January 7, 1997

LBNL-39753
UCB-PTH-96/63
hep-ph/9701244

Cosmology of Supersymmetric Models with Low-energy Gauge Mediation*

André de Gouvêa[1], Takeo Moroi[2] and Hitoshi Murayama[1,2]

[1] *Department of Physics*
University of California, Berkeley, California 94720

[2] *Theoretical Physics Group*
Earnest Orlando Lawrence Berkeley National Laboratory
University of California, Berkeley, California 94720

Abstract

We study the cosmology of supersymmetric models in which the supersymmetry breaking effects are mediated by gauge interactions at about the 10^5 GeV scale. We first point out that the gravitino is likely to overclose the Universe in this class of models. This requires an entropy production, which prefers a baryogenesis mechanism at a relatively low temperature. The Affleck-Dine mechanism for baryogenesis is one of the possibilities to generate enough baryon asymmetry, but the analysis is non-trivial since the shape of the potential for the flat direction differs substantially from the conventional hidden sector case. To see this, we first perform a 2-loop calculation to determine the shape of the potential. By combining the potential with the supergravity contribution, we then find that the Affleck-Dine baryogenesis works efficiently to generate sufficient baryon asymmetry. On the other hand, we also point out that string moduli fields, if present, are stable and their coherent oscillations overclose the Universe by more than 15 orders of magnitude. One needs a very late inflationary period with an e-folding of $N \gtrsim 5$ and an energy density of $\lesssim (10^7 \text{ GeV})^4$. A thermal inflation is enough for this purpose. Fortunately, the Affleck–Dine baryogenesis is so efficient that enough baryon asymmetry can survive the late inflation.

*This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098 and in part by the National Science Foundation under grant PHY-95-14797. HM was also supported by the Alfred P. Sloan Foundation. AdG was supported by CNPq (Brazil).

Disclaimer

1 Introduction

The promise of low-energy supersymmetry (SUSY) is to stabilize the hierarchy between the weak scale and a higher scale of new physics, *e.g.* the Planck scale (see, *e.g.*, [1]). Supersymmetry, however, has to be spontaneously broken because we do not see degenerate pairs of particles and their superpartners. Moreover, there are stringent phenomenological constraints on the spectrum of superparticles such as the degeneracy among squarks or sleptons at the percent level (see, *e.g.*, [2]). Therefore, constructing viable mechanisms of supersymmetry breaking has been regarded as one of the most important issues in supersymmetry model building.

Recently, the idea of generating supersymmetry-breaking masses via gauge interactions has attracted interests (Low-Energy Gauge Mediation, or LEGM) [3, 4, 5]. In this scheme, the supersymmetry breaking effects appear in the supersymmetric Standard Model in the following manner. There is a sector which breaks supersymmetry dynamically at around a 10^7 GeV scale, and it generates supersymmetry breaking effects in the so-called messenger sector at around 10^5 GeV, which further induce supersymmetry breaking masses of order 10^3 GeV in the supersymmetric Standard Model via ordinary gauge interactions. This mechanism guarantees the required degeneracy among squarks, sleptons at a high degree, and also generates the masses for scalars and gauginos at comparable magnitudes as desired phenomenologically. There have been active studies on the phenomenology of such models [6, 7]. On the other hand, there has been little discussion on cosmological consequences of this mechanism, except issues concerning stable particles in the messenger sector [8]. Since the scheme completely differs from the conventional hidden sector scenario at high energies, early cosmology is expected to differ substantially as well.

There are (at least) two ingredients in the LEGM models which may lead to a cosmology different from the hidden sector case. The first is a very small gravitino mass. Since supersymmetry is broken at around 10^7 GeV, compared to around 10^{10} GeV in the hidden sector scenario, the gravitino mass is much lighter: $m_{3/2} \sim 100$ keV compared to 100 GeV. The second is that the supersymmetry breaking effects "shut off" at high energies.* In particular, the flat directions in the supersymmetric Standard Model have very different potentials at large field amplitudes.

In this paper, we study the implications of the LEGM models to cosmology. In Section 2, we first discuss the cosmological constraints on light gravitinos mainly based on the analysis by two of the authors (TM and HM) and Yamaguchi [10]. Then, in Section 3, we pay particular attention to the estimate of the gravitino mass in the LEGM models, and argue that it is highly unlikely to be lighter than $2h^2$keV as required by cosmology. This point implies that there must be a substantial entropy production, which casts a concern on the baryon asymmetry. Therefore, we turn our attention to a possible mechanism of baryogenesis at a relatively low temperature, using the idea by Affleck and Dine [11]. The important point in the LEGM models is that the SUSY breaking effects due to the messenger interaction "shut off" at high energies. Therefore, in Section 4, we performed a 2-loop calculation to determine

*A similar effect was discussed in [9] in the context of the sliding singlet mechanism.

the shape of the potential for the flat direction. In Section 5, we estimate the possible value of the baryon-to-entropy ratio which can be induced by the Affleck-Dine baryogenesis. For a sufficiently large amplitude of the flat direction, the potential is dominated by the supergravity contribution rather than the LEGM contribution, and we will see that the Affleck-Dine baryogenesis works well enough to explain the present value of the baryon asymmetry. Furthermore, in Section 6, we point out that the string moduli, if present within the LEGM models, cause a serious problem because they are stable and their coherent oscillations grossly overclose the Universe. However we also point out a possible solution to the problem. Since the Affleck–Dine baryogenesis is so efficient, the baryon asymmetry can survive the enormous entropy production required to dilute the moduli fields, possibly by thermal inflation [12]. Finally, in Section 7, we summarize our conclusions.

2 Cosmology of a Light Gravitino

In this section, we briefly review the cosmology with a light stable gravitino [10].* If a stable gravitino is thermalized in the early Universe, and if it is not diluted by some mechanism (such as a late inflation and/or a substantial entropy production), its mass density may exceed the closure limit: $\Omega_{3/2} < 1$. Since the number density of the gravitino is fixed once the gravitinos are thermalized, the above argument sets an upper bound on the gravitino mass [13]:

$$m_{3/2} \lesssim 2h^2 \mathrm{keV} \ : \ \text{without dilution}, \tag{2.1}$$

where h is the Hubble constant in units of 100km/sec/Mpc. In other words, if the gravitino mass is heavier than $2h^2$ keV, we need some mechanism to dilute the gravitino in order not to overclose the Universe. Since the gravitinos are produced more at a higher temperature, we obtain an upper bound on the maximal temperature, $T_{\rm max}$, from which the ordinary radiation dominated Universe starts. For example, in the inflationary Universe, $T_{\rm max}$ corresponds to the so-called reheating temperature $T_{\rm RH}$ which is typically higher than $T_{\rm RH} \gtrsim 10^8$ GeV, if there is no significant entropy production after reheating. If $T_{\rm max}$ turns out to be less than 10^8 GeV or so, we judge that one needs a substantial entropy production below $T_{\rm RH}$. It is worth to recall that the recent measurements prefer $h \sim 0.7$ and hence the upper bound is about 1 keV.

The crucial point about the light gravitino is that the interaction of the (longitudinal component of) gravitino becomes stronger as the gravitino mass gets lighter. This is because the longitudinal component of the gravitino behaves like the goldstino, whose interaction is proportional to $\langle F \rangle^{-1} \sim (m_{3/2} M_*)^{-1}$, where $M_* = 2.4 \times 10^{18}$ GeV is the reduced Planck scale. For the light gravitino, the interaction of the longitudinal component of the gravitino ($\sim$ the goldstino) ψ to the chiral multiplet (ϕ, χ) and to the gauge multiplet (A_μ, λ) is given

*In this paper, we assume the absolute conservation of R-parity.

by [14]

$$\mathcal{L} = \frac{im_\lambda}{8\sqrt{6}m_{3/2}M_*}\bar{\psi}[\gamma_\mu\gamma_\nu]\lambda F^{\mu\nu} + \frac{m_\chi^2 - m_\phi^2}{\sqrt{3}m_{3/2}M_*}(\bar{\psi}\chi_{\rm L})\phi^* + {\rm h.c.}, \tag{2.2}$$

where m_ϕ, m_χ, and m_λ represent the masses of ϕ, χ, and λ.† As indicated in Eq. (2.2), the interaction of ψ becomes stronger as the gravitino mass gets smaller.

In the thermal bath, two types of the processes may contribute to overproduce the gravitino: one is the decay of the sparticle $\tilde{X}$ into its superpartner X and the gravitino, $\tilde{X} \rightarrow \psi + X$, and the other is the scattering processes, $x + y \rightarrow \psi + z$, where x, y, z are relevant (s)particles. The decay process is significant especially for the case $m_{3/2} \lesssim$ 100keV. The partial decay width of a sparticle $\tilde{X}$ into the gravitino is estimated as

$$\Gamma(\tilde{X} \rightarrow \psi + X) \sim \frac{1}{48\pi}\frac{m_{\tilde{X}}^5}{m_{3/2}^2 M_*^2}, \tag{2.3}$$

with $m_{\tilde{X}}$ being the mass of $\tilde{X}$, and it becomes large as the gravitino mass gets small. This decay process produces gravitinos as $\dot{n}_{3/2} + 3Hn_{3/2} = \Gamma(\tilde{X} \rightarrow \psi + X)n_{\tilde{X}}$ where H is the expansion rate of the Universe at the given time.‡ Here and below, $n_{3/2}$ is the number density of gravitinos in the Universe at a given time. If the gravitino mass is in the range $2h^2$keV $\lesssim m_{3/2} \lesssim$ 100keV, the decay rate becomes so large that the decay process overproduces the gravitino once the sparticles are thermalized [10]. Thus, if the gravitino mass is in this range, $T_{\rm max}$ should be lower than about $m_{\tilde{X}} \sim$ 100 GeV – 1 TeV depending on the mass spectrum of superparticles, or the Universe is overclosed.§

If the gravitino mass is heavier than O(100keV), the decay process becomes unimportant and the most important production mechanisms of gravitinos are scattering processes. In this case, the Boltzmann equation for the number density of the gravitino, $n_{3/2}$, is given by

$$\dot{n}_{3/2} + 3Hn_{3/2} = \Sigma_{\rm tot} n_{\rm rad}^2, \tag{2.4}$$

where H is the expansion rate of the Universe, $\Sigma_{\rm tot}$ is the thermally averaged total cross section, and $n_{\rm rad} = (\zeta(3)/\pi^2)T^3$. At high energies the first term in Eq. (2.2) becomes more significant than the second one, and hence $\Sigma_{\rm tot}$ is as large as $O(g_3^2 m_{\rm G3}^2/m_{3/2}^2 M_*^2)$. After a detailed calculation, we obtain [10]

$$\Sigma_{\rm tot} \sim 5.9\frac{g_3^2 m_{\rm G3}^2}{m_{3/2}^2 M_*^2}, \tag{2.5}$$

†Here, ψ represents the spin $\frac{1}{2}$ field, though the gravitino has spin $\frac{3}{2}$. In the high energy limit, ψ is related to the longitudinal (helicity $\pm 1/2$) component of the gravitino, $\psi^\mu_{1/2}$, as $\psi^\mu_{1/2} \sim \sqrt{2/3}\partial^\mu\psi/m_{3/2}$.

‡If the gravitino number density is large, there is also a damping term because of the detailed balance, (r.h.s.) $= \Gamma(\tilde{X} \rightarrow \psi + X)n_{\tilde{X}}(1 - n_{3/2}/n_{3/2}^{\rm eq})$, where $n_{3/2}^{\rm eq}$ is the thermal equilibrium value of $n_{3/2}$.

§A similar argument can be applied to the decay process of the particles in the messenger sector or the SUSY breaking sector. In that case, the decay rate becomes much larger since the parent particle is much heavier. Thus, if the particles in those sectors are thermalized, the lower bound on the gravitino mass becomes more stringent than $\sim$ 100keV.

where g_3 and m_{G3} are the gauge coupling constant and the gaugino mass for $SU(3)_C$. Solving Eq. (2.4), and taking account of the effect of the dilution factor, $g_*(T_{max})/g_*(T)$ (where $g_*(T)$ is the number of the relativistic degrees of freedom in the thermal bath with temperature T), the number density of the gravitino is given by

$$\begin{aligned} \frac{n_{3/2}(T)}{n_{\rm rad}} &= \frac{g_*(T)}{g_*(T_{\rm max})} \frac{\Sigma_{\rm tot} n_{\rm rad}}{H}\bigg|_{T=T_{\rm max}} \\ &\sim 3\times 10^{-2} \frac{g_*(T)}{g_*(T_{\rm max})} \left(\frac{m_{3/2}}{100{\rm keV}}\right)^{-2} \left(\frac{m_{\rm G3}}{1{\rm TeV}}\right)^{2} \left(\frac{T_{\rm max}}{10{\rm TeV}}\right). \end{aligned} \tag{2.6}$$

Using $g_*(T \lesssim 1{\rm MeV}) \sim 3.9$, and $g_*(T_{max}) \sim 200$, we obtain

$$\Omega_{3/2} = \frac{m_{3/2} n_{3/2}}{\rho_{\rm c}} \sim 1\times h^{-2} \left(\frac{m_{3/2}}{100{\rm keV}}\right)^{-1} \left(\frac{m_{\rm G3}}{1{\rm TeV}}\right)^{2} \left(\frac{T_{\rm max}}{10{\rm TeV}}\right), \tag{2.7}$$

and the condition $\Omega_{3/2} \leq 1$ sets an upper bound on T_{max}. In summary, the upper bound on T_{max} is given by [10]

$$T_{\rm max} \lesssim \left\{ \begin{array}{lcl} 100{\rm GeV} \ - \ 1{\rm TeV} & : & 2h^2{\rm keV} \lesssim m_{3/2} \lesssim 100{\rm keV} \\ 10{\rm TeV}\times h^2 \left(\frac{m_{3/2}}{100{\rm keV}}\right) \left(\frac{m_{\rm G3}}{1{\rm TeV}}\right)^{-2} & : & m_{3/2} \gtrsim 100{\rm keV} \end{array} \right. . \tag{2.8}$$

The above constraints are summarized in the Fig. 1. As one can see, the upper bound on T_{max} is much lower than the usual reheating temperature after ordinary inflation, $T_{RH} \gtrsim 10^8$. To reduce the number density of the gravitino, therefore, a large entropy production is required.

3 Light Gravitino in the LEGM Models

We discussed cosmological constraints on a light stable gravitino in the previous section, and showed that one needs to dilute gravitinos produced in the early Universe somehow if $m_{3/2} \gtrsim 2h^2$ keV. In this section we estimate the gravitino mass in the LEGM models carefully and find it unlikely to be below $2h^2$ keV.

In the scheme of the LEGM models, there are three sectors: the dynamical SUSY breaking (DSB) sector which originally breaks SUSY, the ordinary sector which consists of the particles in the minimal SUSY standard model (MSSM), and the messenger sector which mediates the SUSY breaking from the DSB sector into the ordinary sector. The scales for these sectors have a large hierarchy, since they are related by loop factors: $\Lambda_{DSB} \gg \Lambda_{mess} \gg M_{SUSY}$, where Λ_{DSB}, Λ_{mess} and M_{SUSY} represent the scales for the dynamical SUSY breaking sector, messenger sector, and the ordinary sector ($\sim$ electroweak scale), respectively.

In the LEGM models a gauge interaction, which becomes strong at the scale Λ_{DSB}, induces a non-perturbative superpotential. Due to non-perturbative effects, F-components of chiral multiplets in the SUSY breaking sector acquire non-vanishing VEVs, $\langle F_0 \rangle \sim \Lambda_{DSB}^2$,

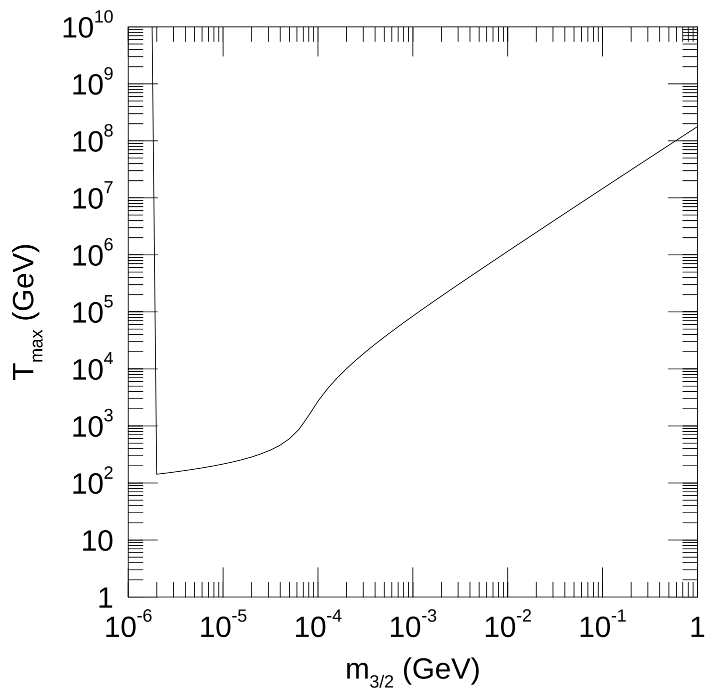

Figure 1: The upper bound on $T_{\rm max}$ as a function of the gravitino mass from the requirement that the relic stable gravitinos do not overclose the Universe. We take the Hubble parameter to be $H_0 = 100$ Mpc/km/sec. There is no constraint below $m_{3/2} = 2$ keV, which is represented by the vertical line. For smaller H_0, the constraints become more stringent. The upper bound on $T_{\rm max}$ shifts towards smaller $T_{\rm max}$ as $(H_0)^2$. The vertical line moves towards smaller $m_{3/2}$ also as $(H_0)^2$. Note that the current data prefer $H_0 \sim 70$ Mpc/km/sec.

and SUSY is dynamically broken. Assuming a vanishing cosmological constant, the gravitino mass in this model is given by

$$m_{3/2} = \frac{\langle F_0 \rangle}{\sqrt{3} M_*} \sim \frac{\Lambda_{\rm DSB}^2}{M_*}. \tag{3.1}$$

In the next stage, the SUSY breaking is fed down to the messenger sector by integrating out the $\mathrm{U}(1)_{\rm mess}$ interaction. The messenger sector contains a gauge singlet S, whose A- and F-component F_S acquire VEVs after minimizing the potential.* The scale of these VEVs are related to the original SUSY breaking scales $\Lambda_{\rm DSB}$ as $\langle S \rangle \sim \langle F_S \rangle^{1/2} \sim O(g_{\rm mess}^2 \Lambda_{\rm DSB}/16\pi^2)$. The ratio of $\langle F_S \rangle$ to $\langle S \rangle$ determines the masses of the sparticles in the ordinary sector. By counting the loop factors, we obtain

$$\Lambda_{\rm mess} \equiv \frac{\langle F_S \rangle}{\langle S \rangle} = \kappa_{\rm mess} \frac{g_{\rm mess}^2}{16\pi^2} \sqrt{m_{3/2} M_*}, \tag{3.2}$$

where $\kappa_{\rm mess}$ is supposed to be of $O(1)$ [5], and $g_{\rm mess}$ is the gauge coupling constant for the $\mathrm{U}(1)_{\rm mess}$ gauge interaction. In the messenger sector, there are also $\mathbf{5} + \bar{\mathbf{5}}$ representation of SU(5), *i.e.* $\mathrm{SU}(2)_{\rm L}$ doublets (L and $\bar{L}$) and $\mathrm{SU}(3)_{\rm C}$ triplets (Q and $\bar{Q}$). These have the superpotential

$$W_{\rm mess} = \lambda_2 S L \bar{L} + \lambda_3 S Q \bar{Q}. \tag{3.3}$$

Once S and F_S acquire VEVs, the scalar components of L and $\bar{L}$ (Q and $\bar{Q}$) have a mass matrix of the form

$$\left(\begin{array}{cc} \lambda^2 \langle S \rangle^2 & \lambda \langle F_S \rangle \\ \lambda \langle F_S \rangle & \lambda^2 \langle S \rangle^2 \end{array} \right), \tag{3.4}$$

while the fermionic components have mass $\lambda \langle S \rangle$. Therefore, SUSY is broken in the mass spectrum of vector-like $\mathbf{5} + \bar{\mathbf{5}}$ messenger fields. By integrating out the messenger fields, the soft SUSY breaking parameters are induced in the ordinary sector. With $N_{\mathbf{5}}$ pairs of vector-like $\mathbf{5} + \bar{\mathbf{5}}$ multiplet, the gaugino masses, $m_{\rm G1}$, $m_{\rm G2}$, and $m_{\rm G3}$ are given by

$$m_{{\rm G}i} = \frac{g_i^2}{16\pi^2} c_i \Lambda_{\rm mess} N_{\mathbf{5}}, \tag{3.5}$$

where $c_3 = c_2 = 1$, and $c_1 = \frac{5}{3}$ in our convention. On the other hand, the masses for the sfermions $m_{\tilde{f}}$ ($\tilde{f} = \tilde{u}_{\rm R}, \tilde{d}_{\rm R}, \tilde{q}_{\rm L}, \tilde{l}_{\rm L}$, and $\tilde{e}_{\rm R}$) are given by [17]

$$m_{\tilde{f}}^2 = 2 \Lambda_{\rm mess}^2 N_{\mathbf{5}} \sum_{i=1}^{3} C_i \left(\frac{g_i^2}{16\pi^2} \right)^2. \tag{3.6}$$

*One actually needs a substantially more complicated messenger sector than the original ones [3, 4, 5] in order to avoid a run-away global minimum [15, 16]. Such details are, however, beyond the scope of this paper.

Here, $C_1 = \frac{5}{3}Y^2$ with Y being the usual hypercharge, and $C_i = \frac{4}{3}$ and $\frac{3}{4}$ if $\tilde{f}$ is in the fundamental representation of $\mathrm{SU}(3)_\mathrm{C}$ and $\mathrm{SU}(2)_\mathrm{L}$, and $C_i = 0$ for the gauge singlets.

Combining the above relations with the experimental bounds on the sparticle masses, we can obtain a lower bound on Λ_mess, and hence the gravitino mass. A lighter state gives us a more stringent constraint. For most parameters, the lightest among the sfermions is the right handed selectron, whose mass $m_{\tilde{e}_\mathrm{R}}$ is given by[†]

$$m_{\tilde{e}_\mathrm{R}}^2 \sim \frac{10}{3}\left(\frac{g_1^2(\Lambda_\mathrm{mess})}{16\pi^2}\right)^2 \Lambda_\mathrm{mess}^2 N_\mathbf{5} - m_Z^2 \sin^2\theta_\mathrm{W} \cos 2\beta. \tag{3.7}$$

For $\tan\beta$ close to 1, the right-handed selectron mass gives us a stringent bound on the messenger scale. Even if we adopt a conservative constraint of $m_{\tilde{e}_\mathrm{R}} \geq 45\mathrm{GeV}$,[‡] we obtain

$$\Lambda_\mathrm{mess} \gtrsim 2\times 10^4 \mathrm{GeV} \times \frac{1}{\sqrt{N_\mathbf{5}}}. \tag{3.8}$$

If $\tan\beta$ is large, the D-term contribution enhances the right handed selectron mass, and $m_{\tilde{e}_\mathrm{R}}$ can be larger than the experimental limit with smaller value of Λ_mess. In that case, however, the sneutrino mass $m_{\tilde{\nu}_\mathrm{L}}$

$$m_{\tilde{\nu}_\mathrm{L}}^2 \sim \left\{ \frac{3}{2}\left(\frac{g_2^2(\Lambda_\mathrm{mess})}{16\pi^2}\right)^2 + \frac{5}{6}\left(\frac{g_1^2(\Lambda_\mathrm{mess})}{16\pi^2}\right)^2 \right\} \Lambda_\mathrm{mess}^2 N_\mathbf{5} + \frac{1}{2} m_Z^2 \cos 2\beta, \tag{3.9}$$

receives a negative contribution from the D-term, and Λ_mess is still constrained to be larger than $\sim 2\times 10^4\mathrm{GeV}$. Therefore, the bound (3.8) holds for all values of $\tan\beta$.

In order to translate the above constraint (3.8) into a lower bound on the gravitino mass, we need information about the gauge coupling constant of the $\mathrm{U}(1)_\mathrm{mess}$; larger g_mess (at the messenger scale) gives us a less stringent constraint on the gravitino mass. However, if g_mess is too large at the messenger scale, it blows up below the Planck scale or even below the GUT scale.[§] By using the 1-loop renormalization group equation, the $\mathrm{U}(1)_\mathrm{mess}$ gauge coupling constant at the messenger scale is constrained by

$$\frac{g_\mathrm{mess}^2(\Lambda_\mathrm{mess})}{16\pi^2} \lesssim \frac{1}{b_\mathrm{mess}\ln(\Lambda_\mathrm{cut}^2/\Lambda_\mathrm{mess}^2)}, \tag{3.10}$$

where Λ_cut is the cutoff scale where the perturbative picture may break down, and $b_\mathrm{mess} = \sum_A Q_{\mathrm{mess},A}^2$ is the sum of the squared charge of the messengers. Typically, $b_\mathrm{mess} \sim 10$. (For

[†]In fact, the selectron mass receives a correction from renormalization effects. However, the correction is less than 10%, and the following arguments are almost unchanged even if we include such effects.

[‡]We are aware that LEP-II has placed stronger limits on the $m_{\tilde{e}}$, but as a function of the neutralino mass. The mass difference between $m_{\tilde{e}}$ and the lightest neutralino $\sim \tilde{B}$ is not large in the LEGM models, and the constraint weakens substantially in this situation. We chose the model-independent LEP bound because of this reason.

[§]There is the logical possibility of employing an asymptotically free non-abelian gauge group as the messenger group, and assume that its scale parameter is very close to Λ_DSB. We, however, consider it as an exponential fine-tuning of parameters.

the model proposed in [4], $b_{\rm mess} = \frac{34}{3}$, and for a model given in [5], $b_{\rm mess} = 14$.) Assuming $b_{\rm mess} = 10$ and $\Lambda_{\rm cut} \sim M_{\rm GUT}$, $g_{\rm mess}$ at the messenger scale is constrained to be smaller than ~ 0.5, and

$$m_{3/2} \gtrsim 70{\rm keV} \times \frac{\kappa_{\rm mess}^{-2}}{N_{\bf 5}} \left(\frac{g_{\rm mess}}{0.5} \right)^{-2} \left(\frac{m_{\tilde{e}_{\rm R}}}{45{\rm GeV}} \right)^2 . \tag{3.11}$$

In the minimal model, $N_{\bf 5} = 1$, and if we assume the perturbative unification of the gauge coupling constants in the MSSM, $N_{\bf 5} \leq 4$ [6]. Therefore, in any case, the lower bound above is about one or two order of magnitude larger than the cosmological upper bound (2.1). Notice that the lower bound on the gravitino mass increases as the experimental lower bound on the sparticle masses increases.¶

Based on the above estimations, we define the canonical set of the parameters for our following analysis:

$$m_{3/2} = 100{\rm keV}, \tag{3.12}$$
$$\Lambda_{\rm mess} = 3 \times 10^4 {\rm GeV}, \tag{3.13}$$
$$\langle F_S \rangle^{1/2} = \langle S \rangle = \Lambda_{\rm mess}. \tag{3.14}$$

Note that it is easy to raise the gravitino mass; we only have to assume a smaller value for the gauge coupling constant for the $\mathrm{U}(1)_{\rm mess}$. In the following analysis, we basically assume the above set of parameters, and we also discuss how our results change if we vary them.

The above estimations are based on perturbative calculations, and one may worry that a strong coupling in the dynamical sector may allow us to lower the gravitino mass. Such a scenario seems unlikely, however. To see this, it is convenient to define the "vacuum polarizations" from the DSB sector for the $\mathrm{U}(1)_{\rm mess}$ gauge multiplet:

$$\text{F.T.} \langle 0 | T(A_\mu A_\nu) | 0 \rangle_{\rm 1PI} = i q^2 \Pi_A(q^2) g_{\mu\nu}, \tag{3.15}$$
$$\text{F.T.} \langle 0 | T(\lambda \lambda) | 0 \rangle_{\rm 1PI} = (-i) \{ \not{q} \Pi_\lambda(q^2) + \Sigma(q^2) \}, \tag{3.16}$$
$$\text{F.T.} \langle 0 | T(DD) | 0 \rangle_{\rm 1PI} = i \Pi_D(q^2), \tag{3.17}$$

where F.T. stands for the four-dimensional Fourier Transform to the momentum space, and 1PI for one-particle irreducible diagrams. At tree level, $\Pi_A = \Pi_\lambda = \Pi_D = \Sigma = 0$. These quantities are radiative corrections of $O(g_{\rm mess}^2 / 16\pi^2)$ if the perturbative calculation is reliable. The messenger scale in the LEGM model is induced by integrating out the SUSY breaking sector and the $\mathrm{U}(1)_{\rm mess}$ gauge multiplet. By using Π_A, Π_λ and Π_D, the SUSY breaking scalar mass in the messenger sector ($\sim$ the messenger scale) is given by

$$m_{\rm mess}^2 \sim g_{\rm mess}^2 \int \frac{d^4 q}{(2\pi)^4 i} \frac{1}{q^2} \{ 3 \Pi_A(q^2) - 4 \Pi_\lambda(q^2) + \Pi_D(q^2) \}. \tag{3.18}$$

¶A collorary to this analysis is that it is unlikely to have a decay of a sparticle into the gravitino inside a collider detector. This casts some doubts on the naturalness of $ll\gamma\gamma$ signature at CDF in the LEGM models. A possible way out is to employ the vector-like model [18] and couple a singlet field directly to the messenger fields in the superpotential [19]. This model, however, probably suffers from a tunneling to a color- and charge-breaking supersymmetric minimum if all coupling constants are $O(1)$ [16].

If we limit ourselves to the physics of the DSB sector, there is no pole in the Π functions at $q^2 = 0$ which, if present, implies the Higgs mechanism for the $U(1)_{\rm mess}$ gauge group. The only singularities in Π functions are, therefore, branch cuts which appear above certain threshold $q^2 \gtrsim \Lambda_{\rm DSB}^2$ which is the only scale in the problem. Then the integrations in Eq. (3.18) can be Wick rotated and we obtain

$$m_{\rm mess}^2 \sim -\frac{g_{\rm mess}^2}{16\pi^2} \int_0^\infty dq_E^2 \{3\Pi_A(-q_E^2) - 4\Pi_\lambda(-q_E^2) + \Pi_D(-q_E^2)\}. \tag{3.19}$$

Now it is clear that the integration is purely Euclidean, and hence all Π functions are far off-shell. Thus, the perturbative result is essentially reliable even when the DSB sector is strongly coupled. It is also useful to recall that similar calculations of vacuum polarization amplitudes in QCD tend to agree with lowest order perturbative results for the running of the fine-structure constant, or the scaled-up QCD estimate of the electroweak S-parameter.[‖] We therefore conclude that there is no significant enhancement of the resulting $m_{\rm mess}^2$, and hence the estimates of the messenger scale and the resulting gravitino mass (3.11) can be trusted.

The constraint (3.11) sets severe bounds on cosmology. In particular, we need some mechanism to generate a dilution factor of $\sim (m_{3/2}/2h^2 \text{keV})$ at a relatively low temperature below the upper bound on the maximum temperature given in Eq. (2.8) and in Fig. 1, if the gravitino mass is larger than $2h^2$keV. Furthermore, even if we adopt such a large entropy production at a low temperature, baryogenesis may still be a problem. The Affleck-Dine mechanism [11] for baryogenesis is one of the possibilities to generate baryons at a relatively low temperature.[**]

However, in the LEGM models, the behavior of the flat direction at large amplitude is quite different from the usual supergravity case. Thus, even if we assume the Affleck-Dine mechanism, it is a non-trivial question whether we can have enough baryon number density. In the following sections, we pursue this possibility, and as a result, we will see that the Affleck-Dine mechanism works sufficiently well, enough to explain the present value of the baryon-to-entropy ratio.

[‖]Note that the S-parameter is defined by the vacuum polarization amplitudes at $q^2 = 0$, and hence more sensitive to the non-perturbative effects than our case (3.19) which smears them over the wide range of q^2. Still, a perturbative estimate of S differs from the scaled-up QCD only by a factor of two. One may also estimate the S-parameter by assuming that it is dominated by the ρ and a_1 poles. Then the result is obtained by the tree-level process. Even so, the coupling of the resonances to the current operator has a factor of $1/4\pi$ and the counting of $1/4\pi$ factors remains the same in as the perturbative one-loop result. HM thanks Christopher D. Carone on this point.

[**] Electroweak baryogenesis may be another possibility to generate baryon asymmetry at relatively low temperatures. However, the resulting baryon-to-photon ratio depends on the details of the complicated dynamics of the phase transition. Furthermore, the generated baryon asymmetry would not be large, if any, and would probably not survive a huge entropy production to dilute the string moduli fields as discussed in Section 6.

4 Flat Directions in the LEGM Models

As discussed in the previous sections, the constraint from the gravitino cosmology is quite severe in models with the LEGM. Therefore, it is preferable to look for baryogenesis scenarios which do not require high temperatures.

We will focus on the Affleck–Dine baryogenesis in the LEGM models in this paper. There is one crucial difference from the hidden sector models: the potential along the MSSM flat directions is not simply parabolic. Therefore we discuss the form of the potential first in this section, and then its implication to the Affleck–Dine baryogenesis in the next section.

In the hidden sector models where the supersymmetry breaking effect is mediated by Planck-scale operators, the soft supersymmetry breaking parameters are actually "hard", in the sense that they renormalize as usual mass terms between the Planck-scale and the weak scale. On the other hand, the supersymmetry breaking scalar mass terms are suppressed beyond the messenger scale in the LEGM models.

This is analogous to the situation in the QCD. The current masses of the quarks renormalize according to the ordinary perturbation theory. They are "hard" masses. However the constituent quark masses are suppressed by a power of the energy: "soft". This is because the constituent quark masses are dynamically generated by the spontaneous chiral symmetry breaking, which is characterized by the order parameter $\langle \bar{q} q \rangle$. The constituent quark mass has to be proportional to this order parameter. At high momentum transfer, a dimensional analysis then tells us that the effective constituent mass behaves as $m_{\rm const}(Q^2) \sim \langle \bar{q} q \rangle / Q^2$.

The same argument applies to the soft supersymmetry breaking masses from the LEGM. Supersymmetry is broken by an F-component of a chiral superfield, $\langle F_S \rangle \neq 0$. The soft supersymmetry breaking scalar mass is necessarily proportional to the order parameter of supersymmetry breaking, *i.e.*, $m^2 \propto \langle F_S \rangle^\dagger \langle F_S \rangle$. A dimensional analysis tells us that it is suppressed at high momentum transfers, $m^2(Q^2) \sim \langle F_S \rangle^\dagger \langle F_S \rangle / Q^2$. Therefore, the supersymmetry breaking parameters "shut off" at high energies.

The potential of a MSSM flat direction is given simply by $V = m^2 |\phi|^2$, where m^2 is a soft supersymmetry breaking mass. A renormalization group improvement gives us $V = m^2(|\phi|^2)|\phi|^2$. In the hidden sector case, $m^2(|\phi|^2)$ has only a logarithmic dependence on $|\phi|^2$ and hence can be taken approximately constant unless it crosses zero at some energy scale. For most cosmological applications, this is a sufficiently good description. In the LEGM models, however, the effective mass $m^2(|\phi|^2)$ exhibits a power dependence on $|\phi|^2$ which cannot be neglected. We expect that $m^2(|\phi|^2)$ behaves as $\langle F_S \rangle^\dagger \langle F_S \rangle / |\phi|^2$ for large $|\phi|$, and hence the potential behaves approximately like a constant for $|\phi| > \langle S \rangle$, which is the mass scale of the messengers.

We performed an explicit two-loop calculation of the effective potential $V(\phi)$ and its details are presented in Appendix A. Here we only quote the result. As expected, the potential behaves parabolically around the origin, while it becomes approximately constant for large $|\phi|$; actually it keeps growing slowly as $(\ln |\phi|^2)^2$. The potential of a MSSM flat direction behaves as

$$V(\phi) \sim \left(\frac{g^2}{(4\pi)^2}\right)^2 \left(\frac{\langle F_S\rangle}{\langle S\rangle}\right)^2 |\phi|^2 + O(|\phi|^4) \tag{4.1}$$

for small $|\phi| \ll \langle S\rangle$, and

$$V(\phi) \sim V_0 \left(\ln \frac{|\phi|^2}{\langle S\rangle^2}\right)^2 \tag{4.2}$$

with

$$V_0 \sim \frac{g^2}{(4\pi)^4} \langle F_S\rangle^2 \sim (3 \times 10^3 \text{GeV})^4, \tag{4.3}$$

for large $|\phi| \gg \langle S\rangle$. Here, g generically refers to standard model gauge coupling constants.*

For extremely large $|\phi|$, however, the contribution from supergravity becomes important. Supergravity generates a contribution to the scalar potentials $\sim m_{3/2}^2 |\phi|^2$ for any $|\phi|$.† To determine the relative importance of the LEGM and supergravity contributions, we compare their derivative V' because this is the quantity which appears in the equation of motion. The derivative of the potential from the LEGM is

$$\frac{\partial V}{\partial \phi} \sim V_0 \frac{2\phi^*}{|\phi|^2} \left(\ln \frac{|\phi|^2}{\langle S\rangle^2}\right), \tag{4.4}$$

which is to be compared with the supergravity contribution $\partial V/\partial\phi = m_{3/2}^2 \phi^*$. The supergravity contribution is more important above a threshold value ϕ_{eq} which is given by

$$\phi_{\text{eq}} \sim \left\{\frac{2V_0}{m_{3/2}^2} \left(\ln \frac{|\phi_{\text{eq}}|^2}{\langle S\rangle^2}\right)\right\}^{1/2} \sim 7 \times 10^{11} \text{ GeV} \times \left(\frac{m_{3/2}}{100\text{keV}}\right)^{-1} \left(\frac{V_0^{1/4}}{3 \times 10^3 \text{ GeV}}\right)^2. \tag{4.5}$$

The motion of the flat direction is determined by the effective potential given in this section and the canonical kinetic term, and there is no need to include the wave function renormalization factor at this order in perturbation theory. See Appendix B for details.

5 Affleck–Dine Baryogenesis in the LEGM Models

In this section, we aim at estimating the size of baryon-to-entropy ratio from Affleck–Dine baryogenesis in the LEGM models. Because of the multiple scales in the problem, the

* The readers may wonder why Eqs. (4.2) and (4.3) have only two powers of gauge coupling constants despite the two-loop-ness of the effective potential. This is the result of an explicit calculation, and we can also explain it in a simple way. When the field value is large, the standard model gauge multiplets acquire large masses of order $g\phi$. The effective potential is generated by the exchange of heavy gauge multiplets, and hence it is suppressed by $1/|g\phi|^2$. This cancels two powers in gauge coupling constants.

†This is true for the minimal supergravity and its variants. This contribution, however, is not there in no-scale supergravity, or in general, supergravity with Heisenberg symmetry [20, 21]. In such a case, ϕ_{eq} must be taken at M_* in the rest of the discussions.

discussion becomes somewhat complicated. The basic conclusion is that the Affleck–Dine baryogenesis works as efficiently as in the hidden sector case, but in a much more non-trivial manner. Finally we discuss the possible dilution of gravitinos via the decay of the Affleck–Dine flat direction, and find that the gravitinos can be diluted below the closure limit if the initial amplitude of the flat direction is sufficiently large.

5.1 Generalities

In Affleck–Dine baryogenesis [11], one assumes that a MSSM flat direction has a large amplitude at the end of the primordial inflation. The mechanism to achieve a large amplitude varies: a negative curvature from a non-minimal Kähler potential [22], or no-scale supergravity [21]. In any case, it tends to be equal to or larger than the expansion rate of the Universe during inflation $H_{\rm inf} \sim 10^{11}$ GeV – 10^{13} GeV depending on inflationary scenarios. We phenomenologically parameterize it just as the initial amplitude ϕ_0.

A typical assumption is that there is a baryon-number violating Kähler potential term $K \sim l^* q^* u^c d^c / M_*^2$, where M_* is the reduced Planck scale.* The supersymmetry breaking effects from the LEGM generates a term in the potential†

$$\begin{aligned} \mathcal{O} &\sim \int d^4\theta \frac{g^4}{(4\pi)^4} \frac{|\theta^2 F_S|^2}{g^2|\phi|^2} \left(\ln \frac{|\phi|^2}{\langle S \rangle^2} \right)^2 \frac{1}{M_*^2} l^* q^* u^c d^c + h.c. \\ &= \frac{V_0}{|\phi|^2} \left(\ln \frac{|\phi|^2}{\langle S \rangle^2} \right)^2 \frac{1}{M_*^2} \tilde{l}^* \tilde{q}^* \tilde{u}^c \tilde{d}^c + h.c., \end{aligned} \tag{5.1}$$

while the supergravity effect induces an operator in the scalar potential

$$\mathcal{O} \sim \int d^4\theta \frac{|\theta^2 F_0|^2}{M_*^2} \frac{1}{M_*^2} l^* q^* u^c d^c + h.c. = \frac{m_{3/2}^2}{M_*^2} \tilde{l}^* \tilde{q}^* \tilde{u}^c \tilde{d}^c + h.c. \tag{5.2}$$

The LEGM operator is dominant if $|\phi_0| \lesssim \phi_{\rm eq}$, while the supergravity one dominates if $|\phi_0| \gtrsim \phi_{\rm eq}$. Therefore, we discuss the two cases separately below. In either case, the size of the baryon-number violating operator is much smaller than in the hidden sector case (Appendix E). It turns out, however, that the baryogenesis proceeds efficiently with these operators.

Below we generically refer to the fields as ϕ without distinguishing various species. The baryon number in the scalar sector is given by

$$n_B = i(\dot{\phi}^* \phi - \dot{\phi} \phi^*), \tag{5.3}$$

*It could also well be the GUT-scale $M_{\rm GUT}$. However, in this paper, we assume this form of the baryon number violating operator for simplicity. The extension to the case with $K \sim l^* q^* u^c d^c / M_{\rm GUT}^2$ is trivial, and one can easily estimate the resulting baryon-to-entropy ratio.

†We have not calculated this explicitly. This form is expected based on the analogy to the calculation of the effective potential in the previous section. The only difference is that the previous one arises from the kinetic term $\phi^* \phi$ in the Kähler potential rather than from a non-renormalizable term $l^* q^* u^c d^c / M_*^2$ here.

while the baryon number violating operator is written as

$$\mathcal{O} \sim \left[m_{3/2}^2 + \frac{V_0}{|\phi|^2} \left(\ln \frac{|\phi|^2}{\langle S \rangle^2} \right)^2 \right] \frac{1}{M_*^2} (\phi^4 + \phi^{*4}). \tag{5.4}$$

5.2 $|\phi_0| \gtrsim \phi_{\rm eq}$

For sufficiently large $|\phi_0|$ ($|\phi_0| \gtrsim \phi_{\rm eq}$), the supergravity contribution is initially important, and the field begins to roll down the potential when the expansion rate of the Universe H is comparable to $H \sim m_{3/2}$. Let us first estimate the primordial baryon number asymmetry in this case, $|\phi_0| \gtrsim \phi_{\rm eq}$.

A rough estimation of the baryon asymmetry, which is generated just after the start of the oscillation of the ϕ field can be done only by using simple order of magnitude arguments. With the above baryon number violating operator, the time evolution of the baryon number is given by

$$\dot{n}_B + 3Hn_B = i \left(\frac{\partial \mathcal{O}}{\partial \phi} \phi - \frac{\partial \mathcal{O}}{\partial \phi^*} \phi^* \right). \tag{5.5}$$

When the field begins to roll down the potential, its initial motion is slow, and one can neglect the $\dot{n}_B$ term in the equation (see Appendix D). Then the resulting baryon number can be estimated by [23]

$$n_B \sim \frac{i}{3H} \left(\frac{\partial \mathcal{O}}{\partial \phi} \phi - \frac{\partial \mathcal{O}}{\partial \phi^*} \phi^* \right). \tag{5.6}$$

Using the approximate order of magnitude of the operator and $H \sim m_{3/2}$, we obtain

$$n_B \sim \frac{m_{3/2} {\rm Im}(\phi_0^4)}{M_*^2}. \tag{5.7}$$

It depends on the imaginary part of the initial amplitude. The entropy of the radiation at this stage is given by $s \sim g_* T^3$ while the energy density $\rho_{\rm rad} \sim g_* T^4 \sim m_{3/2}^2 M_*^2$. By putting them together, we can estimate the baryon-to-entropy ratio,

$$\frac{n_B}{s} \sim g_*^{-1/4} \frac{{\rm Im}(\phi_0^4)}{m_{3/2}^{1/2} M_*^{7/2}} \sim 4 \times 10^{10} \times \left(\frac{|\phi_0|}{M_*} \right)^4 \left(\frac{m_{3/2}}{100 {\rm keV}} \right)^{-1/2} \sin 4\theta_0, \tag{5.8}$$

where the initial amplitude is parameterized as $\phi_0 = |\phi_0| e^{i\theta_0}$. As one can see, a large baryon asymmetry can be generated, if the initial amplitude of ϕ is not too small. Therefore, a large enough baryon number can remain in this scenario, even if there is a substantial entropy production after the Affleck-Dine baryogenesis. Note that one obtains exactly the same expression in the hidden sector models, but with a different $m_{3/2}$.

The present baryon-to-entropy ratio is also given by the above formula, if there is no significant entropy production. However, in a realistic situation, there can be entropy production. In particular, the decay of the Affleck-Dine field ϕ may produce a large amount of entropy. Furthermore, for $m_{3/2} \gtrsim 2h^2$ keV, we need a non-negligible entropy production to dilute the primordial gravitino. If there is an entropy production after the Affleck-Dine baryogenesis, the primordial baryon number density is also diluted. In the following, we discuss how an entropy production affects the results.

We estimate the entropy production due to the decay of the flat direction. As discussed in the previous section, ϕ starts to oscillate when $T = T_0 \sim g_*^{-1/4}\sqrt{m_{3/2}M_*}$, if $|\phi_0| \gtrsim \phi_{\rm eq}$. During $|\phi| \gtrsim \phi_{\rm eq}$, the potential for ϕ is dominated by the supergravity contribution, and hence

$$|\phi|^2 R^3 = \text{const.} \quad (\phi_{\rm eq} \lesssim |\phi| \lesssim |\phi_0|). \tag{5.9}$$

Thus, the temperature at $|\phi| \sim \phi_{\rm eq}$, which we denote $T_{\rm eq}$, is estimated as

$$T_{\rm eq} \sim T_0 \left(\frac{\phi_{\rm eq}}{|\phi_0|} \right)^{2/3}. \tag{5.10}$$

For $|\phi| \lesssim \phi_{\rm eq}$, the potential for the flat direction is dominated by the LEGM piece, and the evolution for ϕ does not obey the relation (5.9). By using the virial theorem, the evolution of the flat direction can be estimated, and is given by

$$|\phi| R^3 = \text{const.} \quad (\langle S \rangle \lesssim |\phi| \lesssim \phi_{\rm eq}). \tag{5.11}$$

(See Appendix C.) By using the above relations, we obtain the dilution factor due to the decay of the flat direction.

Now an important question is at what field amplitude ϕ decays into radiation. When the motion is dominated by a parabolic term, we know the time dependence of the oscillation (just a harmonic oscillator), and we can calculate the rate of particle production in such a background. The result is known to be the same as the single particle decay rate, if the amplitude is not too large compared to the oscillation frequency. Once the amplitude becomes comparable to $\langle S \rangle$, the potential is almost parabolic, and we find that the coherent oscillation decays into radiation rapidly. On the other hand, a corresponding calculation is difficult when the potential is dominated by the logarithmic term. In our analysis, we regard the decay amplitude, $\phi_{\rm dec}$, as a free parameter, and discuss the $\phi_{\rm dec}$-dependence of the results.[‡]

The temperature of the background radiation at the decay time of the flat direction, $T_{\rm dec}$, is given by

$$T_{\rm dec} \sim T_{\rm eq} \left(\frac{\phi_{\rm dec}}{\phi_{\rm eq}} \right)^{1/3} \sim T_0 \left(\frac{\phi_{\rm dec}}{\phi_{\rm eq}} \right)^{1/3} \left(\frac{\phi_{\rm eq}}{|\phi_0|} \right)^{2/3}. \tag{5.12}$$

[‡]For a canonical parameter, we take $\phi_{\rm dec} \sim 10^5$ GeV for estimating dilution factors. Since ϕ decays at $\phi \sim \langle S \rangle \sim 3 \times 10^4$ GeV at latest, this choice gives us the minimum estimate of the baryon asymmetry. If ϕ decays earlier, the dilution factor is less and the baryon asymmetry is larger. We likely overestimate the dilution factor with this choice. We will come back to this point later when we discuss a possible dilution of gravitinos from the decay of the flat direction.

On the contrary, the energy density of the flat direction is

$$\rho_{\rm flat} \sim V_0. \tag{5.13}$$

Then, if $\rho_{\rm flat} \gtrsim \rho_{\rm rad}$, the dilution factor from the decay is given by

$$\begin{aligned} D &\sim \left(\frac{\rho_{\rm flat}}{\rho_{\rm rad}}\right)^{3/4} \sim \left(\frac{V_0}{g_* T_{\rm dec}^4}\right)^{3/4} \\ &\sim 6\times 10^8 \left(\frac{|\phi_0|}{M_*}\right)^2 \left(\frac{\phi_{\rm dec}}{10^5{\rm GeV}}\right)^{-1} \left(\frac{m_{3/2}}{100{\rm keV}}\right)^{-1/2} \left(\frac{V_0^{1/4}}{3\times 10^3~{\rm GeV}}\right). \end{aligned} \tag{5.14}$$

Note that $D \sim 1$ if $\rho_{\rm flat} \lesssim \rho_{\rm rad}$, or in terms of the initial amplitude, $\phi_0 \lesssim 10^{14}$ GeV.

Combining the above dilution factor with the estimation of the primordial baryon number density given in Eq. (5.8), we obtain the present baryon number asymmetry. In order to make a pessimistic estimate of the resulting baryon asymmetry, we assume that the flat direction decays only when its amplitude is as small as $\langle S \rangle$. This assumption maximizes the entropy production, and hence, gives us the minimum value for the baryon asymmetry. If it decays earlier, then the entropy production is less and hence the baryon asymmetry is larger. With this caveat in mind, we can make an estimate of the resulting baryon-to-entropy ratio, and in the case with entropy production ($D > 1$), the resulting baryon-to-entropy ratio is given by

$$\frac{n_B}{s} \sim D^{-1} g_*^{-1/4} \frac{{\rm Im}(\phi_0^4)}{m_{3/2}^{1/2} M_*^{7/2}} \sim 70 \times \left(\frac{|\phi_0|}{M_*}\right)^2 \left(\frac{\phi_{\rm dec}}{10^5~{\rm GeV}}\right) \left(\frac{V_0^{1/4}}{3\times 10^3~{\rm GeV}}\right) \sin 4\theta_0. \tag{5.15}$$

Note that the result is independent of the gravitino mass. It is intriguing that the final result is more or less the same as in the hidden sector case Eq. (E.9).

5.3 $|\phi_0| \lesssim \phi_{\rm eq}$

Next, we discuss the case of $|\phi_0| \lesssim \phi_{\rm eq}$.[§] In this case, the potential for the flat direction is dominated by the LEGM-piece, and the flat direction starts to move when $H \sim \sqrt{|V'(\phi_0)|/|\phi_0|} \sim \sqrt{V_0 (\ln |\phi_0|^2/\langle S \rangle^2)}/|\phi_0|$. The temperature at this stage, T_0, is estimated as

$$T_0^2 \sim g_*^{-1/2} M_* \frac{V_0^{1/2}}{|\phi_0|} \left(\ln \frac{|\phi_0|^2}{\langle S \rangle^2}\right)^{1/2}. \tag{5.16}$$

[§]As noted before, the no-scale supergravity does not generate potential term proportional to $m_{3/2}^2$ and hence the evolution of the flat direction is always dominated by the LEGM piece. Then the formulae presented in this subsection must be used even for a larger $|\phi_0| \sim M_*$. Such a large $|\phi_0|$ is indeed expected in the no-scale case because the flat directions remain flat even during the inflation [21].

Then, by using Eq.(5.6), we can estimate the resulting baryon number density, and hence the baryon-to-entropy ratio. Note that the baryon-number-violating operator $\mathcal{O}$ is different from the previous case.

Following exactly the same steps as in the previous case, we find

$$n_B \sim \frac{V_0^{1/2}\mathrm{Im}(\phi_0^4)}{M_*^2|\phi_0|}\left(\ln\frac{|\phi_0|^2}{\langle S\rangle^2}\right)^{3/2} = \frac{V_0^{1/2}|\phi_0|^3}{M_*^2}\left(\ln\frac{|\phi_0|^2}{\langle S\rangle^2}\right)^{3/2}\sin 4\theta_0, \tag{5.17}$$

when the flat direction starts to move. Therefore, the baryon-to-entropy ratio is given by

$$\begin{aligned}\frac{n_B}{s} &\sim g_*^{-1/4}\frac{|\phi_0|^{9/2}}{V_0^{1/4}M_*^{7/2}}\left(\ln\frac{|\phi_0|^2}{\langle S\rangle^2}\right)^{3/4}\sin 4\theta_0. \\ &\sim 6\times 10^{-14}\times\left(\frac{|\phi_0|}{10^{12}\ \mathrm{GeV}}\right)^{9/2}\left(\frac{V_0^{1/4}}{3\times 10^3\ \mathrm{GeV}}\right)^{-1}\sin 4\theta_0\end{aligned} \tag{5.18}$$

which is typically too small. It is useful to note that the above formula is larger by a factor of the logarithm than the corresponding formula Eq. (5.8) for the case $\phi_0 \gtrsim \phi_{\mathrm{eq}}$ when we substitute $\phi_0 = \phi_{\mathrm{eq}}$. Of course such a discontinuity cannot exist. It simply means that there is a transition region at $\phi_0 \sim \phi_{\mathrm{eq}}$ where there is a slight rise in n_B/s when we cross $\phi_0 \sim \phi_{\mathrm{eq}}$ from above.

The dilution factor can be estimated also along the lines of the previous case. We have

$$T_{\mathrm{dec}} \sim T_0\left(\frac{\phi_{\mathrm{dec}}}{\phi_0}\right)^{1/3}, \tag{5.19}$$

and hence (if $\rho_{\mathrm{flat}} > \rho_{\mathrm{rad}}$),

$$\begin{aligned}D &\sim \left(\frac{\rho_{\mathrm{flat}}}{\rho_{\mathrm{rad}}}\right)^{3/4} \sim \left(\frac{V_0}{g_* T_{\mathrm{dec}}^4}\right)^{3/4} \sim \frac{|\phi_0|^{5/2}}{M_*^{3/2}\phi_{\mathrm{dec}}(\ln|\phi_0|^2/\langle S\rangle^2)^{3/4}} \\ &\sim 2\times 10^{-4}\left(\frac{|\phi_0|}{10^{12}\ \mathrm{GeV}}\right)^{5/2}\left(\frac{\phi_{\mathrm{dec}}}{10^5\ \mathrm{GeV}}\right)^{-1}.\end{aligned} \tag{5.20}$$

(More correctly, the dilution factor is $D = ((\rho_{\mathrm{flat}} + \rho_{\mathrm{rad}})/\rho_{\mathrm{rad}})^{3/4}$ and cannot be less than unity.) Therefore, the dilution factor is much less important than in the previous case.

5.4 Numerical Analysis

A more detailed behavior of the baryon-to-entropy ratio can be studied by a numerical calculation. Here, to see the behavior of the results, we show the resulting baryon-to-entropy ratio for a particular set of parameters: $V_0 = (3\times 10^3\mathrm{GeV})^4$, $\langle S\rangle = 3\times 10^4$ GeV, $\phi_{\mathrm{dec}} = 10^5$

GeV, and $\sin 4\theta_0 = 1$. The results for other sets of parameters can be easily estimated by using Eqs.(5.8) and (5.15). First, we solved the equation of motion for the flat direction,

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi^*} = 0, \tag{5.21}$$

with the potential¶

$$V = m_{3/2}^2 |\phi|^2 + V_0 \left(\ln \frac{|\phi|^2}{\langle S \rangle^2} \right)^2 - \left[m_{3/2}^2 + \frac{V_0}{|\phi|^2} \left(\ln \frac{|\phi|^2}{\langle S \rangle^2} \right)^2 \right] \frac{1}{M_*^2} (\phi^4 + \phi^{*4}). \tag{5.22}$$

We start our calculation at a temperature much higher than T_0 ($T = 10T_0$), and follow the evolution of the flat direction as well as the temperature of the thermal bath. With the initial value $\phi_0 = 0.2 M_* e^{i\pi/8}$, the initial motion is shown in Fig. 2. As one can see, ϕ starts an elliptical motion due to the baryon-number violating term in the potential. This means that a non-vanishing baryon number is generated once ϕ starts to oscillate. We indeed found the generated baryon number to be consistent with our estimates in the previous subsections within a factor of a few.

With the motion of the flat direction, we can calculate the baryon number by using Eq.(5.3), and hence the baryon-to-entropy ratio. After several cycles of oscillation, the baryon-to-entropy ratio becomes almost constant. Then, the evolution of ϕ and T can be easily traced by using Eqs.(5.9) and (5.11) with $RT =$ const. Finally, we calculate the dilution factor, $D = ((\rho_{\rm rad} + \rho_\phi)/\rho_{\rm rad})^{3/4}$, at the decay time of ϕ, and multiply the primordial baryon-to-entropy ratio by D^{-1} to obtain the resulting baryon asymmetry.

In Fig. 3, we show the $|\phi_0|$ dependence of the present baryon-to-entropy ratio, n_B/s. From the figure, we can see that the results based on the order of magnitude estimations provide us good approximations. For a sufficiently large ϕ_0 such that the entropy production is significant, the resultant baryon-to-entropy ratio is independent of the gravitino mass, and is proportional to $|\phi_0|^2$. We also checked that the approximate formula (5.15) reproduces the behavior for the large $|\phi_0|$ region. For a smaller value of $|\phi_0| \lesssim 10^{14}$ GeV, the entropy production from the decay becomes negligible. The result then is proportional to $|\phi_0|^4$ (Eq. (5.8)). For an even smaller $|\phi_0| \lesssim \phi_{\rm eq} \sim 7 \times 10^{11}$ GeV $\times$ (100 keV$/m_{3/2}$), the behavior goes over to $|\phi_0|^{9/2}$ (Eq. (5.18)). As noted in the paragraph below Eq. (5.18), there is a transition region from $\sim 10\phi_{\rm eq}$ to $\phi_{\rm eq}$, where the curves fall less steeply because a logarithmic enhancement factor comes in. In any case, the baryon-to-entropy can clearly be sufficiently large in this scenario, as required by the standard big-bang nucleosynthesis $n_B/s \sim 10^{-10}$, if the initial amplitude is larger than 10^{13-14} GeV.

A more precise estimate of the baryon asymmetry requires the specification of the flat direction, the relevant operator, and the size of the initial amplitude. The usual caveat concerning the $B - L$ invariance applies: If one employs an operator which preserves $B - L$

¶The potential given in Eq.(5.22) is unbounded-below for $\phi \gtrsim M_*$, and higher-dimension operators are supposed to stabilize it. However, we only consider the initial amplitude to be less than $\sim M_*$ in our analysis, and hence the postulated potential is a good enough approximation.

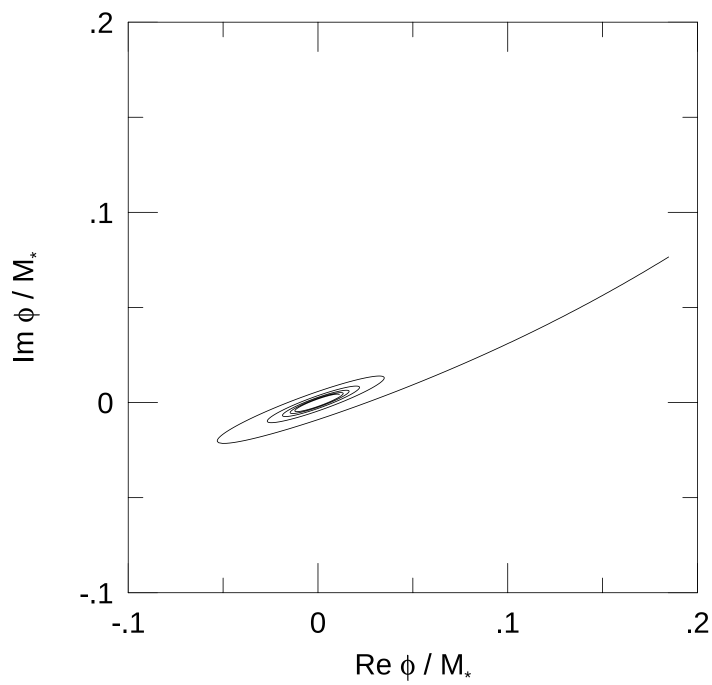

Figure 2: The initial motion of the flat direction with the potential given in Eq. (5.22). Here, we take $m_{3/2} = 100$ keV, and $\phi_0 = 0.2 M_* e^{i\pi/8}$.

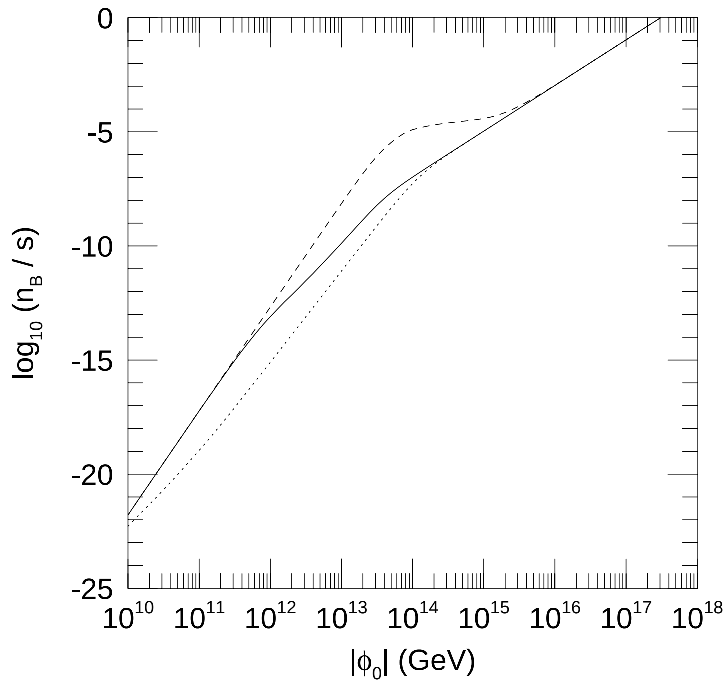

Figure 3: The resulting baryon-to-entropy ratio as a function of the initial amplitude ϕ_0. The parameters are taken to be $V_0 = (3 \times 10^3 \mathrm{GeV})^4$, $\theta_0 = \pi/8$, $\phi_{\mathrm{dec}} = 10^5$ GeV, and $m_{3/2} = 1$ keV (dotted line), $m_{3/2} = 100$ keV (solid line), and $m_{3/2} = 10$ MeV (dashed line).

symmetry like that generated by SU(5) grand unified models, it may be wiped out again by the electroweak sphaleron effect [25]. One needs to either preserve the B and L asymmetries using a Bose condensate [24], or by generating a $B-L$ asymmetry [26, 27]. In view of the discussions of the next section, we find the protection of B via a Bose condensate is a likely scenario.

5.5 Diluting Gravitinos

In the LEGM models, the mass of the gravitino is about 100 keV, and its mass density exceeds the closure limit if $T_{\rm max}$ is larger than $(1-10)$ TeV, as discussed in Section 2. Since the reheating after the primordial inflation raises the temperature typically above $T_{\rm RH} \gtrsim 10^8$ GeV or so, we assume that the gravitinos were once thermalized, which is of course the worse case scenario. We will discuss whether the decay of the flat direction can generate a large enough entropy to dilute gravitinos below the closure limit.

Before discussing the implication of the entropy production to the gravitino, it is useful to estimate the freeze-out temperature of the gravitino, $T_{\rm freeze}$, which is the temperature at which the expansion rate of the Universe becomes comprable to the production rate of the gravitino: $H(T_{\rm freeze}) \sim \Sigma_{\rm tot} n_{\rm rad}(T_{\rm freeze})$. By using the gravitino production cross section given in Eq. (2.5), we obtain

$$T_{\rm freeze} \sim 200{\rm TeV} \times \left(\frac{m_{3/2}}{100{\rm keV}} \right)^2 \left(\frac{m_{\rm G3}}{1{\rm TeV}} \right)^{-2}. \tag{5.23}$$

Below $T_{\rm freeze}$, the expansion rate of the Universe becomes larger than the production rate of the gravitino, and hence the gravitino cannot be thermalized. Therefore, an entropy production at $T \lesssim T_{\rm freeze}$ dilutes gravitinos produced before the entropy production. On the contrary, even if the entropy is produced when $T \gtrsim T_{\rm freeze}$, the gravitino is thermalized again, and its number density is determined by the thermal distributions. If the energy density of the flat direction dominates the energy density of the Universe, decay of the flat direction ϕ reheats the Universe. The reheating temperature is estimated as $T_{\rm R} \sim (V_0)^{1/4} \sim 1-10$ TeV. Comparing this reheating temperature to Eq. (5.23), we can see that the gravitino cannot be thermalized after the decay of ϕ. In other words, gravitinos produced before the decay of ϕ are diluted with a dilution factor given in Eq. (5.14), if the decay of ϕ produces the entropy.

If the gravitino mass is larger than ~ 1 keV, we need a substantial entropy production; otherwise, the Universe is overclosed by the mass density of the gravitino. By assuming that the gravitino is thermalized, we can estimate the number density of the gravitino as

$$n_{3/2}(T) = \frac{3}{2} \frac{g_*(T)}{g_*(T_{\rm freeze})} n_{\rm rad}(T). \tag{5.24}$$

Then, requiring $\Omega_{3/2} = D^{-1} m_{3/2} n_{3/2} / \rho_{\rm c} \leq 1$, we obtain

$$D \gtrsim 50 \times h^{-2} \left(\frac{m_{3/2}}{100{\rm keV}} \right). \tag{5.25}$$

Comparing the above constraint with Eq. (5.14), we can see that the decay of the flat direction can produce enough entropy to dilute the gravitinos away. For example, for $\phi_{\rm dec} \sim 10^5$ GeV and $m_{3/2} \sim 100$ keV, the dilution factor is large enough, if $|\phi_0| \gtrsim 10^{14-15}$ GeV. Even with such a large dilution, the present baryon-to-entropy ratio can be sufficiently large (see Eq. (5.15)).

If the flat direction decays at an amplitude larger than $\sim 10^5$ GeV, the dilution factor given in Eq. (5.14) becomes smaller, and the entropy production due to the decay of ϕ may not be enough to decrease the gravitino density. In addition, for $m_{3/2} \lesssim 100$ keV, the reheating temperature $T_{\rm R} \sim V_0^{-1/4} \sim 1 - 10$ TeV may be higher than the freeze-out temperature of the gravitino. In these cases, we have to assume an extra source of the entropy production of $O(m_{3/2}/2h^2 {\rm keV})$ after the reheating. Even in this case, the estimation of the primordial baryon-to-entropy ratio (5.8) is still valid, and the final baryon asymmetry can be as large as the one estimated by Eq. (5.15) *and* an additional dilution factor Eq. (5.25) required to dilute the gravitinos. Therefore, Affleck-Dine baryogenesis can generate a big enough baryon asymmetry to explain the present value of the baryon-to-entropy ratio.

In fact, we can crudely estimate the decay amplitude $\phi_{\rm dec}$ even when it is in the region of the logarithmic potential, if its motion is circular rather than elliptic. The change from the original Affleck–Dine estimate of the decay rate Eq. (E.7) is that the rotation frequency of the ϕ field is given by $(V_0 \ln(|\phi|^2/\langle S\rangle^2))^{1/2}/|\phi|$ rather than $m_{3/2}$. Since we are interested in a dilution factor, we assume that the flat direction dominates the Universe, and the field decays when

$$\Gamma_\phi \sim \left(\frac{\alpha_s}{\pi}\right)^2 \frac{1}{|\phi_{\rm dec}|^2} \left(\frac{V_0 \ln(|\phi_{\rm dec}|^2/\langle S\rangle^2)}{|\phi_{\rm dec}|^2}\right)^{3/2} \sim H \sim \frac{V_0^{1/2}}{M_*} \ln \frac{|\phi_{\rm dec}|^2}{\langle S\rangle^2}. \tag{5.26}$$

and

$$|\phi_{\rm dec}| \sim \left[\left(\frac{\alpha_s}{\pi}\right)^2 V_0 M_* \left(\ln \frac{|\phi_{\rm dec}|^2}{\langle S\rangle^2}\right)^{1/2}\right]^{1/5} \sim 8 \times 10^5 \text{ GeV} \times \left(\frac{V_0}{(3 \times 10^3 \text{ GeV})^4}\right)^{1/5}. \tag{5.27}$$

Therefore, the decay amplitude does not change much from the value assumed before. On the other hand, the case with an elliptic orbit is more difficult to deal with. We are not aware of any study on the decay rate of ϕ for an arbitrary elliptic motion even for the parabolic potential. The other limit of almost linear motion is discussed in the literature and tends to give a larger decay rate, and hence a larger $\phi_{\rm dec}$ [28]. However, we believe the motion of the ϕ field in the case of our interest here, namely for $|\phi_0| \gtrsim 10^{14-15}$ GeV to dilute gravitinos, to be quite far from a linear one, and $\phi_{\rm dec}$ is not likely to be much larger than our estimate. We conclude that $\phi_{\rm dec}$ is not much larger than the minimum possible value 10^5 GeV which we used in most of our discussions.

6 Cosmology of String Moduli

We point out in this section that the moduli fields in the string theory, if they acquire masses in the LEGM models, are stable and drastically overclose the Universe.

According to a general analysis [29], string moduli acquire masses comparable to the gravitino mass $m_{3/2}$. Their initial amplitude is likely to be of the order of the string or Planck scale because it is the only scale in the problem. The cosmological problem of the moduli fields is discussed extensively in the literature in the context of hidden sector models (for the original paper, see [30]). There, the moduli fields acquire masses of the order of 1 TeV, and decay after nucleosynthesis, thereby spoiling the success of the nucleosynthesis theory. Even if one pushes the mass to 10 TeV so that the moduli fields decay before nucleosynthesis, the enormous production of entropy with a dilution factor of order $M_*/m_{3/2} \sim 10^{14}$ wipes out all pre-existing baryon asymmetry. This problem may be solved by adopting the Affleck-Dine baryogenesis [31], or by the thermal inflation [12] (see Appendix E for more discussions).

In the LEGM models, the situation is completely different.* The string moduli are *stable* within the cosmological time scale, and are still oscillating around their potential minima. A dimensional analysis gives the decay rate of a moduli field to be $\Gamma \sim m_{3/2}^3/8\pi M_*^2 \sim (3 \times 10^{18}\text{ years})^{-1}$, for $m_{3/2} \sim 100$ keV. Therefore there is a problem concerning its energy density.

The estimation of the moduli energy density is straight-forward. When a moduli field begins to oscillate, the expansion rate is $H \sim m_{3/2}$. The entropy at this stage is given by $s \sim g_*^{1/4}(m_{3/2}M_*)^{3/2}$. Assuming the initial amplitude of order M_*, the ratio of the moduli energy density to the entropy is given by

$$\frac{\rho_{\text{moduli}}}{s} \sim g_*^{-1/4}(m_{3/2}M_*)^{1/2} \sim 1.3 \times 10^6\text{ GeV}\left(\frac{m_{3/2}}{100\text{ keV}}\right)^{1/2}. \tag{6.1}$$

Since both the energy density of the moduli and the entropy are diluted by the expansion with the same rate R^{-3}, the ratio remains constant until now unless there is entropy production. On the other hand, the total energy density is bounded from above by the critical density ρ_c,

$$\frac{\rho_{\text{moduli}}}{s} \leq \frac{\rho_c}{s_{\text{now}}} = 3.6 \times 10^{-9} h^2\text{ GeV}. \tag{6.2}$$

where s_{now} is the present value of the entropy density. The predicted ratio Eq. (6.1) is in gross conflict with the constraint Eq. (6.2).

It is not clear how such an enormous energy density can be diluted. First of all, the necessary dilution factor is at least 10^{15}. Furthermore, one needs such an entropy production at a very late stage of the Universe, with $H \leq m_{3/2}$, or equivalently, $T \leq g_*^{-1/4}(m_{3/2}M_*)^{1/2} = 1.3 \times 10^6$ GeV for $m_{3/2} = 100$ keV. One needs to create a baryon asymmetry either after

*It was argued that the problem does not exist [32] if SUSY is broken dynamically, which is true for scalar fields which directly participate in the dynamical supersymmetry breaking. However, the string moduli fields were not considered in this discussion.

such an enormous entropy production at a very late stage, or large enough to survive the enormous entropy production.

Actually, the Affleck–Dine mechanism may create a large enough baryon asymmetry to survive the enormous entropy production which dilutes the string moduli below the critical density as we will show below.

A quantity which remains constant over an entropy production is the ratio of the baryon number and the moduli energy density, because both of them scale as R^{-3}. We estimated in the previous section that the initial baryon number density is $n_B \sim m_{3/2} \mathrm{Im}\phi_0^4/M_*^2$, at the time when the flat direction begins to oscillate, *i.e.*, $H \sim m_{3/2}$. In fact, this is the same time as when the moduli fields begin to oscillate, and the energy density of the moduli is $\rho_{\rm moduli} \sim m_{3/2}^2 M_*^2$. Therefore, their ratio is determined at this stage:

$$\frac{\rho_{\rm moduli}}{n_B} \sim m_{3/2} \left(\mathrm{Im} \frac{M_*^4}{\phi_0^4} \right). \tag{6.3}$$

or equivalently,

$$\frac{n_B}{s} \sim \frac{\rho_{\rm moduli}}{s} \times m_{3/2}^{-1} \left(\frac{|\phi_0|}{M_*} \right)^4 \sin 4\theta_0. \tag{6.4}$$

Combining the above equation with the constraint (6.2), we find

$$\frac{n_B}{s} \lesssim \frac{\rho_c}{s_{\rm now}} \times m_{3/2}^{-1} \left(\frac{|\phi_0|}{M_*} \right)^4 \sin 4\theta_0 \sim 4 \times 10^{-5} h^2 \left(\frac{m_{3/2}}{100 \text{ keV}} \right)^{-1} \left(\frac{|\phi_0|}{M_*} \right)^4 \sin 4\theta_0. \tag{6.5}$$

As one can see, if $|\phi_0| \gtrsim 10^{17}$ GeV, the baryon-to-entropy ratio may be larger than $\sim 10^{-10}$ even if we assume a large entropy production to dilute the moduli field.

The important question is whether one can have a brief period of inflation at such a late stage of Universe to dilute string moduli in the LEGM models. The inflationary expansion rate $H_{\rm inf}$ must be less than $H_{\rm inf} \lesssim m_{3/2} \sim 100$ keV, *i.e.* the energy density of the inflation $\rho_{\rm inf} \lesssim (m_{3/2} M_*)^2 \sim (10^7 \text{ GeV})^2$. Moreover, the e-folding should not exceed 20 or so in order to keep the primordial density fluctuation generated by a "standard" inflation with $H_{\rm inf} \sim 10^{11}$–10^{13} GeV [34]. On the other hand, an e-folding of $N \gtrsim 5$ is sufficient to dilute the string moduli by 10^{-15}. A thermal inflation [12] may offer a natural solution to these questions.

Fortunately, we apparently do not need to introduce new energy scales into the model in the framework of the thermal inflation. Suppose a positive mass squared of $m^2 \sim (100 \text{ GeV})^2$ is generated for a scalar field χ due to higher order loops at the energy scale of $\Lambda_{\rm DSB} \sim 10^7$ GeV. The renormalization group running of the mass squared may drive it negative at a scale slightly below $\Lambda_{\rm DSB}$. If the scalar field is a flat direction of both F- and D-terms in the potential, it develops a minimum at $v \lesssim \Lambda_{\rm DSB}$. This is an ideal potential for a thermal inflation. The scalar field may initially be stuck at the origin because of the thermal effects, giving a cosmological constant. As the radiation gets red-shifted, the thermal effects turn off and the field rolls down the potential to its true minimum $\chi = v$. The e-folding in this case is roughly $N \simeq \frac{1}{2}\ln(v/m) \sim \frac{1}{2}\ln(\Lambda_{\rm DSB}/m) \sim 5$ [12] which is exactly what is needed to dilute the string moduli below the critical density.

It may be interesting to compare this result with the case of the hidden sector SUSY breaking scenario with the Polonyi field or with the string moduli. (Hereafter, we call them generically as "Polonyi fields". See Appendix E for the estimation of baryon asymmetry in this case.) Note that Eq. (6.4) is valid. Thus, the question is the constraint on the energy density of the Polonyi field, ρ_z. In this case, the Polonyi field decays much faster than the LEGM case since its mass is larger, and a typical lifetime for the Polonyi field is given by

$$\tau_z \sim \left(\frac{N_{\rm ch}}{4\pi} \frac{m_{3/2}^3}{M_*^2} \right)^{-1} \sim 10^3 {\rm sec} \times \left(\frac{m_{3/2}}{1{\rm TeV}} \right)^3, \tag{6.6}$$

where $N_{\rm ch} \sim O(10)$ is the number of the decay channel. Thus, it does not contribute to the mass density of the present Universe, and the constraint (6.2) cannot be applied. However, it may affect the great success of the standard big-bang nucleosynthesis (BBN) scenario.

The mass density of the Polonyi field speeds up the expansion rate of the Universe when the neutron decouples from the thermal bath (*i.e.*, $T \sim 1$MeV), which may result in an over production of ^{4}He. Furthermore, the radiative decay of the Polonyi field induces cascade photons which cause the photofission process and change the primordial abundances of the light nuclei. The constraint on the primordial density of the Polonyi field strongly depends on its lifetime τ_z [33]. If $\tau_z \lesssim 10^4$ sec, nucleosynthesis requires $\rho_z/s \lesssim 10^{-5}$ GeV. For a Polonyi field with a longer lifetime, the constraint becomes more stringent. For a Polonyi field with $\tau_z \lesssim 10^{(4-5)}$ sec, which is the case for the Polonyi mass typically larger than a few TeV, its mass density is constrained as $\rho_z/s \lesssim 10^{-7}$ GeV. These constraints on ρ_z/s are compared to the estimate of the baryon-to-entropy ratio (E.14) which holds irrespective of the presence of a substantial dilution of Polonyi field by, *e.g.*, a late inflation,

$$\frac{n_B}{s} \sim \frac{\rho_z}{s} \times m_{3/2}^{-1} \left(\frac{|\phi_0|}{M_*} \right)^4 \sin 4\theta_0.$$

Thus, for this range of the Polonyi mass, the resulting baryon-to-entropy ratio may still be as large as 10^{-10} if $|\phi_0| \sim M_*$, and hence the Affleck-Dine scenario may provide us a reasonable value for the baryon-to-entropy ratio. However, if the Polonyi field has a longer lifetime, as for a sub-TeV Polonyi mass as usually expected, the constraint on ρ_z becomes even more stringent. In particular, for the case with $\tau_z \gtrsim 10^7$ sec, which typically corresponds to $m_z \lesssim 100$ GeV, $\rho_z/s \lesssim 10^{-13}$ GeV. In this case, the result is too small to be identified as the present baryon asymmetry of the Universe.

7 Conclusion

We studied the cosmology of the LEGM models. We first estimated the lower bound on the gravitino mass, and saw that the bound conflicts with the cosmological constraint if the primordial gravitino is not diluted. This fact indicates a huge entropy production at a relatively low temperature, and the conventional scenario of baryogenesis may not work well.

In this case, the Affleck-Dine baryogenesis is one interesting possibility. The size of the baryon number violating operators is much smaller than in the hidden sector models. However the flat direction begins to move at a much later stage which in turn increases the baryon number. The dilution factor due to the decay of flat direction also has a complicated dependence on parameters. After putting all the effects together, we found that the Affleck–Dine baryogenesis works efficiently for an initial amplitude of the flat direction, $|\phi_0| \gtrsim 10^{13}$ GeV. We also discussed that the decay of the MSSM flat direction may provide enough entropy to dilute the primordial gravitino for a relatively large initial amplitude of the flat direction, $|\phi_0| \gtrsim 10^{14-15}$ GeV. Therefore, the gravitino problem in the LEGM models may be solved if we assume such a large initial amplitude.

We also discussed the cosmological implication of the moduli fields in the string theory. Their masses are of the order of the gravitino mass, and their lifetime is much larger than the present age of the Universe in the LEGM models. The mass density of the moduli field may overclose the Universe. To dilute the moduli fields, a very late inflation is needed. We found that the baryon asymmetry generated by Affleck-Dine baryogenesis can be large enough to survive such a late inflation for $|\phi_0| \gtrsim 10^{17}$ GeV, even if we assume a huge entropy production to dilute the primordial moduli field below the critical density.

Acknowledgment

This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098 and in part by the National Science Foundation under grant PHY-95-14797. HM was also supported by the Alfred P. Sloan Foundation. AdG was supported by CNPq (Brazil).

A Computing the Effective Potential

The effective potential for the flat direction ϕ can be computed by the following usual procedure. We let it to have an expectation value, and calculate the vacuum energy in the presence of ϕ background. The vacuum energy is identically zero if we do not pick the effect of supersymmetry breaking. The lowest order contribution is from two-loop diagrams where the standard model gauge multiplets couple to the vector-like messenger fields whose mass spectrum breaks supersymmetry. The gauge multiplets acquire masses because of the ϕ background and hence the result depends on ϕ.

The mass spectrum of the messenger sector is M for fermions, and $M_\pm^2 = M^2 \pm MB$ for scalars. We assume one vector-like multiplet with a unit U(1) charge, and calculate

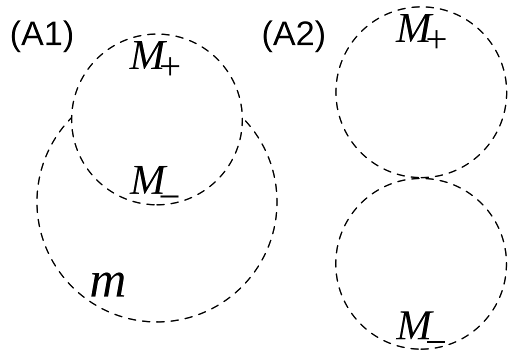

Figure 4: Feynman diagrams which contribute to the vacuum energy in the background of the flat direction $\phi = \bar{\phi}$. The vertices are due to the D-term potential. The scalar field with mass $m = 2g\langle\phi\rangle$ is the scalar component of the massive gauge multiplet in the presence of the background ϕ. The scalar fields with masses M_+ and M_- are the messenger scalars.

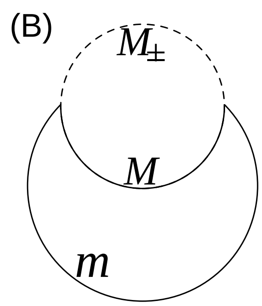

Figure 5: A Feynman diagram with the gaugino of mass m, the messenger fermion of mass M, and the messenger scalars of mass $M_\pm$.

the contribution from a U(1) gauge multiplet exchange. This U(1) gauge group is the toy-model version of the standard model gauge groups. The flat direction ϕ, $\bar{\phi}$ also has ± 1 charge under U(1), with D-flatness condition $\phi = \bar{\phi}$. The result can be easily generalized to arbitrary gauge groups and messenger multiplets. We refer to the U(1) gauge coupling constant as g. The U(1) gauge multiplet acquires a mass $m = 2g|\phi|$. The task is to calculate the vacuum energy as a function of M, B, g and m.

The Feynman diagrams are shown in Figs. 4–6. In all our calculations we expand the amplitudes in terms of MB/M^2 and keep only the leading non-trivial terms of $O(MB)^2$. We need $MB/M^2 < 1$ to avoid a color- or charge-breaking vacuum, and this expansion is known to be a good approximation for the supersymmetry breaking mass squared for the flat direction unless MB is very close to M^2 [36].

We start with diagrams (A1) and (A2) in Fig. 4,

$$(\text{A1}) \quad = \quad ig^2 m^2 \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4k}{(2\pi)^4} \frac{1}{p^2 - m^2} \left[\frac{1}{(k^2 - M_+^2)((p+k)^2 - M_-^2)} \right], \qquad \text{(A.1)}$$

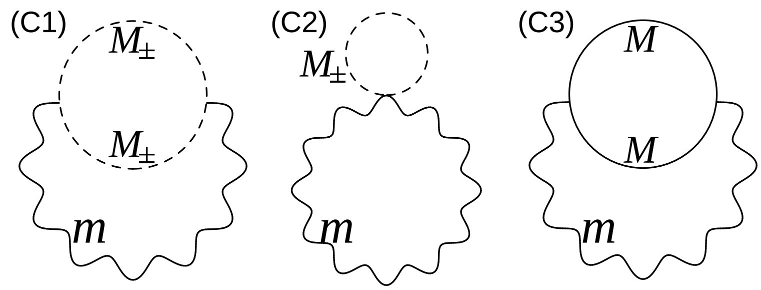

Figure 6: Feynman diagrams with vacuum polarization due to (C1) messenger scalar loops, (C2) "seagull" diagram with messenger scalars, and (C3) messenger fermions.

$$(\text{A2}) \quad = \quad ig^2 \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4k}{(2\pi)^4} \left[\frac{1}{(k^2 - M_+^2)(p^2 - M_-^2)} \right]. \tag{A.2}$$

Since we know that the sum of all diagrams vanishes in the supersymmetric limit $MB \to 0$, we subtract the corresponding amplitude in the supersymmetric limit from each diagrams. The diagrams (A1) and (A2) give after the subtraction:

$$\begin{aligned}(\text{A1})_s + (\text{A2})_s \quad &= \quad -ig^2 \int_0^1 dx \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4k}{(2\pi)^4} \frac{p^2}{p^2 - m^2} \\ &\left[\frac{1}{(k^2 + x(1-x)p^2 - M^2 + (1-2x)MB)^2} - (MB \to 0) \right]. \end{aligned} \tag{A.3}$$

Here and hereafter, the subscript s refers to the subtraction of amplitudes in the supersymmetric limit.

We expand the integrand in powers of MB/M^2. The linear terms in MB vanish upon x integration, and we are left with the following expression, to $O((MB)^2)$ in the integrand,

$$(\text{A1})_s + (\text{A2})_s = ig^2 \int_0^1 dx \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4k}{(2\pi)^4} \frac{p^2}{p^2 - m^2} \frac{(4-1)(1-2x)^2(MB)^2}{(k^2 + x(1-x)p^2 - M^2)^4} + O(B^4). \tag{A.4}$$

We follow the same strategy as above to compute the contribution from the diagram (B) in Fig. 5 containing the messenger fermions,

$$\begin{aligned}(\text{B})_s \quad &= \quad -i(\sqrt{2}g)^2 \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4k}{(2\pi)^4} \frac{2(k \cdot p)}{(p^2 - m^2)(k^2 - M^2)} \\ &\left(\frac{1}{(p+k)^2 - M_+^2} + (M_+^2 \to M_-^2) - 2(M_+^2 \to M^2) \right) \\ &= \quad -4ig^2 \int_0^1 dx \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4k}{(2\pi)^4} \frac{xp^2}{p^2 - m^2} \\ &\left[\frac{1}{(k^2 + x(1-x)p^2 - M^2 - xMB)^2} + (MB \to -MB) - 2(MB \to 0) \right] \end{aligned}$$

$$= -4ig^2 \int_0^1 dx \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4k}{(2\pi)^4} \frac{p^2}{p^2 - m^2} \frac{(MB)^2 6x^3}{(k^2 + x(1-x)p^2 - M^2)^4} + O(B^4). \tag{A.5}$$

Finally the diagrams (C1), (C2), (C3) with the gauge boson loop in Fig. 6. We first calculate the vacuum polarization diagrams of messenger fields. Note that the contribution of messenger fermions (C3) is the same as the one in the supersymmetric limit, and hence cancels after the subtraction. The scalar loop gives

$$\begin{aligned}(\mathrm{C1})_s &= g^2 \int \frac{d^4k}{(2\pi)^4} (2k+p)^\mu (2k+p)^\nu \\ & \left[\frac{1}{k^2 - M_+^2} \frac{1}{(k+p)^2 - M_+^2} + (M_+^2 \to M_-^2) - 2(M_+^2 \to M^2) \right],\end{aligned} \tag{A.6}$$

and the "seagull" diagram gives

$$(\mathrm{C2})_s = -2g^2 g^{\mu\nu} \int \frac{d^4k}{(2\pi)^4} \left[\frac{1}{k^2 - M_+^2} + (M_+^2 \to M_-^2) - 2(M_+^2 \to M^2) \right]. \tag{A.7}$$

Their sum is

$$(\mathrm{C1})_s + (\mathrm{C2})_s = g^2(-g^{\mu\nu}p^2 + p^\mu p^\nu) \int \frac{d^4k}{(2\pi)^4} \int_0^1 dz \frac{6(1-2z)^2 (MB)^2}{(k^2 + z(1-z)p^2 - M^2)^4} + O(B^4). \tag{A.8}$$

Now including the gauge boson loop, the total contribution of the vacuum polarization diagrams is

$$(\mathrm{C})_s = i9g^2(MB)^2 \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4p}{(2\pi)^4} \int_0^1 dz \frac{p^2}{p^2 - m^2} \frac{(1-2z)^2}{(k^2 + z(1-z)p^2 - M^2)^4}. \tag{A.9}$$

Adding all diagrams, we obtain $-iV_{\mathit{eff}}(m^2) = (\mathrm{A})_s + (\mathrm{B})_s + (\mathrm{C})_s$ up to $O(MB)^2$,

$$V_{\mathit{eff}}(m^2) = -12g^2(MB)^2 \int_0^1 dx \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4k}{(2\pi)^4} \frac{p^2}{p^2 - m^2} \left[\frac{(1-2x)^2 - 2x^3}{(k^2 + x(1-x)p^2 - M^2)^4} \right]. \tag{A.10}$$

After a Wick rotation in the k and p spaces, the d^4k integration can be carried out trivially. Note also that the denominator is symmetric under the interchange of $x \leftrightarrow (1-x)$. We therefore symmetrize the x integration by substituting the polynomial in the numerator $P(x) = (1-2x)^2 - 2x^3$ by $\frac{1}{2}(P(x) + P(1-x)) = -x(1-x)$, finding

$$V_{\mathit{eff}}(m^2) = \frac{-g^2(MB)^2}{128\pi^4} \int_0^1 dx \int_0^\infty p^2 d(p^2) \left[\frac{x(1-x)p^2}{(p^2 + m^2)(x(1-x)p^2 + M^2)^2} \right]. \tag{A.11}$$

The p^2 integral is logarithmically divergent. Fortunately, the divergent piece is m^2 independent, and hence is the renormalization of the cosmological constant. We subtract $V_{\mathit{eff}}(0)$

from the above expression and redefine it as V_{eff}. The final integral is convergent for any $m^2 \in [0, \infty)$:

$$V_{\mathit{eff}}(m^2) = \frac{g^2(MB)^2 m^2}{128\pi^4} \int_0^1 dx \int_0^\infty p^2 d(p^2) \left[\frac{x(1-x)}{(p^2+m^2)(x(1-x)p^2+M^2)^2} \right]. \qquad \text{(A.12)}$$

The p integration can be computed using the following tricks. First, change the integration variable to $q^2 = x(1-x)p^2$. Then the q^2 integration can be done in the standard way, and we obtain

$$V_{\mathit{eff}}(z^2) = \frac{g^2(MB)^2}{128\pi^4} \int_0^1 dx \left[\frac{1/z^2 - x(1-x) + x(1-x)\ln[x(1-x)z^2]}{(1/z^2 - x(1-x))^2} \right]. \qquad \text{(A.13)}$$

Here and below, we use $z^2 \equiv m^2/M^2$.

We can further perform the x integration using dilogarithms. Using the roots of the denominator $a \equiv \left(1 - \sqrt{1 - 4/z^2}\right)/2$ and $1-a$,

$$V_{\mathit{eff}}(a) = \frac{g^2(MB)^2}{128\pi^4} \int_0^1 dx \left[\frac{(x-a)(x-1+a) + x(1-x)\ln\left(\frac{x(1-x)}{a(1-a)}\right)}{(x-a)^2(x-1+a)^2} \right]. \qquad \text{(A.14)}$$

After the final integral is carried out we are left with an expression for the effective potential as a function of a:

$$\begin{aligned} V_{\mathit{eff}}(a) &= \frac{g^2(MB)^2}{64\pi^4} \left\{ \frac{\ln(a(1-a))}{(1-2a)^2} + \right. \\ & \left. \frac{1-2a(1-a)}{(1-2a)^3} \left[\frac{1}{2}\ln^2(a) - \frac{1}{2}\ln^2(1-a) - \mathrm{Li}_2(a) + \mathrm{Li}_2(1-a) \right] \right\}. \qquad \text{(A.15)} \end{aligned}$$

The form of the effective potential is shown in Fig. 7 as a function of z as a solid line.

The expression is manifestly real for $z^2 \geq 4$. In the limit $z \to \infty$, $a \approx 1/z^2$ and the potential behaves as

$$\begin{aligned} V_{\mathit{eff}} &= \frac{g^2(MB)^2}{64\pi^4} \left[\frac{1}{2}\ln^2(z^2) - \ln(z^2) + \frac{\pi^2}{6} + O\left(\frac{1}{z^2}\ln^2 z^2\right) \right] \\ &\sim \frac{g^2(MB)^2}{128\pi^4} \left(\ln \frac{4g^2|\phi|^2}{eM^2} \right)^2. \qquad \text{(A.16)} \end{aligned}$$

This asymptotic form of the effective potential is also shown in Fig. 7 as a dotted line.

In the case $z^2 < 4$, a is complex, $a = 1/2 + i\sqrt{4/z^2 - 1}$. We can make the effective potential manifestly real, using the following dilog relations:

$$\begin{aligned} \mathrm{Li}_2(a) + \frac{1}{2}\ln^2(a) &= -\mathrm{Li}_2\left(\frac{1}{a}\right) - i\pi\ln(a) + \frac{\pi^2}{2} - \zeta(2), \qquad \text{(A.17)} \\ \mathrm{Li}_2(1-a) + \frac{1}{2}\ln^2(1-a) &= -\mathrm{Li}_2\left(\frac{1}{1-a}\right) + i\pi\ln(1-a) + \frac{\pi^2}{2} - \zeta(2). \qquad \text{(A.18)} \end{aligned}$$

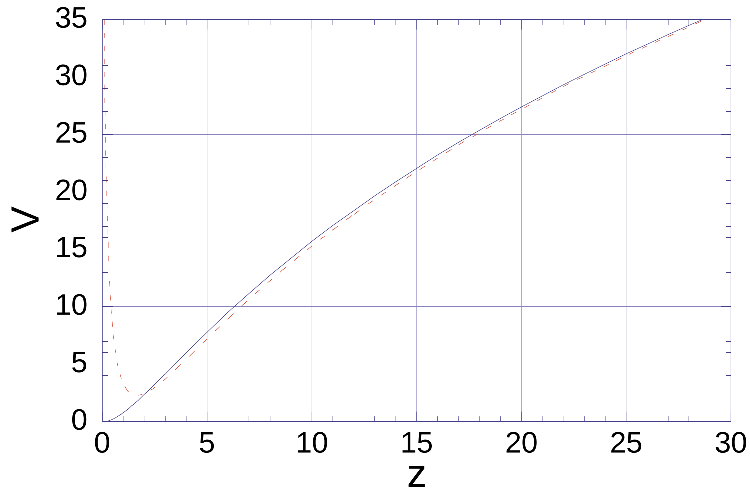

Figure 7: A plot of the effective potential Eq. (A.15) for $z \equiv 2g|\phi|/M < 30$, in the unit of $g^2(MB)^2/(128\pi^4)$. The solid line is the exact result, and the dotted line shows the asymptotic form Eq. (A.16) valid for large z.

We find

$$V_{\textit{eff}}(z^2 < 4) = \frac{g^2(MB)^2}{64\pi^4}\left\{-\frac{\ln\left(\frac{1}{z^2}\right)}{\frac{4}{z^2}-1}\right.$$
$$\left.+\frac{\left(1-\frac{2}{z^2}\right)}{\left(\frac{4}{z^2}-1\right)^{\frac{3}{2}}}\left[-\ln\left(\frac{1}{z^2}\right)\left(\pi+\arctan\left(-\sqrt{\frac{4}{z^2}-1}\right)\right)+i\left(\mathrm{Li}_2\left(\frac{1}{a}\right)-\mathrm{Li}_2\left(\frac{1}{1-a}\right)\right)\right]\right\}. \tag{A.19}$$

Note that $\mathrm{Li}_2(z)-\mathrm{Li}_2(z^*)$ is pure imaginary. In the above form, it is simple to take the limit $z^2 \to 0$, and we obtain

$$\begin{aligned} V_{\textit{eff}} &= \frac{g^2(MB)^2}{64\pi^4}\left[\frac{z^2}{4}\ln(z^2)+\left(\frac{z^3}{8}-\frac{z}{4}\right)\left(z\ln(z^2)+\frac{iz^2}{2}-z-\frac{iz^2}{2}-z\right)+O(z^3\ln(z^2))\right] \\ &= \frac{g^2(MB)^2}{64\pi^4}\left(\frac{z^2}{2}+O(z^3\ln(z^2))\right) \qquad \text{(A.20)} \\ &= \frac{g^2m^2}{128\pi^4}\left(\frac{(MB)^2}{M^2}\right)+O(m^4) \\ &= 4\left(\frac{\alpha}{4\pi}\right)^2\left(\frac{MB}{M}\right)^2(|\phi|^2+|\bar{\phi}|^2)+O(\phi^4). \qquad \text{(A.21)} \end{aligned}$$

The approximate form Eq. (A.20) truncated at $O(z^2)$ is shown in Fig. 8 as a dotted line together with the exact form Eq. (A.15) (or equivalently, Eq. (A.19)) as a solid line. From

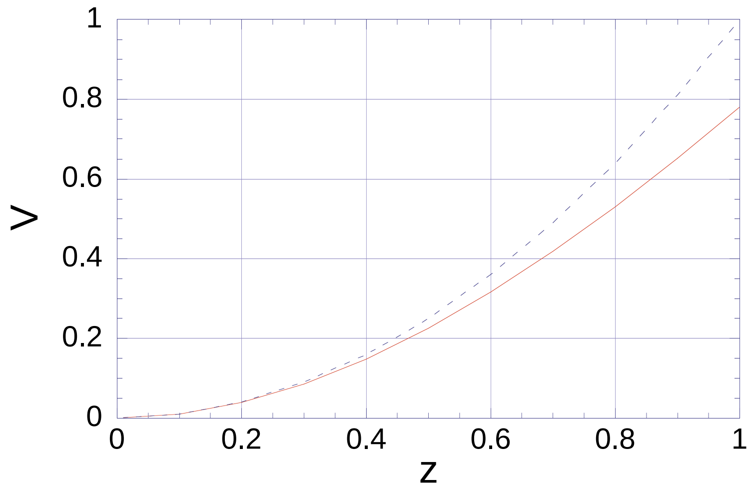

Figure 8: A plot of the effective potential Eq. (A.15) (or equivalently, Eq. (A.19)) for the small field amplitude, $z \equiv 2g|\phi|/M < 1$, in the unit of $g^2(MB)^2/(128\pi^4)$. The solid line is the exact result, and the dotted line shows the approximate form Eq. (A.20) valid for small z.

the last expression (A.21) with $\alpha \equiv g^2/4\pi$, we can read off the mass of the flat direction. For messengers in $\mathbf{5}+\mathbf{5}^*$ representation, we multiply the final result by a group theory factor $T^aT^b\mathrm{tr}(T^aT^b) = \frac{1}{2}C_f$ where the trace is taken over the messenger fields and C_f is the second order Casimir for the flat direction. We obtain $m_\phi^2 = 2C_f\left(\frac{\alpha}{4\pi}\right)^2\left(\frac{MB}{M}\right)^2$, which agrees with that in Ref. [5]

B Effective Potential and Wave-function Renormalization

When one computes an effective potential, one can determine the location of the minimum. It is well-known that one also needs to evaluate the wave-function renormalization $Z(\phi)(\partial\phi)^2$ in order to discuss the time evolution of the scalar field in general. Fortunately, such a calculation is not necessary in our case.

Let us recall the simple fact that the effective potential in our case is at 2-loop order: $V \sim (\alpha/4\pi)^2$. Since it is a flat direction in the supersymmetric limit, this is the *only* term in the potential. The equation of motion in the flat space is

$$\ddot{\phi} + \frac{Z'(\phi)}{Z(\phi)}(\dot{\phi})^2 + \frac{1}{Z(\phi)}V'(\phi) = 0. \tag{B.1}$$

Here we dropped the friction term $3H\dot{\phi}$, but the essence of the following discussions does not depend on such simplifying assumptions.

Because V' is of order $(\alpha/4\pi)^2$, the motion is suppressed by a power in the coupling constant. Note that $Z'(\phi)$ is *at most* order $(\alpha/4\pi)$. By factoring out the coupling constant factors,

$$V = \left(\frac{\alpha}{4\pi}\right)^2 v(\phi) \tag{B.2}$$

$$Z = 1 + \left(\frac{\alpha}{4\pi}\right) \zeta(\phi), \tag{B.3}$$

we find

$$\ddot{\phi} + \left(\frac{\alpha}{4\pi}\right) \frac{\zeta'(\phi)}{Z(\phi)} (\dot{\phi})^2 + \frac{1}{Z(\phi)} \left(\frac{\alpha}{4\pi}\right)^2 v'(\phi) = 0. \tag{B.4}$$

It is convenient to rescale the time variable t by

$$\tau \equiv \frac{\alpha}{4\pi} t, \tag{B.5}$$

and we find

$$\frac{\partial^2 \phi}{\partial \tau^2} + \left(\frac{\alpha}{4\pi}\right) \frac{\zeta'(\phi)}{Z(\phi)} \left(\frac{\partial \phi}{\partial \tau}\right)^2 + \frac{1}{Z(\phi)} v'(\phi) = 0. \tag{B.6}$$

It is clear that the leading terms in the equation of motion are given by $\partial^2\phi/\partial\tau^2 + v'(\phi) = 0$, and all dependences on the wave function renormalization occur only at higher orders in perturbation theory. Therefore, the calculation of the effective potential is enough for our purpose, and we do not need $Z(\phi)$.

C Time Evolution of the Flat Direction

The evolution of the flat direction ϕ is interesting in the LEGM models. Once the amplitude is dominated by the gauge-mediated piece, the potential is approximately proportional to $(\ln|\phi|^2)^2$, and the dilution of the coherent oscillation occurs much slower than in the parabolic potential case. In this Appendix, we investigate the evolution of the flat direction by using the virial theorem.

The virial theorem tells us that,

$$2\langle K \rangle = \left\langle \frac{\partial V}{\partial \phi} \phi + \frac{\partial V}{\partial \phi^*} \phi^* \right\rangle, \tag{C.1}$$

where $K = \dot{\phi}^* \dot{\phi}$ is the kinetic energy. In our case, $V \sim V_0 (\ln|\phi|^2/\langle S \rangle^2)^2$ with $\langle S \rangle \sim 3 \times 10^4$ GeV, and

$$\langle K \rangle = \left\langle \frac{2}{\ln|\phi|^2/\langle S \rangle^2} V \right\rangle. \tag{C.2}$$

For $\ln|\phi|^2/\langle S \rangle^2 \gg 1$, the energy density of the field is potential dominated.

The field equation is

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi^*} = 0. \tag{C.3}$$

Multiplying it by $\dot{\phi}^*$ and using the energy density $\mathcal{E} = K + V$,

$$0 = \dot{\mathcal{E}} + 6HK \sim \dot{\mathcal{E}} + 6H\frac{2}{\ln|\phi|^2/\langle S\rangle^2}\mathcal{E}. \tag{C.4}$$

Since the energy density is dominated by the potential term, we can write down the approximate evolution equation of the amplitude $|\phi|$,

$$\frac{d|\phi|}{dt} \sim -3H|\phi|, \tag{C.5}$$

and hence

$$|\phi(t)|R(t)^3 \sim \text{constant} \tag{C.6}$$

This formula is valid when $\ln|\phi|^2/\langle S\rangle^2 \gg 1$.

D Estimation of the Primordial Baryon Asymmetry

In this Appendix, we justify Linde's formula, Eq. (5.6) in our case. In fact, the validity of the formula depends on the nature of the operator $\mathcal{O}$ and the time evolution of the flat direction ϕ. We clarify the reason why the formula is valid in the cases of our interest. We start from the equation of motion for the baryon number density, Eq. (5.5),

$$\dot{n}_B + 3Hn_B = i\left(\frac{\partial\mathcal{O}}{\partial\phi}\phi - \frac{\partial\mathcal{O}}{\partial\phi^*}\phi^*\right). \tag{D.1}$$

It is useful to rewrite the equation in terms of baryon-to-entropy ratio $Y_B \equiv n_B/s$, to find

$$\dot{Y}_B = \frac{1}{s}i\left(\frac{\partial\mathcal{O}}{\partial\phi}\phi - \frac{\partial\mathcal{O}}{\partial\phi^*}\phi^*\right), \tag{D.2}$$

where we used the relation $sR^3 =$ constant. Assuming a vanishing initial value $Y_B(t_0) = 0$, we obtain

$$Y_B(\infty) = \int_{t_0}^{\infty} dt\frac{1}{s}i\left(\frac{\partial\mathcal{O}}{\partial\phi}\phi - \frac{\partial\mathcal{O}}{\partial\phi^*}\phi^*\right). \tag{D.3}$$

A crucial question is whether the t integral is dominated at $t \sim t_0$ or $t \sim \infty$.

In the following analysis, we assume the Universe to be radiation dominated when the field begins to roll down the potential, $R \propto t^{1/2}$. Another assumption is that the baryon-number violating operator $\mathcal{O}$ can be treated as a small perturbation to the evolution of the field ϕ.

In the case $\phi_0 \gtrsim \phi_{\rm eq}$, or in the general hidden sector case, we have $\mathcal{O} \propto \phi^4$. The evolution of ϕ is essentially determined by $m_{3/2}^2 \phi^2$ by assumption and hence $\phi \propto R^{-3/2} \propto t^{-3/4}$. On the other hand, $s \propto R^{-3} \propto t^{-3/2}$. The integrand in Eq. (D.3) therefore behaves as $t^{-3/2}$ and hence it is dominated by $t \sim t_0$. By putting them together,

$$\begin{aligned} Y_B(\infty) &= \int_{t_0}^{\infty} dt \frac{1}{s_0} i \left(\frac{\partial \mathcal{O}}{\partial \phi} \phi - \frac{\partial \mathcal{O}}{\partial \phi^*} \phi^* \right) \frac{t^{-3/2}}{t_0^{-3/2}} \\ &= \frac{1}{2} t_0 \times \frac{8 m_{3/2}^2 \mathrm{Im}(\phi_0^4)}{s_0 M_*^2} = \frac{2 m_{3/2}^2 \mathrm{Im}(\phi_0^4)}{s_0 M_*^2 H_0}, \end{aligned} \tag{D.4}$$

which essentially justifies Eq. (5.6).

In the other case of interest, $\phi_0 \lesssim \phi_{\rm eq}$, both the behavior of the operator and time-evolution are completely different as discussed in detail in Section 5. We have $\mathcal{O} \propto \phi^2$, while $\phi \propto R^{-3} \propto t^{-3/2}$. Then the integrand behaves as $t^{-3/2}$, which is unexpectedly the same as in the previous case. By putting them together, we obtain

$$\begin{aligned} Y_B(\infty) &= \int_{t_0}^{\infty} dt \frac{1}{s_0} i \left(\frac{\partial \mathcal{O}}{\partial \phi} \phi - \frac{\partial \mathcal{O}}{\partial \phi^*} \phi^* \right) \frac{t^{-3/2}}{t_0^{-3/2}} \\ &= \frac{1}{2} t_0 \times \frac{8 V_0 \mathrm{Im}(\phi_0^4)}{s_0 |\phi_0|^2 M_*^2} = \frac{2 V_0 \mathrm{Im}(\phi_0^4)}{s_0 |\phi_0|^2 M_*^2 H_0}, \end{aligned} \tag{D.5}$$

which again essentially justifies Eq. (5.6).

As it is clear from above the derivations, Eq. (5.6) is not necessarily valid if the integral is dominated at $t \sim \infty$ rather than $t \sim t_0$. We have not seen an explicit discussion on this point in the literature.

E Affleck-Dine Baryogenesis in Hidden Sector Scenario

In this appendix, we discuss the Affleck-Dine baryogenesis based on the scenario with SUSY breaking in the hidden sector. In this case, the gravitino mass is much larger than in the LEGM case, and all the scalar fields also have the SUSY breaking masses of the order of the gravitino mass.* In particular, the potential for the flat direction is always given by the supergravity contribution, which is essentially parabolic with a curvature of the order of the gravitino mass,

$$V(\phi) \sim m_{3/2}^2 |\phi|^2, \tag{E.1}$$

with $m_{3/2} \sim 1$ TeV. Due to this fact, the evolution of the flat direction is much simpler than in the LEGM case.

*In this Appendix, we denote all the soft SUSY breaking masses for the scalar fields by $m_{3/2}$ for simplicity.

Even if the gravitino mass is about 1 TeV, Eq.(5.6) is still valid since the baryon number is generated when ϕ starts to oscillate. With the baryon number violating operator (5.4), we obtain

$$n_B|_{H\sim m_{3/2}} \sim \frac{m_{3/2}|\phi_0|^4}{M_*^2}\sin 4\theta_0. \tag{E.2}$$

and hence

$$\left.\frac{n_B}{s}\right|_{H\sim m_{3/2}} \sim g_*^{-1/4}\frac{|\phi_0|^4}{m_{3/2}^{1/2}M_*^{7/2}}\sin 4\theta_0. \tag{E.3}$$

If there is no entropy production after this stage, the above formula gives us the resulting baryon-to-entropy ratio.

If there is entropy production, the primordial baryon number is diluted. The primary source of the entropy is the decay of the flat direction. Here, the potential for the flat direction is always parabolic, and ϕ starts to oscillate when $T = T_0 \sim g_*^{-1/4}\sqrt{m_{3/2}M_*}$, as discussed in Section 5. Then, by using the relation $|\phi|^2T^{-3}$ = const., the background temperature at the ϕ decay is given by

$$T_{\rm dec} \sim g_*^{-1/4}\sqrt{m_{3/2}M_*}\left(\frac{\phi_{\rm dec}}{|\phi_0|}\right)^{2/3}, \tag{E.4}$$

where $\phi_{\rm dec}$ is the amplitude of the flat direction when it decays. Furthermore, the reheating temperature due to the decay of ϕ, $T_{\rm R}$, is given by

$$T_{\rm R} \sim g_*^{-1/4}\sqrt{m_{3/2}\phi_{\rm dec}}. \tag{E.5}$$

Then, the dilution factor is given by

$$D \sim \frac{T_{\rm R}^3}{T_{\rm dec}^3} \sim \frac{|\phi_0|^2}{\phi_{\rm dec}^{1/2}M_*^{3/2}}. \tag{E.6}$$

Usually, ϕ decays when the expansion rate of the Universe, H, becomes comparable to the decay rate of ϕ, Γ_ϕ. In Ref.[11], Γ_ϕ is estimated as

$$\Gamma_\phi \sim \left(\frac{\alpha_s}{\pi}\right)^2\frac{m_{3/2}^3}{|\phi|^2}, \tag{E.7}$$

and hence $H \sim \Gamma_\phi$ results in

$$\phi_{\rm dec} \sim \left(\frac{\alpha_s}{\pi}\right)^{2/3} m_{3/2}^{2/3}M_*^{1/3}. \tag{E.8}$$

Combining the above results, we obtain

$$\frac{n_B}{s} \sim \left(\frac{\alpha_s}{\pi}\right)^{1/3} \left(\frac{M_*}{m_{3/2}}\right)^{1/6} \left(\frac{|\phi_0|}{M_*}\right)^2 \sin 4\theta_0 = 120 \times \left(\frac{|\phi_0|}{M_*}\right)^2 \left(\frac{\alpha_s}{0.1}\right)^{1/3} \left(\frac{m_{3/2}}{1\ \mathrm{TeV}}\right)^{-1/6} \sin 4\theta_0. \tag{E.9}$$

Another potential source of entropy is the Polonyi field related to the SUSY breaking, or the moduli fields in the string theory, which also have masses of order $m_{3/2}$. The critical difference between the flat direction and the Polonyi field z is the formula of their decay width; since the Polonyi field couples to particles in the observable sector only through interactions suppressed by the gravitational scale, its decay width Γ_z is much smaller than the width of ϕ. As discussed in Section 6, Γ_z is estimated as

$$\Gamma_z \sim \frac{N_{\mathrm{ch}}}{4\pi} \frac{m_{3/2}^3}{M_*^2}. \tag{E.10}$$

Even with this decay rate, we can apply an argument similar to the case of the entropy production due to ϕ decay; Eqs.(E.2) and (E.3) are still valid, and we also obtain equations similar to Eqs.(E.4) – (E.6) where ϕ's are replace by z's. The remainder is to evaluate the amplitude of z at its decay time, z_{dec}, by using the relevant formula for Γ_z. By solving the equation $H \sim \Gamma_z$ with $H \sim m_{3/2} z/M_*$, we obtain

$$z_{\mathrm{dec}} \sim \frac{m_{3/2}^2}{M_*}. \tag{E.11}$$

Then, assuming the initial amplitude of z to be $z_0 \sim M_*$, the dilution factor is given by $D \sim M_*/m_{3/2}$, and hence

$$\frac{n_B}{s} \sim D^{-1} \frac{m_{3/2} \mathrm{Im}(\phi_0^4)}{M_*^2} \sim \left(\frac{m_{3/2}}{M_*}\right)^{1/2} \left(\frac{|\phi_0|^4}{M_*^4}\right) \sin 4\theta_0. \tag{E.12}$$

Thus, the baryon-to-entropy ratio may be larger than $\sim 10^{-10}$ even after the decay of the Polonyi field.

However the reheating temperature after the decay of z is likely to be too low. By using Eq. (E.10), the reheating temperature is estimated as

$$T_{\mathrm{R}} \sim g_*^{-1/4} \sqrt{\Gamma_z M_*} \sim 1\mathrm{MeV} \times \left(\frac{m_{3/2}}{10\mathrm{TeV}}\right)^{3/2}. \tag{E.13}$$

Thus, if the gravitino mass is heavier than about 10 TeV, the Polonyi field may decay before the big-bang nucleosynthesis (BBN), and the scenario which gives Eq.(E.12) may be viable.† However, for a favorable range of the gravitino mass ($m_{3/2} \lesssim$ 1TeV), the reheating

†In fact, even if $m_{3/2} \gtrsim$ 10TeV, there may still be a problem since the lightest superparticle produced by the decay of the Polonyi field may overclose the Universe [31, 35]. To solve this difficulty, we may have to accept a much larger gravitino mass, or a scenario in which the lightest superparticle in the MSSM sector is unstable.

temperature is less than 100 keV which is lower than the temperature where the big-bang nucleosynthesis (BBN) starts. This means that the decay of z significantly affects the results of the standard BBN scenario. In this case, we need some mechanism to reduce the energy density of the Polonyi field. A thermal inflation [12] is an interesting candidate for it. The baryon-to-entropy ratio in this case is discussed in Section 6. By using the fact that the ratio of n_B to ρ_z is constant in time, we obtain

$$\frac{n_B}{s} \sim \frac{\rho_z}{s} \times m_{3/2}^{-1} \left(\frac{|\phi_0|}{M_*} \right)^4 \sin 4\theta_0. \tag{E.14}$$

Thus, once we fix the ratio ρ_z/s after the late inflation, the baryon-to-entropy ratio can be estimated.

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