferred way. For combined-equalization the design criterion at Tx is optimal power assignment with respect to CSI rather than avoidance of multiple-access interference (MAI), whereas MAI cancellation followed by maximum ratio combining (MRC) of the almost interference free signal is the main task at Rx. Thus, combined-equalization offers two degrees of freedom - at Tx and Rx - in order to resolve the two contrary design goals of optimal power allocation with respect to the fading channel and MAI suppression/avoidance. As a result, improved single-user bounds for MC-CDMA systems transmitting over fading channels are achieved in the case of combined-equalization. Simulation results show that these improved single-user bounds can be approached very closely even for fully-loaded systems if the combined pre- and post-equalization techniques are chosen accordingly.

I. INTRODUCTION

Post-equalization at Rx for uplink MC-CDMA systems in the form of single-user detection (SUD) or multi-user detection (MUD) has been investigated by several authors [1] [2] [3] [4] [5]. Recently, pre-equalization at Tx for uplink time division duplex (TDD) MC-CDMA has attained increased interest and has been investigated in detail [2] [6] [7]. In this contribution, the novel concept of combined-equalization [8] is further developed and investigated for coded uplink MC-CDMA. It is shown that the proposed uplink MC-CDMA concept significantly outperforms other known uplink MC-CDMA concepts.

In [8], combined-equalization is introduced and investigated within an uncoded uplink MC-CDMA system. Investigations in this paper show that MAI can be suppressed much more efficiently in a coded system than in an uncoded system. In addition, a novel pre-equalization technique named generalized pre-equalization (G-pre-eq) is introduced which is especially tailored for combined-equalization and achieves additional improvements of several dB over the techniques investigated in [8].

Combined-equalization operates on the premise that the fading channel is known at both Tx and Rx. This knowledge can be made available for example by exploiting TDD to gather CSI at Tx needed for pre-equalization [7] and by performing channel estimation at Rx. Combined-equalization combines pre- and post-equalization in a preferred way. For combined-equalization the design criterion for pre-equalization at Tx is optimal power assignment with respect to the fading channel rather than avoidance of MAI, whereas MAI cancellation with MRC in the final detection stage of the almost interference free received signal is the main task at Rx. Such design enables an additional degree of freedom within uplink MC-CDMA systems by which only signal-to-noise ratio (SNR) maximization is done at Tx, while signal-to-noise-plus-interference ratio (SINR) maximization followed by MAI elimination is performed at Rx.

The paper is structured as follows. In Section II, the uplink MC-CDMA transmitter and the different pre-equalization techniques are introduced. The uplink MC-CDMA receiver and post-equalization techniques are described in Section III. The concept of combined-equalization within an MC-CDMA system and its potential benefits are investigated in Section IV. The bit error rate (BER) performances of the observed uplink MC-CDMA systems are shown and compared in Section V. Finally, in Section VI some conclusions are given.

II. UPLINK MC-CDMA TRANSMITTER

Under investigation are synchronous uplink MC-CDMA systems. The block diagram of the kth uplink MC-CDMA transmitter, \( k = 1, \ldots, K \), with pre-equalization is shown in Fig. 1(a). \( K \) is the number of mobile users which are simultaneously active within the considered mobile radio cell. After channel coding, outer interleaving \( \Pi_{\text{out}} \), and symbol mapping, the complex-valued data symbol \( d^{(k)} \) is spread by a spreading sequence \( c^{(k)} \). The spreading process results in the sequence \( s^{(k)} \) given by

\[
s^{(k)} = c^{(k)}d^{(k)} = (s_1^{(k)}, \ldots, s_L^{(k)})^T, \tag{1}
\]

where \((.)^T\) denotes transposition and \(c^{(k)} = (c_1^{(k)}, \ldots, c_L^{(k)})^T\) represents the kth spreading sequence of length \( L \) out of a set.
of orthogonal spreading sequences, e.g., Walsh-Hadamard sequences [9].

After the inner interleaving operation $\Pi_{in}$, the resulting sequence is pre-equalized according to

$$\tilde{s}^{(k)} = G_{pre}^{(k)} s^{(k)} = (\tilde{s}_1^{(k)}, \ldots, \tilde{s}_L^{(k)})^T,$$

(2)

where $G_{pre}$ is a diagonal $L \times L$ pre-equalization matrix with diagonal elements $G_{pre,l,l}^{(k)}$, $l = 1, \ldots, L$. The elements of the pre-equalization matrix $G_{pre}$ are calculated from the CSI, which has to be known at Tx. Different pre-equalization techniques lead to different pre-equalization matrices.

The focus of the investigations performed in this paper is on constrained pre-equalization, where the transmit power is the same as in the case without pre-equalization [2] [7], i.e.,

$$\sum_{l=1}^{L} |G_{pre,l,l}^{(k)} s_l^{(k)}|^2 = \sum_{l=1}^{L} |s_l^{(k)}|^2.$$  

(3)

In this contribution the novel concept of G-pre-eq is introduced which is especially designed for combined-equalization. G-pre-eq corrects the phase of the fading channel and weights the transmission signal with a coefficient proportional to $|H_{l,l}^{(k)}|^{p+1}$, where $H_{l,l}^{(k)}$, $l = 1, \ldots, L$, represents the fading coefficient on the $l$th subcarrier of the $k$th fading channel and $p$ is any real number. The pre-equalization coefficients are given by

$$G_{pre,l,l}^{(k)} = |H_{l,l}^{(k)}|^{p} H_{l,l}^{(k)*} \sqrt{\frac{L}{\sum_{n=1}^{L} |H_{n,n}^{(k)}|^2 p + 2}}.$$  

(4)

G-pre-eq is an unified approach to pre-equalization and comprises several well-known pre-equalization techniques, such as pre-equalization maximum ratio combining (pre-eq MRC), pre-equalization zero forcing (pre-eq ZF), and pre-equalization equal gain combining (pre-eq EGC). Table I summarizes the special cases which are comprised in G-pre-eq and gives the corresponding value $p$. For more information about pre-eq MRC, pre-eq ZF, and pre-eq EGC please refer to [7]. Note, G-pre-eq reduces to pre-eq MRC in the case $p = 0$. However, setting $p > 0$ allows to invest more available power on good subcarriers and less on weak subcarriers than in the case of pre-eq MRC. Thus, choosing $p > 0$ and combining G-pre-eq with post-equalization MRC (post-eq MRC) improved single-user bounds are obtained, since the SNR at Rx is improved compared to the case where pre-eq MRC and post-eq MRC are combined [10]. More details about G-pre-eq and the concept of its combination with post-equalization will be given in Section IV.

After pre-equalization, the resulting sequence $\tilde{s}^{(k)}$ is orthogonal frequency-division multiplexing (OFDM) modulated onto the $N_c$ subcarriers of the Tx antenna and transmitted. OFDM comprises inverse fast Fourier transform and addition of a guard interval by cyclic extension of the OFDM symbol. Depending on the size $I_{in}$ of the interleaver $\Pi_{in}$, spreading in frequency and/or time is performed [1]. Note, by choosing $L \ll N_c$ the Rx complexity is reduced, and several sequences $s^{(k)}$ can be modulated in parallel. For simplicity of notation but without loss of generality the interleaver $\Pi_{in}$ is omitted from further analysis, and it is presumed that $N_c = L$. Previous simplifications are made only for purposes of simpler notation, while simulation results are given for a simulation environment which takes into account the effects of the interleaver $\Pi_{in}$.

### III. UPLINK MC-CDMA RECEIVER

The block diagram of an uplink MC-CDMA receiver with data detection is shown in Fig. 1(b). After inverse OFDM (IOFD) operation the received signal at Rx results in

$$r = \sum_{k=1}^{K} H^{(k)} s^{(k)} + n = (r_1, \ldots, r_L)^T,$$

(5)

where $H^{(k)}$ represents the $L \times L$ diagonal channel matrix with the diagonal elements $H_{l,l}^{(k)}$, $l = 1, \ldots, L$. The vector $n = (n_1, \ldots, n_L)^T$ represents the AWGN with variance $\sigma^2$. As shown in Fig. 1(b), the received IOFD demodulated signal $r$ is deinterleaved, detected, and demapped.

The received signal $r$ given in Eq. (5) can be represented in a more convenient form as

$$r = \sum_{k=1}^{K} \tilde{H}^{(k)} s^{(k)} + n.$$  

(6)

The $L \times L$ diagonal matrix $\tilde{H}^{(k)}$ is introduced for convenience, and is given by its diagonal elements

$$H_{l,l}^{(k)} = H_{l,l}^{(k)} G_{pre,l,l}^{(k)}, \ l = 1, \ldots, L.$$  

(7)

Considering a frequency-selective channel for the uplink MC-CDMA transmission the orthogonality among the different user signals is destroyed and MAI occurs. Although
pre-equalization can partly reduce MAI and fading channel influence, significant residual interference remains. The residual interference can be reduced by applying efficient data detection techniques at Rx.

In general, data detection techniques can be divided into SUD and MUD techniques. Whereas MUD exploits knowledge about MAI, this knowledge is not taken into account by SUD. SUD and MUD techniques considered in this contribution will be described in the following subsections. Note, the detection block comprises the despreading process as well.

The detected signal is demapped. The symbol demapper outputs the real-valued soft decided bit $\omega$. The optimum soft decided information which can be exploited in a Viterbi decoder is the log-likelihood ratio (LLR) [11]. In this contribution, a simple LLR approximation is used given by

$$LLR \approx \omega. \quad (8)$$

Finally, the sequence of LLR values is soft decoded by applying the Viterbi algorithm and the vector of detected source bits is output from the channel decoder.

1) Single-User Detection: SUD is realized by an adaptive one-tap equalizer to combat the phase and amplitude distortions imposed on the received signal during the transmission. SUD does not take into account any information about MAI. Due to the bad performance in a fully-loaded uplink MC-CDMA system, SUD techniques applied alone are of minor interest, but they represent an essential part of interference cancellation techniques and, therefore, are considered here. The optimal SUD technique for the single-user case is post-eq MRC. The equalization coefficients are given by

$$G_{\text{post},l,l} = \hat{H}_{l,l}^* \quad (9)$$

Post-eq MRC is an important part of the interference cancellation process, since after some iterations the received signal is almost MAI free and, therefore, post-eq MRC is the optimal technique in that case. Other SUD techniques like EGC or ZF equalization are not considered in this paper.

2) Multi-User Detection: In this subsection, the two MU techniques considered in this contribution are briefly reviewed, namely minimum mean-square error multi-user detection (MMSE-MUD) and parallel interference cancellation (PIC).

Minimum Mean Square Error Multi-User Detection: MMSE-MUD is described by the $K \times L$ post-equalization matrix

$$G_{\text{post}} = (A^H A + \sigma^2 I_L)^{-1} A^H \quad (10)$$

where $(\cdot)^H$ denotes the Hermitian transposition and $I_L$ is an $L \times L$ identity matrix [1]. $A$ is an $L \times K$ matrix with coefficients

$$A_{l,k} = \hat{H}_{l,l}^{(k)} e_l^{(k)}, \quad l = 1, \ldots, L; \quad k = 1, \ldots, K. \quad (11)$$

Parallel Interference Cancellation: The block diagram of an uplink MC-CDMA receiver with soft PIC [5] is illustrated in Fig. 2. The interference is reconstructed by first detecting, demapping, deinterleaving and decoding the interfering symbols. In addition to the output of the decoded source bits, the soft-in/soft-out channel decoder delivers LLRs of the re-encoded and re-interleaved bits which are transformed into soft bits by [12]

$$\omega_{\text{sb}} = \tanh \left( \frac{LLR}{2} \right) \quad (12)$$

and used for the soft PIC process. The soft bits are mapped onto soft symbols such that the reliability information included in the soft bits is retained. The obtained complex-valued data symbols are spread with their specific spreading sequence and the resulting chips are once again pre-distorted accordingly. For each user the corresponding total MAI is reconstructed and subtracted from the received signal $r$. After cancelling the interference, the desired data symbol is detected with the optimal SUD technique, i.e., post-eq MRC.

In this paper, interference cancellation is performed in such a way that in the initial cancellation stage MMSE-MUD is applied and the corresponding MAI part of the resulting signal is subtracted from the received signal, while in following cancellation stages post-eq MRC is involved in the data detection process.

IV. COMBINED-EQUALIZATION FOR UPLINK MC-CDMA SYSTEMS

In this section, the novel concept of combined-equalization for MC-CDMA systems is investigated. Combined-equalization combines pre- and post-equalization in a preferred way. For combined-equalization the design criterion for pre-equalization at Tx is optimal power assignment with respect to the fading channel rather than avoidance of MAI, whereas MAI cancellation with MRC in the final detection stage of the almost interference free received signal is the main task at Rx. This design approach enables an additional degree of freedom within uplink MC-CDMA systems by which only SNR maximization is done at Tx, while SINR maximization followed by MAI elimination is performed at Rx. In [10], the single-user bounds for combined-equalization applying pre-eq MRC and post-eq MRC (MRC-MRC bounds) are determined which are up to 3dB better than the conventional MF bounds proving the potential benefit of combined-equalization. Assigning even more power to good subcarriers and less to weak subcarriers further improves single-user bounds. However, considering a multi-user environment, severe MAI occurs. To overcome
this, the new G-pre-eq technique has been introduced in Section II which allows to achieve the optimal design trade-off between power assignment at Tx and efficient MAI suppression at Rx. A theoretical approach towards optimal combined-equalization with respect to BER performance is difficult to follow, since the optimal power assignment at Tx depends on the MAI suppression capabilities at Rx and, thus, a combined optimization of Tx and Rx is required. The proposed pragmatic approach is to use G-pre-eq which introduces the parameter \( p \) for optimizing power assignment with the goal to achieve the minimal BER for a given receiver structure.

The idea of G-pre-eq is to optimize \( p > 0 \) in such a way that in a fully-loaded uplink MC-CDMA system the minimal BER performance is achieved. By enlarging \( p \) the power is re-distributed over the available subcarriers in a more unequal way. Compared to pre-eq MRC even more available power is invested on good subcarriers and less on weak subcarriers. Therefore, by enlarging \( p \) the single-user bounds are further improved compared to the MRC-MRC bound, since the corresponding SNR at Rx is improved as well [10]. The task of MAI elimination is then completely left to Rx. If Rx is able to eliminate almost all MAI, the corresponding combined-equalization single-user bounds will be approached closely. The data detection techniques at Rx have to cope with additional MAI as \( p \) grows, due to the additionally introduced distortions. Therefore, there will be an optimal value \( p \) for which the BER performance is minimized.

Note, G-pre-eq is not the only solution capable to re-distribute power more unequally than pre-eq MRC. There are many possible solutions which can achieve a similar effect. Nevertheless, G-pre-eq is an interesting pre-equalization technique since it achieves the desired effect, unifies several well-known pre-equalization techniques, and has a simple mathematical interpretation.

V. Simulation Results

In this section, simulation results of various uplink MC-CDMA systems with pre-, post- or combined-equalization are compared for an independent Rayleigh fading channel. Fading channel is normalized to 1.

The considered system parameters are as follows. The transmission bandwidth is 20 MHz and the carrier frequency is fixed to 5 GHz. The number of subcarriers is \( N_c = 1024 \), while Walsh-Hadamard sequences of length \( L = 16 \) are used for spreading. The resulting MC-CDMA symbol duration is 51.2 \( \mu \)s. The guard interval duration exceeds the maximal delay of the mobile radio channel. The MC-CDMA frame consists of 24 subsequent MC-CDMA symbols. The depth of the inner interleaver \( \Pi_{in} \) is equal to 24 subsequent MC-CDMA symbols, which enables time and frequency interleaving. For channel coding, a convolutional code with memory 6 and code rate of \( R = 1/2 \) is used. The data symbols are mapped with QPSK. The maximal achievable bit rate when all subcarriers and all spreading codes are assigned to one user is equal to 20 Mb/s, while the achievable data rate in the case of a fully-loaded system with \( K = L = 16 \) users is 1.25 Mb/s.

Note, the following simulation results are given in terms of the BER versus SNR in \( E_b/N_0 \) where \( E_b/N_0 \) is the ratio between transmitted energy and noise spectral density at Rx. Moreover, throughout the simulations only fully-loaded systems \( (K = 16) \) are investigated.

The BER versus the SNR of an uplink MC-CDMA system with different pre-equalization techniques is shown in Fig. 3. As references the AWGN performance and the MF bound for \( L = 16 \) are given. It can be seen that pre-eq EGC performs better than pre-eq MRC and pre-eq ZF.

In Fig. 4, the BER versus SNR of uplink MC-CDMA systems which use only post-equalization is shown. In addition to the AWGN curve, the MF performance is also given as reference. It can be seen that PIC is able to eliminate almost all MAI after some iterations. After 3 iterations performance improvements are no longer significant and, thus, are not shown here. A BER of \( P_b = 10^{-4} \) is achieved for an SNR of around 4 dB which is about 4.5 dB better than for pre-eq EGC.

Fig. 5 shows the BER of uplink MC-CDMA with combined-equalization applying G-pre-eq together with soft PIC at Rx. Simulation results are given for 3 and 5 PIC iterations and for a fixed SNR, while the parameter \( p \) is varied in order to find the optimal value which minimizes the BER. It can be seen that in both cases the minimal BER is achieved by setting \( p \in [2,3] \). The SNR is chosen to be \( SNR = 0 \) dB. This value leads to an approximate BER working point of \( 10^{-4} \leq P_b \leq 10^{-4} \).

The performance of an uplink MC-CDMA system with combined-equalization applying G-pre-eq with \( p = 3 \) together with soft PIC at Rx is shown in Fig. 6. Up to 5 PIC iterations are considered. As reference, the AWGN performance, the MF bound for \( L = 16 \), and the single-user bound for the combination of G-pre-eq \( (p = 3) \) with post-eq MRC are given. For this combined-equalization concept a BER of \( P_b = 10^{-4} \) is already achieved for an SNR around 0 dB. Compared to the BER performances obtained with other known uplink MC-CDMA based on post- and pre-equalization applied alone an improvement of 4 dB and more is realized. Moreover, the corresponding single-user bound of this combined-equalization concept is approached quite closely after 3 iterations while applying 5 iterations does not result in significant improvements.

VI. Conclusions

The benefits of exploiting the channel knowledge at both sides, Tx and Rx, for uplink MC-CDMA are investigated. It is shown that by using such knowledge and by combining post- and pre-equalization the SNR at Rx can be improved and, thus, new improved single-user bounds for MC-CDMA are obtained. Moreover, simulation results show, that by proper combination of pre-equalization at Tx together with
SNR in dB

Bit Error Rate

pre-eq MRC
pre-eq ZF
pre-eq EGC
MF bound, L=16
AWGN theory

pre-eq MRC
pre-eq ZF
pre-eq EGC
MF bound, L=16
AWGN theory

Fig. 3. BER vs. SNR of uplink MC-CDMA for pre-eq MRC, pre-eq ZF and pre-eq EGC; QPSK, $L = 16$, $K = 16$, $R = 1/2$.

Fig. 4. BER vs. SNR of uplink MC-CDMA for post-equalization applying MMSE-MUD and PIC, QPSK, $L = 16$, $K = 16$, $R = 1/2$.

post-equalization at Rx these bounds can be closely approached. It is noticeable that a BER of $P_b = 10^{-4}$ is already achieved for a SNR around 0 dB even in the case of a fully-loaded system. The performed analysis shows that the proposed uplink MC-CDMA systems with CSI at both Tx and Rx can outperform other known uplink MC-CDMA systems by several dB.

REFERENCES


