O.R. Applications

Retail price markup commitment in decentralized supply chains

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Abstract

We investigate the operational decisions and resulting profits for a supply chain facing price-dependent demand under a policy where there is an ex-ante commitment made on the retail price markup. We obtain closed-form solutions for this policy under the assumption of a multiplicative demand function and we analytically compare its performance with that of a traditional price-only policy. We compare these results to results obtained when demand follows a linear additive form. These formulations are shown to be qualitatively different as the manufacturer’s wholesale pricing decision is independent of the retail price markup commitment in the multiplicative case, but not when demand is linear additive. We demonstrate that the ex-ante commitment can lead to Pareto-improving solutions under linear additive demand, but not under the multiplicative demand function. We also consider the effect of pricing power in the supply chain by varying who determines the retail price markup.

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1. Introduction

Many traditional operations management models assume that retail prices are set exogenously. This can be a very limiting assumption that is not reflected in many industries where oligopolies and non-perfect market conditions exist. In many scenarios price is a decision variable that can be used to influence demand and, by extension, operational costs and revenues. Recent advances in research relating pricing and inventory decisions in decentralized models have been put forth by Emmons and Gilbert (1998), Wang et al. (2004), Cachon and Lariviere (2005) and others. We add to this body of research by examining the effect of ex-ante commitments to retail price markup in decentralized supply chains facing price-dependent demand. We also extend this area of investigation to include the effect of the division of pricing power between players in the supply chain, where pricing power refers to the player’s ability to influence the price markup in the supply chain.

We develop models for a retail fixed markup (RFM) policy where an ex-ante commitment is made to the retail price markup before wholesale prices have been set. Such a policy may arise in practice for a variety of reasons. Some retailers choose to follow a fixed markup pricing policy for convenience. Many retailers believe that there is a minimum markup over wholesale price that they must receive in order to be profitable. In order to keep the retail price as low as possible, the retailer may choose to simply increase the wholesale price by this minimum markup. Furthermore, it can be difficult for a retailer with thousands of products in stock to investigate the demand curve and find...
the optimal retail price for each product (although such initiatives for supply chain coordination are gaining momentum). Thus, the retailer may simply apply a fixed markup to a category of products, which is either known or can be historically deduced by the manufacturer (e.g., a “keystone markup” where a retailer simply doubles the cost to arrive at the retail price, see American Marketing Association, 2004). In fact, Monroe (1990) claims that fixed markup pricing is the most common form of pricing used by retailers. Retail markups are directly regulated in some countries, for example in pharmaceuticals in India and some countries in Europe (see Kanavos and Reinhardt, 2003 and PMPRB, 2002).

Fixed markups may also be imposed by a powerful manufacturer in the supply chain as a form of vertically restrictive pricing. Vertically restrictive pricing involves the influence of retail pricing by upstream supply chain players. Resale price maintenance (RPM), minimum advertised pricing (MAP) and manufacturer suggested retail prices (MSRP) are common examples of the use of vertically restrictive pricing in practice. The legality of such practices is an open source of debate, but there is ample evidence that such agreements exist and that their use is even protected by law in many instances. While vertically restrictive pricing can be questioned as violating antitrust laws, the Federal Trade Commission states that

The antitrust laws, however, give a manufacturer latitude to adopt a policy regarding a desired level of resale prices and to deal only with retailers who independently decide to follow that policy. A manufacturer also is permitted to stop dealing with a retailer who breaches the manufacturer’s resale price maintenance policy (see http://www.ftc.gov/bc/compguide/illegal.htm).

A variety of court cases have also been used to defend the practice of vertically restrictive pricing including United States v. Colgate & Co., State Oil Co. v. Khan and Atlantic Richfield Co. v. USA Petroleum Co., and cases involving Monsanto and Business Electronics (see Arquit, 1991 and Klein, 1999). Models and analysis have also shown that vertically restrictive pricing agreements can reduce inefficiencies in the supply chain by limiting what is known as the ‘free-rider’ ability of retailers – the tendency for retailers to enjoy the benefits of supplier efforts such as marketing campaigns and building brand identity without proper compensation. Several of these studies are summarized by a report from the Organisation for Economic Cooperation and Development published in 1976. Gurnani and Xu (2006), Deneckere et al. (1996) and Liu et al. (2006) provide examples of price-control agreements used in practice from industries such as software, grocery items and gasoline resellers.

In this paper, we examine several forms of fixed markup pricing policies, including cases where the price markup is chosen by the retailer, by the manufacturer and through strategic negotiation. This allows us to investigate both the effect of the overall ex-ante commitment to a price markup and the effect of pricing power, or who controls the ability to choose the price markup. In our framework, a price-only contract represents one extreme where the retailer has full control to pick retail price after observing wholesale price. A scenario where the manufacturer chooses the wholesale price and price markup, and hence the retail price, represents another extreme where the manufacturer has considerable power to control pricing. An intermediate scenario is represented by the situation where the retailer chooses the ex-ante price markup, which is known to the manufacturer, and the manufacturer responds with a wholesale price. We examine all of these scenarios in our analysis, as well as scenarios where the markup is chosen through strategic negotiation to characterize additional divisions of pricing power (see Srivastava et al., 2000 for a discussion on modeling price and margin negotiations in the supply chain). In particular, we seek to answer the following questions in this research: (1) What is the effect of an ex-ante commitment to retail price markup and how does this affect the overall supply chain and individual agents’ performance? (2) What effect does the form of the demand function have on performance? (3) What effect does the division of relative pricing power have on supply chain and individual agent performance?

Our analysis shows that both supply chain and individual agent performance under an ex-ante commitment to retail price markup (as represented by RFM) is highly dependent on the functional form of demand. RFM results in minimal improvement over traditional price-only contracts when demand follows a multiplicative form and one agent controls price markup. However, if pricing power is more evenly distributed between retailer and manufacturer (i.e., neither can select the retail markup unilaterally), performance improvement can be greater. Furthermore, if demand follows a linear additive form, then the performance improvement can be quite significant. We show that Pareto-improving solutions, where both the retailer and the manufacturer are better off under RFM than under price-only, are possible only when demand follows a linear additive demand function. This difference in agent performance under the two demand formulations can be linked to the fact that the manufacturer’s wholesale pricing decision is independent of the retail price markup commitment in the multiplicative case, but not when demand is linear additive. Thus, adjusting the fixed markup commitment value in the linear additive demand case forces the manufacturer to choose different wholesale prices which can lead to Pareto-improving solutions under RFM, but this does not happen when demand is multiplicative and iso-price-elastic. One consequence of our findings is that the retailer may, in some situations, be better off revealing certain information (e.g., revealing her price markup if demand is linear additive), but in other situations (when demand is multiplicative and iso-price-elastic) the retailer will prefer to keep this information hidden.

The remainder of this paper is organized as follows: in Section 2, we review the related literature; in Sections 3
and 4, we analyze models under multiplicative and linear additive demand functions; conclusions and future research are summarized in Section 5.

2. Literature review

In this section we summarize the existing research that is most closely related to our work. We begin with a general review of price-dependent demand models where both order quantity and price are decision variables. We then discuss some related models that consider decentralized supply chains, but where retail price is exogenous. We also list some existing coordination mechanisms for both exogenous and endogenous pricing models and we close with an overview of the existing literature that considers power issues in the supply chain.

Models where both retail price and order quantity are decision variables have become increasingly popular in operations management research. These models assume that demand is a function of price, generally in what is referred to as a multiplicative form (commonly defined as $D(p) = Ap^{-b}e$ where $D(\cdot)$ is demand, $p$ is retail price, $A$ and $b$ are given parameters and $e$ is a random variable) or a linear-additive form (e.g., $D(p) = A - bp + e$). The first such example is Whitin (1955) who examines a linear-additive functional demand form. Whitin's work is extended by Mills (1959) who also considers a linear-additive functional form for demand and shows that the optimal price is lower in the presence of demand uncertainty than when demand is deterministic. Thowsen (1975) and Lau and Lau (1988) present related works for linear additive functional forms. Contrary to Mills' finding, Karlin and Carr (1962) examine a multiplicative demand form and show that the optimal price under uncertainty is higher than when demand is deterministic. Zabel (1970) also considers a multiplicative form of demand and examines the specific cases when the random component of demand is uniformly and exponentially distributed. Zabel (1972) extends this work to a multi-period setting. Federgruen and Heching (1999) focus on multi-period models for centralized systems facing price-dependent demand and Dana and Petruzzi (2001) examine a scenario where demand can depend on both price and inventory level.

Several researchers have more thoroughly examined the contradictory findings for the effect of uncertainty on optimal price under linear-additive and multiplicative demand forms. Young (1978) verifies the findings of Mills and Karlin and Carr. Petruzzi and Dada (1999) provide an excellent summary of related models as well as probably the best insight into why the different functional forms of demand lead to different behavior of optimal retail prices. Petruzzi and Dada explain that the differences arise due to the different effects price has on variation under multiplicative and linear-additive demand (see Petruzzi and Dada, 1999, p. 187). While we also are interested in observing what effect different functional forms of demand have on our models, we are most interested in decentralized supply chains where each player operates independently.

Many researchers have examined decentralized supply chains under exogenous retail pricing. Tsay et al. (1999), Cachon (2003) and Yano and Gilbert (2005) provide extensive reviews of the related literature, which includes papers that examine coordinating agreements such as buyback contracts (Pasternack, 1985; Kandel, 1996), quantity discounts (Jeuland and Shugan, 1983), quantity flexibility agreements (Anupindi and Bassok, 1999) and others. However, as Cachon (2003) notes, most of these agreements fail to coordinate the supply chain under price-dependent demand. Several policies that have been shown to provide channel coordination under price-dependent demand are discussed below; however, there are significant difficulties in implementation and enforcement of these policies.

Weng (1997) presents one of the first agreements that can coordinate the supply chain under price-dependent demand. The author proposes a two-part tariff to coordinate a supply chain subject to information asymmetry. Hu (2001) proposes three different coordination policies under price-dependent demand: quantity fixing, two-part linear pricing and a return policy with price fixing. Yao et al. (2004) introduce a contract that can coordinate the channel, but it requires wholesale price to be a function of retail price coupled with a buyback policy where the buyback value is also a function of retail price. Bernstein and Federgruen (2005) consider a multiple retailer setting. The authors show that it is not possible to coordinate the supply chain with a buyback policy and constant wholesale price, but they introduce a “price-discount sharing” policy that can coordinate the channel where the buyback value is a function of retail price. While these contracts are shown to achieve channel coordination, most are difficult to implement and/or expensive to monitor. For example, two-part tariffs assign risk solely to one player to achieve coordination; thus, they are typically not self-enforcing since the manufacturer always has incentive to charge a wholesale price higher than marginal cost. Marvel and Peck (1995) claim that such difficulties limit the use of two-part tariffs in practice. Return (i.e., buyback) and revenue sharing policies may also be problematic as they require the manufacturer to monitor retail sales, which can be quite difficult and expensive. Cachon and Lariviere (2005) specifically mention that a significant limitation of revenue sharing is “the administrative burden it imposes on the firms” and that this burden may explain why it is not more widely implemented in industry.

Our goal is not to design a contract that will necessarily coordinate the channel. We examine ex-ante markup commitment through an RFM policy because: (1) such policies exist in practice and we wish to see what effect they have on the supply chain; (2) RFM is easy to implement and verify for both players since it requires the manufacturer to observe only retail price and not retail inventory or sales; and (3) our results indicate that, in many scenarios, RFM can result in Pareto-improving solutions with significant
supply chain savings. Other researchers have also chosen to focus on agreements that can greatly improve supply chain efficiency, but do not lead to channel coordination. This includes Emmons and Gilbert (1998) who consider a return policy under price-dependent demand with a multiplicative demand function where the mean demand is defined as $a(p - k); p$ is the retail price, $a$ and $k$ are given parameters. They argue that the buyback will benefit both players when the wholesale price is higher than some threshold value. Granot and Yin (2005) extend the model proposed by Emmons and Gilbert and analytically prove that the performance of a buyback contract is bounded at 75% of the efficiency of an integrated scenario and that a buyback contract offers only limited improvement over price-only in this scenario. Raz and Porteus (2003) compare price-only, buyback and revenue sharing agreements for a decentralized system facing a demand distribution where there are a discrete number of market states and demand in each state is piece-wise linear. Song et al. (forthcoming) analyze a return policy under relatively general demand functions. Under multiplicative demand and constraints on demand elasticity, they show that the profit functions are well behaved and that unique optimal solutions are obtained for both the manufacturer and the retailer in a price-only scenario. Interestingly, the authors show that the optimal return policy is equivalent to a price-only contract under an iso-elastic demand function. Our findings are similar for the RFM policy when the retailer can control the price markup and demand follows an iso-price-elastic, multiplicative form. Wang et al. (2004) consider a consignment contract with revenue sharing. The consignment contract they consider is similar in some respects to the RFM contracts considered here; however, their consignment contracts require the retailer to pay only for items sold to the end-user, while our RFM models require the retailer to pay for all items procured from the supplier. Furthermore, we consider aspects related to pricing power, which are not considered in Wang et al.’s models. Similar to our work, Wang et al. consider both multiplicative and linear additive demand forms. However, they find that the functional form of demand has little effect on the outcome of their suggested policies, while we show that the form of the demand function greatly affects the performance of RFM. Liu et al. (2006) also consider RFM policies, but they consider only linear additive demand models and they focus on RFM as a mechanism for enforcing vertically restrictive pricing policies. We expand on Liu et al. (2006) by considering both linear additive and multiplicative demand functions and by investigating the effect of pricing power in the supply chain. In contrast to Liu et al. (2006), our models do not include holding cost and penalty cost terms, which allows us to generate additional analytical results and insights.

One critical issue in the development and implementation of supply chain policies is the idea of relative “power” in the supply chain. As Cachon (2003) notes, “Power, like beauty, can be in ‘the eye of the beholder’, or it can be more concrete.” Here, we attempt to make the consideration of a particular type of supply chain power, which we refer to as “pricing power”, somewhat more concrete. Lee and Staelin (1997) provide a general framework on the relationship of vertical strategic interaction and the optimality of channel price leadership, which is related to our definition of pricing power in the supply chain. The authors examine a special case of RFM where the retailer chooses the ex-ante price markup. We relax this by allowing the manufacturer or joint decision-making to choose the markup, but we do not consider horizontal competition as in some of their models. Lee and Staelin claim that price leadership is valuable under vertical strategic substitutability (which in our model translates into a linear demand function under), but that a price leader is worse off under vertical strategic complementarity (the multiplicative demand function in our model). The authors show that the form of the demand function and the resulting channel interactions are crucial to channel performance; however, Lee and Staelin limit their analysis to a deterministic demand setting. We extend their models by including stochastic demand and by examining a specific form of vertical interaction, namely ex-ante markup commitment through RFM. Our results verify that the functional form of demand greatly influences the performance of the entire supply chain as well as the individual agents, a counterpart to their findings under deterministic demand. Lariviere and Porteus (2001) also discuss retail power, but they define it as the ability to extract a given reservation utility, which is different than our consideration here. Furthermore, retail price is considered to be exogenous in their models and demand is assumed to be independent of retail price. We develop and use the phrase “pricing power” to refer to the ability of an agent in the supply chain to control the retail price markup.

3. Model and analysis under multiplicative demand function

In this paper, we consider a single period model of a supply chain with one manufacturer and one retailer. Because we are most concerned with short life cycle products, we assume there is no salvage value after the sales season ends (and no returns are allowed between the retailer and the supplier), no holding costs are charged on leftovers, and no additional penalty costs are incurred for lost sales other than lost revenue (see Wang et al., 2004 for similar assumptions). There is a linear cost of manufacturing incurred by the manufacturer and the wholesale cost for the retailer is independent of order quantity. Both players in the supply chain are risk neutral and have full and symmetric information on demand and costs.

We use the following notation in our model description:

\[ p = \text{retail price per unit}, \, p > 0; \]
\[ w = \text{wholesale price per unit}, \, c < w < p; \]
\[ c = \text{manufacturing cost per unit}, \, 0 < c < w; \]
\[ A = \text{the deterministic portion of demand}, \, A > 0; \]
$b$ = the elasticity (multiplicative case) or sensitivity (linear additive case) of demand to price, $b > 0$ for linear additive case and $b \geq 2$ for multiplicative case to avoid trivial results;

$\epsilon$ = the nonnegative random variable of the demand with PDF $f(\cdot)$ and CDF $F(\cdot)$. We assume $\epsilon$ has non-decreasing failure rate, $r(\epsilon) = \frac{f(\epsilon)}{1-F(\epsilon)}$;

$q$ = the retailer’s order size;

$z$ = the stocking factor for the stochastic demand $\epsilon$;

$x$ = the markup of the retailer under RFM, such that $x = 1 - w/p$.

$\Pi_i$ = the expected profit of agent $j$ in supply chain type $i$. The meanings of $i$ and $j$ are summarized in the tables below.

<table>
<thead>
<tr>
<th>Value of $i$</th>
<th>Supply chain type</th>
<th>Value of $j$</th>
<th>Agent</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Integrated</td>
<td>M</td>
<td>Manufacturer</td>
</tr>
<tr>
<td>P</td>
<td>Price-only</td>
<td>R</td>
<td>Retailer</td>
</tr>
<tr>
<td>R</td>
<td>RFM</td>
<td>Omitted</td>
<td>Entire supply chain</td>
</tr>
</tbody>
</table>

In this section, we assume that demand follows an iso-price-elastic, multiplicative functional form such that $D(p) = Ap^{-b}\epsilon$, where $A > 0$, $b > 2$ (see Petruzzi and Dada, 1999 and Wang et al., 2004 for similar demand models).

3.1. Analytical results for integrated supply chain scenario

We first develop a model for the integrated supply chain, which we use as a benchmark. In an integrated supply chain, a single decision maker determines the optimal price and order quantity simultaneously. Thus, the objective function for the integrated supply chain is

$$\Pi_I(p, q) = pE[\min(q, Ap^{-b}\epsilon)] - eq.$$  \hspace{1cm} (1)

Following Petruzzi and Dada (1999), we use the concept of a stocking factor, $z \equiv \frac{b}{A\epsilon}$. Substituting the stocking factor, $z$, into (1), the objective function is

$$\Pi_I(p, z) = Ap^{-b}[(p - c)z - pA(z)]$$  \hspace{1cm} (2)

where $A(z) = \int_0^z (z-x)f(x)dx$. Petruzzi and Dada (1999) and Wang et al. (2004) analyze the integrated supply chain case under the same demand function. Thus, we borrow their results (particularly Theorem 1 from Wang et al.) to state the following.

**Lemma 1.** For any fixed $z$, the unique optimal price is $p^*(z) = \frac{bwz}{b - 1 - z - A(z)}$.

**Theorem 1.** When $\epsilon$ follows a non-decreasing failure rate distribution, there is a unique optimal stocking factor, $z^*$, which maximizes the expected profit of the supply chain and where

$$F(z^*) = \frac{z^* + (b-1)A(z^*)}{bz^*}.$$  \hspace{1cm} (3)

3.2. Analytical results for supply chain under price-only contract

In the decentralized supply chain under a price-only contract, the manufacturer first offers the wholesale price, $w$, to the retailer. The retailer then determines the order size ($q$) and retail price ($p$). Since both players possess full and symmetric information regarding costs and demand, this process is a Stackelberg game in which the manufacturer is the leader and the retailer is the follower. The objective functions of the manufacturer and the retailer, respectively, are

$$\Pi_{Pm}(w) = (w-c)q$$  \hspace{1cm} (4)

$$\Pi_{Pq}(p, q) = pE[\min(q, Ap^{-b}\epsilon)] - wq.$$  \hspace{1cm} (5)

Substituting the stocking factor, $z$, into (5), yields

$$\Pi_{Pq}(p, z) = Ap^{-b}[(p - w)z - pA(z)].$$  \hspace{1cm} (6)

From (6), the retailer’s profit function has the same structure as the integrated supply chain in Eq. (2). Thus, we introduce the following **Corollary 1.**

**Corollary 1.** When demand follows a multiplicative form with iso-price-elasticity and $\epsilon$ has a non-decreasing failure rate,

(1) For any given stocking factor $z$, there is a unique optimal retail price determined by

$$p^*(z) = \left(\frac{bw}{b - 1}\right)\left(\frac{z}{z - A(z)}\right);$$  \hspace{1cm} (7)

(2) There is a unique stocking factor, $z^*$, that optimizes the retailer’s expected profit, which is the same as in the integrated supply chain determined by (3). Thus, $z^*$ is independent of wholesale price, $w$.

**Proof.** Similar to that of Lemma 1 and Theorem 1.

It is interesting to note that the retailer will select the same stocking factor as is chosen in the integrated supply chain. However, in a decentralized setting, the retailer will select a retail price higher than that in an integrated channel. □

We are next interested in determining the optimal wholesale price for the manufacturer under a price-only contract. By considering the retailer’s optimal response, the manufacturer’s profit function is

$$\Pi_{Pm}(w) = Az^*\left[\left(\frac{b}{b - 1}\right)\left(\frac{z^*}{z^* - A(z^*)}\right)\right]^{-b} \left[w^{-b}(w - c)\right]$$  \hspace{1cm} (8)

where $z^*$ is the retailer’s choice of stocking factor, which is not a function of the wholesale price. In (8), the first term, $Az^*\left[\left(\frac{b}{b - 1}\right)\left(\frac{z^*}{z^* - A(z^*)}\right)\right]^{-b}$, is independent of wholesale price. Thus, the manufacturer can only influence the second term of the profit function – a result that leads us to Theorem 2.
Theorem 2. In the decentralized supply chain under a price-only contract, there is a unique optimal wholesale price to maximize the manufacturer’s profit, which is given by $w^* = \frac{bc}{b-1}$.

Proof. It is easy to verify that the manufacturer’s profit function is unimodular. Thus, there is a unique optimal solution found by solving the first order condition. □

From Theorem 2 the unit profit margin of the manufacturer is $\frac{bc}{b-1}$, which is equivalent to the deterministic case. Thus, the manufacturer’s pricing decision is not influenced by demand uncertainty if demand is multiplicative and iso-price-elastic. However, he will encounter a lower order quantity in the stochastic case since the retailer will always charge a higher retail price under uncertainty than when demand is deterministic (the equivalent result was first shown for an integrated supply chain by Karlin and Carr, 1962). Therefore, the manufacturer and the retailer share the risk and costs of uncertainty through smaller order quantities.

3.3. Analytical results for supply chain under RFM policy

In this section, we examine the RFM policy, where the retail price is a direct function of the wholesale price and a fixed markup, i.e., $w = p(1 - z)$. Lee and Staelin (1997) investigate a similar model, but only under deterministic demand. Liu et al. (2006) examine an RFM policy, but investigate a similar model, but only under deterministic price-only contract, which are also independent of price. Since there is a unique optimal stocking factor for the retailer, we can determine the manufacturer’s optimal price by examining his response function. The response function is obtained by substituting the optimal stocking factor into the first order condition for the manufacturer under both price-only and RFM.

Theorem 3. Under an RFM policy, when demand follows a multiplicative form with iso-price-elasticity, there is a unique optimal wholesale price, $w^* = \frac{bc}{b-1}$, that maximizes the manufacturer’s profit, and the corresponding retail price is $p^* = \frac{bc}{(1 - z)(b - 1)}$.

Proof. From (13), the only portion of $\Pi_{RM}(p)$ dependent on retail price is $T(p) = p^b F^{-1}(z)(1 - z)p - c$. The first-order condition for $T(p)$ is

$$\frac{dT(p)}{dp} = p^{b-1}[(1 - z)(1 - b)p - bc] = 0.$$  (14)

The solution to (14) is $p^* = \frac{bc}{(1 - z)(b - 1)}$. The second derivative of $T(p)$ at $p^*$ is

$$\frac{d^2T(p)}{dp^2} \bigg|_{p=p^*} = -\frac{(b - 1)^2(1 - z)^2}{bc} \left(\frac{bc}{(1 - z)(b - 1)}\right)^{-b} < 0.$$  (15)

$T(p)$ is increasing for $p < p^*$ and decreasing for $p > p^*$. Thus, it is unimodular and $p^*$ is the unique optimal solution. The resulting wholesale price is $w^* = p^*(1 - z) = \frac{bc}{b-1}$. □

At optimality, the manufacturer’s profit and the expected profit at the retailer, respectively, are

$$\Pi_{RM}^* = \frac{Ac}{b-1} \left(\frac{bc}{b-1}\right)^{-b} F^{-1}(z)(1 - z)^b \quad \text{and} \quad (16)$$

$$\Pi_{RR}^* = A \left(\frac{bc}{b-1}\right)^{1-b} \left[z F^{-1}(z) - A(F^{-1}(z))\right](1 - z)^{b-1}. \quad (17)$$

Note that the wholesale price charged by the manufacturer under RFM is the same as the wholesale price charged under a price-only contract. Thus, the manufacturer will earn the same unit margin under RFM as under price-only; RFM changes only the retailer’s stocking factor and corresponding order quantity. Since the unit margin is the same for the manufacturer under both price-only and RFM.
polices, the manufacturer will prefer RFM only if it leads to a higher order quantity, which generally means a lower retail price and a lower markup for the retailer. However, the lower markup will discourage the retailer from ordering more inventory to cover the uncertain demand. Thus, even if the manufacturer has considerable pricing power and can move more inventory to cover the uncertain demand, the lower markup will discourage the retailer from ordering a lower markup for the retailer. However, to a higher order quantity, which generally means a lower markup. Similarly, a powerful retailer will balance the tradeoffs from high and low values of markup. Thus, even if the retailer unilaterally sets the optimal markup, the values $x_M$ and $x_R$ found in Theorem 4 do not necessarily optimize total supply chain profits. Theorem 4 pertains to a supply chain in which either the manufacturer or the retailer holds complete (or nearly complete) pricing power and thus can solely set the retail price markup. In the following section, we show that supply chain performance can be improved by distributing the relative pricing power between the manufacturer and the retailer (i.e., choosing the retail markup through strategic negotiation). Also note that $x_M$ and $x_R$ depend on the distribution of the random variable. Since a closed-form expression does not exist for the optimal stocking factor $(z^*)$ under a price-only contract, we limit our analysis in the following section to a uniform distribution for $e$, and we conduct numerical studies to compare the performance of the different pricing policies.

### 3.4. Supply chain performance under uniform distribution

In this section, we assume that the random component of demand follows a uniform distribution, $e \sim U(0, M)$, where $M > 0$ is the upper bound of the uniform distribution (see Emmons and Gilbert (1998) and Wang et al. (2004) for similar assumptions). Thus, $f(x) = 1/M$, $F(x) = x/M$, and $A(x) = x^2/2M$. We first consider the scenario where the manufacturer is able to choose the retailer’s markup as in a vertically restrictive pricing policy. We then allow the markup to be chosen by the retailer or through a joint negotiation process.

In Table 1, we demonstrate the optimal policies for the integrated and decentralized supply chains when the random component of demand follows a uniform distribution and the manufacturer selects the fixed markup value (the optimal markup, $x_M^*$, is found directly from Theorem 4, Part 1). From the results in Table 1, we can compare the retailer prices, stocking factors, profits of the manufacturer, the retailer, and the entire supply chain under price-only and RFM. This is done in the following Propositions 1 and 2.

**Proposition 1.** When $e \sim U(0, M)$ and the manufacturer selects the optimal markup, $x_M^* = 1/(b + 1)$, unilaterally:

1. the optimal stocking factors are the same under the price-only and the integrated scenarios;
2. the optimal retail prices are the same in the RFM and the integrated scenarios.

**Proof.** Follows directly from results in Table 1. □
Table 1
Optimal decisions when $\varepsilon \sim U(0, M)$

<table>
<thead>
<tr>
<th>Optimal decisions</th>
<th>Integrated</th>
<th>Price-only</th>
<th>RFM [$a^*_M = 1/(b + 1)$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wholesale price, $w^*$</td>
<td>–</td>
<td>$bc(b - 1)$</td>
<td>$bc(b - 1)$</td>
</tr>
<tr>
<td>Retail price, $p^*$</td>
<td>$(b + 1)(b - 1)$</td>
<td>$b(b + 1)\varepsilon/(b - 1)^2$</td>
<td>$b(b + 1)$</td>
</tr>
<tr>
<td>Stocking factor, $z^*$</td>
<td>$2M(b + 1)$</td>
<td>$2M(b + 1)$</td>
<td>$2M(b + 1)$</td>
</tr>
<tr>
<td>Profit–supply chain</td>
<td>$(2aM[b(b + 1)]/(b + 1) - b)^{1/b}$</td>
<td>$(2aM[b(b + 1)]/(b + 1) - b)^{1/b}$</td>
<td>$(2aM[b(b + 1)]/(b + 1) - b)^{1/b}$</td>
</tr>
<tr>
<td>Manufacturer</td>
<td>–</td>
<td>$(2aM[b(b + 1)]/(b + 1) - b)^{1/b}$</td>
<td>$(2aM[b(b + 1)]/(b + 1) - b)^{1/b}$</td>
</tr>
<tr>
<td>Retailer</td>
<td>–</td>
<td>$(2aM[b(b + 1)]/(b + 1) - b)^{1/b}$</td>
<td>$(2aM[b(b + 1)]/(b + 1) - b)^{1/b}$</td>
</tr>
</tbody>
</table>

Proposition 2. When $\varepsilon \sim U(0, M)$ and the manufacturer selects the optimal markup, $z^*_M$, unilaterally, the supply chain profit under RFM achieves 75% of the profits available under the integrated setting. Furthermore, supply chain profit under RFM is at least equal to the profit under a price-only contract, but is never more than 2% greater.

Proof. See Appendix.

Proposition 2 shows analytically that the supply chain will always enjoy higher profits under RFM than a price-only contract when the manufacturer is allowed to choose the fixed markup. RFM always achieves 75% of the integrated supply chain profits, while profits under a price-only contract are between 73.6% and 75% of the integrated supply chain profit. Thus, the supply chain’s profit under RFM exceeds price-only profits by less than 2%.

The results above may lead one to surmise that RFM does not lead to particularly large savings over price-only contracts in this scenario. However, these results apply on the manufacturer selecting the optimal markup value, $z^*_M$. Larger gains from RFM may be realized under alternative retail markup choices. Define $z^*_S$ and $z^*_R$ as the optimal markup chosen to maximize the profits for the retailer and the total supply chain, respectively. Proposition 3 specifies the values of these markings.

Proposition 3. Under RFM when $\varepsilon \sim U(0, M)$:

1. The values $z^*_S$ and $z^*_R$ are unique such that $z^*_S < z^*_R$.

Proof. See Appendix.

Propositions 1 and 3 demonstrate that there are separate, unique markup values that can be chosen to maximize (1) the manufacturer’s profit; (2) the retailer’s profit; and (3) total supply chain profit. Each of these situations can be interpreted as distinct points on a relative spectrum of the division of pricing power. The value $z^*_M$ ($z^*_S$) represents the case where the manufacturer (retailer) chooses the price markup; $z^*_S$ represents a division of pricing power where neither the manufacturernor the retailer can choose the markup unilaterally and instead the supply chain optimal markup is chosen, possibly through strategic negotiation. Part (2) of Proposition 3 states that these values are ordered such that $z^*_M < z^*_S < z^*_R$. From Theorem 4, it has already been shown that the retailer prefers a greater markup than the manufacturer, but Proposition 3 also shows that the optimal supply chain markup is between the optimal markups for the manufacturer and the retailer.

The following figures further illustrate the effect of pricing power in an RFM scenario. Fig. 1 plots the optimal markup choices for the manufacturer ($z^*_M$), the retailer ($z^*_R$) and the supply chain ($z^*_S$) under RFM. We find that $z^*_S$ is much closer to $z^*_R$ than $z^*_M$. Thus, a relatively powerful manufacturer will enforce retail markups much closer to the supply chain optimal markup than a powerful retailer. Fig. 2 displays the supply chain profit margins of RFM to price-only for different markup values ($z^*_M$, $z^*_S$ and $z^*_R$) where profit ratio is expressed as $\frac{M}{M_0}$. As shown in Proposition 2, when $\alpha = z^*_M$, RFM offers only marginal improvement for the supply chain (less than 2%). We find that $\alpha = z^*_S$ offers greater improvement for the supply chain under RFM — over 5% in this example. Also notice that $\alpha = z^*_R$ offers no improvement over a price-only contract. As shown in Corollary 2, here the retailer recreates the price-only scenario when she is allowed to choose the markup amount.

Our analysis provokes the following question: why would a retailer ever subscribe to RFM if she is guaranteed to do no better than under price-only? We offer several explanations for the existence of RFM in practice. The first
4. Model and analysis under linear additive demand function

All of our previous results rely on a multiplicative demand model with iso-price-elasticity. However, we are also interested in the robustness of those results to other demand functional forms. In this section we consider a linear additive demand function, \( D(p) = A - bp + \varepsilon \), where \( A, b > 0 \). To avoid trivial results, we limit our analysis to scenarios where \( A > bc \) and \( \tau(z^0) \geq 1/A \) where \( z^0 \geq 0 \) is the lower bound of the support of \( \varepsilon \). All other assumptions are the same as those under a multiplicative demand function. Models related to those considered here exist in previous works (e.g., Petruzzi and Dada, 1999; Liu et al., 2006) when demand follows a linear additive form. We rely on those previous results when possible and develop new results for the specific RFM scenario considered here.

4.1. Analytical results for integrated and price-only supply chain scenarios

For an integrated supply chain, the decision maker’s objective function is

\[
\Pi_i(p, q) = pE[\min(q, A - bp + \varepsilon)] - cq.
\]

We employ a transformation of stocking factor, \( z = q - A + bp \), to rewrite the objective function as

\[
\Pi_i(p, z) = (p - c)(A - bp + z) - pA(z). \tag{18}
\]

As explained in Petruzzi and Dada (1999), while the form of the stocking factor changes in the linear additive case, the effect is the same as in the multiplicative scenario. If \( z > \varepsilon \), we will have units leftover; if \( z \leq \varepsilon \), we will have unfulfilled demand. Petruzzi and Dada also show that there are unique solutions for the optimal choice of stocking factor and retail price to maximize (18) – see Petruzzi and Dada (1999, Theorem 1).

The profit functions for the price-only scenario can be written as

\[
\Pi_{pr}(w) = (w - c)q \quad \text{and} \quad \Pi_{rg}(p, z) = (p - w)(A - bp + z) - pA(z). \tag{19} \tag{20}
\]

After employing the same transformation to stocking factor, \( z \). Liu et al. (2006) consider a very similar model to that shown here, although they also include holding cost and loss-of-goodwill cost terms for the retailer. Their results for a price-only contract can be easily extended to show that there exist optimal retail price, \( p^* \), and stocking factor, \( z^* \), for the retailer under a price-only contract for any given wholesale price, \( w \). Furthermore, the optimal wholesale price, \( w^* \), for the manufacturer given the retailer’s optimal responses is guaranteed to exist within a given interval and can be found through a simple numerical search.

4.2. Analytical results for supply chain under RFM policy

When the retail fixed markup (RFM) policy is used in the supply chain, the retailer’s objective function is

\[
\Pi_{rg}(z|p) = \alpha p(A - bp + z) - pA(z). \tag{21}
\]

Similar to our results in the multiplicative demand case, Lemma 3 states that an optimal stocking factor can be found and that it is independent of retail price.
Lemma 3. When demand has a linear additive form, the unique optimal stocking factor for the retailer under RFM is given by $z^* = F^{-1}(x)$.

Proof. The second derivative of (21) with respect to $z$ is $-p^2(1-z) < 0$, so it is concave in the stocking factor, $z$. Solving the first order condition yields $z^* = F^{-1}(x)$. □

Using the result of Lemma 3, the manufacturer’s objective function is

$$\Pi_{RM}(p) = [(1-x)p - c][A - bp + F^{-1}(x)].$$

(22)

The optimal pricing policy for the manufacturer under RFM is given in Theorem 5.

Theorem 5. When demand has a linear additive form, there is a unique optimal retail price, $p^*$, and associated wholesale price, $w^*$ for the manufacturer under RFM. The optimal retail price is

$$p^* = \frac{(1-x)[A + F^{-1}(x)] + bc}{2b(1-x)}$$

and the associated wholesale price is

$$w^* = \frac{(1-x)[A + F^{-1}(x)] + bc}{2b}.$$  

(23)

Proof. Because $\frac{d\Pi_{RM}(p)}{dp} = -2b(1-x) < 0$, $\Pi_{RM}(p)$ is concave in $p$. Solving $\frac{d\Pi_{RM}(p)}{dp} = 0$ yields (23) and using the relation $w = p(1-x)$ yields (24). □

We can compare the optimal wholesale prices found in the multiplicative and linear additive demand cases to generate several interesting insights. Notice from Theorem 3 that the wholesale price, $w$, is independent of $x$ in the multiplicative demand scenario. This leads to our finding in Part 1 of Corollary 2 that the retailer will recreate a price-only scenario when she is given full power to choose $x$. Because the retailer cannot influence the wholesale price under RFM when demand has a multiplicative form, her best choice is to recreate her profit under a price-only contract and the ex-ante commitment to a fixed markup is of no consequence. Theorem 5 shows that the wholesale price, $w$, is clearly a function of $x$ under linear additive demand. Thus, the retailer can alter the manufacturer’s choice of wholesale price by varying $x$. As we can see in our numerical examples, this leads to significant differences in the performance of RFM when demand follows a linear additive form – namely both the manufacturer and the retailer may enjoy greater profits under RFM than under a price-only contract, which is impossible in the multiplicative demand setting with iso-price-elasticity.

When the manufacturer charges the optimal wholesale price, $w^*$, the optimal order quantity from the retailer is $q^* = \frac{1-(1-x)[A + F^{-1}(x)] + bc}{2b(1-x)}$. Thus, the manufacturer’s profit is $\Pi_{RM} = \frac{1-(1-x)[A + F^{-1}(x)] + bc}{2b(1-x)}$. Proposition 4 demonstrates that there exists an upper bound on the feasible values of $x$ under RFM in the linear additive scenario.

Proposition 4. When demand has a linear additive form and $q \geq 0$:

1. the optimal retail price, $p^*$, is increasing in $x$;
2. if $bc [f(z^0)] < 1$, $q^*$ is unimodal in $x$ with a unique maximizer, $x^*_q$; otherwise, $q^*$ monotonically decreases in $x$;
3. the value of $x$ under RFM will be such that

$$0 < x \leq x = \frac{A + F^{-1}(\beta) - bc}{4F^{-1}(\beta)}.$$  

Proof. See Appendix. □

Part 1 of Proposition 4 confirms intuition. Part 2 is used in the proof of Part 3 and states that $q \rightarrow 0$ as $x$ becomes large. For Part 3, it can be shown that as $x \rightarrow 0$, both the manufacturer and the retailer’s profits approach zero since the order quantity approaches zero; as $x \rightarrow 0$ the retailer’s profit clearly approaches zero. Thus, in reality the actual markup will likely be set through strategic negotiation such that $x \in [0, \bar{x}]$ and will reflect a division of pricing power between retailer and supplier.

4.3. Numerical examples under uniform distribution

Here we examine the relative benefit of the RFM policy when demand follows a linear additive form and the random component of demand has the same distribution as in Section 3.4 ($x \sim \mathcal{U}(0, M)$). RFM leads to much greater benefits under linear additive demand than in the multiplicative demand scenario.

Figs. 3 and 4 show the profits of the manufacturer and the retailer under both RFM and price-only policies for a range of $x$ values. Fig. 3 shows an example with lower demand uncertainty, which results in the manufacturer’s profit under RFM ($\Pi_{RM}$) being monotone in $x$. Fig. 4 represents a case with higher demand uncertainty and shows $\Pi_{RM}$ to be concave in $x$. When the price-dependent component dominates the demand function, the manufacturer always benefits by offering a lower fixed markup to capture
more of the deterministic demand. However, when the random component is a significant portion of demand, the manufacturer prefers a moderate value of $\alpha$ and relies on the retailer’s safety stock order to compensate for lower deterministic demand. Unfortunately, we cannot describe closed-form solutions for the manufacturer’s and the retailer’s optimal choices of $\alpha$ ($\alpha_M^*$ and $\alpha_R^*$) as we did in Section 3 for the multiplicative demand case, but these results (and our numerous other trials) suggest that unique values exist and can be found through a search procedure for the linear additive demand case.

Fig. 5 shows that the fixed markup that maximizes the supply chain profits, $\alpha^*$, all fall within the Pareto-improving region. However, for a wide range of sensitivity values ($b > 1$), $\alpha^*_M$, $\alpha^*_R$, and $\alpha^*$ all fall within the Pareto-improving region. Thus, in these situations, RFM will lead to Pareto-improving solutions regardless of who selects the markup. Fig. 6 examines the effect of price sensitivity when $M = 40$. Notice in these cases that $\alpha^*_M$ and $\alpha^*_R$ consistently fall outside of the Pareto-improving region. However, for a wide range of sensitivity values ($b > 1$), $\alpha^*$ is still in the Pareto-improving area. This suggests a need for strategic negotiation in choosing the markup value at lower levels of demand uncertainty in order to create self-enforceable RFM policies.

So, what can we infer about the actual performance of a supply chain operating under RFM when demand follows a linear additive functional form? And how does the relative pricing power of the manufacturer and retailer affect the performance of the supply chain? Assuming that RFM will only be practiced when it benefits all parties involved (compared to a price-only setting), we define

![Fig. 4. Performance comparison ($M = 100$).](image)

![Fig. 5. Fixed markup vs. demand uncertainty.](image)

![Fig. 6. Fixed markup vs. price sensitivity.](image)
The value of the retailer (manufacturer), who sets the markup to maximize his
profit, is a function of the demand function. The markup chosen by the retailer
is dependent on the form of the demand function as well as
the benefit from the division of pricing power in terms of who chooses the
markup. Therefore, the enforcement of vertically restrictive pricing by a powerful
manufacturer, through a retailer who chooses RFM for convenience or as the result of strategic negotiation. We
compare the performance of the supply chain as a whole and the individual players under RFM with those under
a traditional price-only contract. We analytically specify optimal policies for the retailer and manufacturer decisions
under iso-price-elastic, multiplicative demand and we outline partial results for linear additive demand. We rely on
numerical results for those cases where we cannot completely specify analytical policies. Our results indicate that
the benefit from an ex-ante markup commitment is highly dependent on the form of the demand function as well as the
division of pricing power in terms of who chooses the fixed markup.

When demand follows an iso-price-elastic, multiplicative demand function, and the manufacturer chooses the
markup, RFM offers some benefit to the supply chain. However, numerical results indicate that the system profit
improvement can be considerably greater when pricing power is distributed and the markup is set through strategic negotiation. We also find that the retailer will never prefer RFM to a price-only contract under iso-price-elastic, multiplicative demand regardless of who chooses the markup.

We attribute this to the fact that the wholesale price is independent of the markup in this case, thus, the retailer cannot influence the manufacturer’s decision by committing to a markup. Therefore, the ex-ante commitment to a fixed markup has no value to the retailer in this scenario as she receives no benefit from being the first mover and revealing her markup. In fact, if the retailer reveals any markup \( \bar{z} \neq \bar{z} \), she will be worse off than under price-only. Therefore, some side-benefit or subsidies may be required in order to entice retailer participation when demand has

5. Conclusions and future research

A rapidly emerging global economy and continued technological innovations have greatly increased competition and reduced the lifecycle and margins derived from products in many markets. For firms facing such conditions, the simultaneous consideration of pricing and operational decisions is crucial to improving performance. As noted by Rice and Hoppe (2001), shrinking product lifecycles and advances in information technology also “increases the need for higher and deeper levels of coordination (alliances) among . . . companies” in a supply chain. Thus, we believe that firms will continue to search for collaborative supply chain policies that are simple to implement and administer to gain competitive advantages.

We examine the effect of an ex-ante commitment to retail price markup, which we refer to as a retail fixed markup (RFM) policy. Such a policy can arise through the enforcement of vertically restrictive pricing by a powerful manufacturer, through a retailer who chooses RFM for convenience or as the result of strategic negotiation. We compare the performance of the supply chain as a whole and the individual players under RFM with those under a traditional price-only contract. We analytically specify optimal policies for the retailer and manufacturer decisions under iso-price-elastic, multiplicative demand and we outline partial results for linear additive demand. We rely on numerical results for those cases where we cannot completely specify analytical policies. Our results indicate that the benefit from ex-ante markup commitment is highly dependent on the form of the demand function as well as the division of pricing power in terms of who chooses the fixed markup.

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\[ x_L = \max(x_H, \bar{z}), \quad x_H = \min(x_M, \bar{z}) \quad \text{and} \quad x_{\text{MID}} = (x_L + x_H)/2. \]

Thus, \( x_L (x_H) \) can be thought of as a powerful manufacturer’s (retailer’s) profit, yet still entice retailer (manufacturer) participation. The value \( x_{\text{MID}} \) represents a midpoint compromise between \( x_L \) and \( x_H \).

Figs. 7 and 8 compare the supply chain efficiency of RFM and price-only (defined as \( \frac{H}{100}\% \) and \( \frac{M}{100}\% \), respectively). Here we see that RFM leads to significantly more efficient supply chains in all scenarios. RFM performs particularly well from a supply chain point of view when both demand uncertainty and demand sensitivity are not very high; however, the increase in efficiency remains high even when demand becomes more uncertain or more price sensitive. We also note that the markup chosen by the retailer (\( x_H \)) tends to result in better performance than that chosen by the manufacturer (\( x_L \)). This is contrary to our findings in the iso-price-elastic, multiplicative demand scenario, where a powerful manufacturer is preferred from a supply chain perspective. Thus, we again see evidence of the importance of the functional form of demand. However, even when the manufacturer chooses \( z \) under linear additive demand, RFM still results in much more efficient supply chains than a price-only agreement. Furthermore, we again see instances where a division

\[ \alpha = 6 - \frac{\bar{z}}{\bar{z}}. \]
a multiplicative form. Our results are quite different for a linear additive demand function. We show the existence of Pareto-improving solutions where both the manufacturer and the retailer prefer RFM to price-only. The retailer’s increased profit is achieved through her ability to influence the wholesale price by committing to a fixed markup. We find that a Pareto-improving region exists in a variety of scenarios; hence, the retailer may enjoy a direct benefit from committing to an \textit{ex-ante} price markup commitment when demand follows a linear additive form.

Several managerial insights can be drawn from our research. First, we observe that the decision of whether or not a retailer should reveal and commit to a price markup before wholesale price is chosen depends on the functional form of demand. The retailer can garner higher profits by committing to a markup if demand follows a linear additive form but she is never better off by doing so if demand is multiplicative and iso-price-elastic. Second, we note that the benefit of RFM to the supply chain is highly dependent on the distribution of pricing power. In many cases – and guaranteed in the iso-price-elastic, multiplicative case – neither the manufacturer nor the retailer can bring the greatest improvement to the entire supply chain if one decides the markup unilaterally. The exact effects of the distribution of pricing power are dependent on the functional form of demand. A relatively powerful manufacturer leads to greater channel improvement under iso-price-elastic, multiplicative demand, while a powerful retailer is more beneficial to the channel in the linear additive demand case. We also find that RFM is more robust to differences in pricing power when demand follows a linear additive form, although a unilaterally chosen markup (either by the manufacturer or the retailer) may still not lead to Pareto-improvement. Finally, our results indicate that while RFM does not coordinate the channel, it can lead to supply chain savings over a price-only agreement in a variety of scenarios. Supply chain benefits are present under both multiplicative and linear-additive demand forms, but the benefits are significantly greater for the linear additive demand scenario. Furthermore, RFM has an advantage over many other supply chain collaboration schemes because it is easy to implement and verify since only retail price must be observed and it does not require the retailer to disclose actual sales or inventory values.

There are several directions for future research. The most direct extension is to allow for the holding of inventory in multiple time period models. Multi-period models for decentralized supply chains under price-dependent demand are generally intractable to analytical analysis, but we are currently investigating this extension. Models with multiple manufacturers, multiple retailers or both may capture more insights since horizontal competition will also be considered. Models including factors influencing demand other than price, such as service and retail effort, may give a more comprehensive understanding of organizational behaviors under price-dependent demand. Finally, the consideration of asymmetric information where one or both players have incomplete information regarding demand or the other player’s cost structure could lead to interesting models that are representative of many practical scenarios.

Appendix

Proof of Theorem 4

Part 1: From (16), the first term of the manufacturer’s profit, \( \frac{d}{d \alpha} \left( \frac{h^b}{m-1} \right) A^b \), is independent of retailer’s markup, \( \alpha \). Thus, the manufacturer can only control \( \Omega(x) = F^{-1}(\alpha(x)(1 - \alpha)^b) \), so we examine the properties of \( \Omega(x) \). From (12), \( z^* = F^{-1}(x) \) so \( \frac{dz}{dx} = \frac{1}{f[F^{-1}(x)]} \). Thus, the first-order condition is

\[
\frac{d\Omega(x)}{dx} = (1 - \alpha)^{-1} \left\{ -bF^{-1}(x) + \frac{1 - \alpha}{f[F^{-1}(x)]} \right\}.
\]  

(A1)

Since \( (1 - \alpha)^{-1} > 0 \), the sign of first-order condition is purely determined by \( \Psi(x) = -bF^{-1}(x) + \frac{1 - \alpha}{f[F^{-1}(x)]} \). We also know that \( \lim_{x \to 0^+} \Psi(x) = \frac{1}{\frac{dF}{dx}} > 0 \), and \( \lim_{x \to 1^-} \Psi(x) = -b2 < 0 \), where \( z^0 \) and \( z^\infty \) are the lower and upper bound of the random variable \( x \). Thus, the maximum value of \( x \) must be in the interval \((0, 1)\).

Note that

\[
\frac{d\Psi(x)}{dx} = -\frac{1}{f^3[F^{-1}(x)]} \left\{ (b + 1)f^2[F^{-1}(x)] + (1 - x)f'[F^{-1}(x)] \right\}.
\]  

(A2)

Since \( f(i) > 0 \) for distributions with non-decreasing failure rate, we only need to verify the sign of \( K(x) = (b + 1)f^2[F^{-1}(z)] + (1 - x)f'[F^{-1}(z)] \). When \( \varepsilon \) has a non-decreasing failure rate, \( r'(z) \geq 0 \), where

\[
r'(z) = \frac{f'(z) + f(z)(1 - F(z))}{f(z)[1 - F(z)]} \geq 0. 
\]

Because \( F(z) < 1 \) when \( z < z^\infty \),

\[
f^2(z) + f'(z)[1 - F(z)] \geq 0.
\]

(A3)

From (12), \( z^* = F^{-1}(x) \), so \( x = R(z^*) \). Thus,

\[
K(z^*) = b f^2(z^*) + f'(z^*)[1 - F(z^*)] \geq b f^2(z^*) > 0
\]

from (A3). Let \( z_{x^*}^* \) be the solution of the first order condition (A1). Then, \( \frac{d\Psi(x)}{dx} < 0 \) at \( z_{x^*}^* \), and hence \( \Omega(x) \) is unimodal with a mode at \( z_{x^*}^* \), which is the solution to (A1). Furthermore, \( \frac{dF}{dx} > 0 \) for \( x < z_{x^*}^* \) and \( \frac{dF}{dx} < 0 \) for \( x > z_{x^*}^* \). Thus, \( x = z_{x^*}^* \) is the unique maximizer.

Part 2: From (17) we know that the retailer can only influence the second term, \( \mathcal{V}(z) \), in her profit function, where \( \mathcal{V}(z) = xzF^{-1}(z) - \Lambda(F^{-1}(z)) \). Thus, we only examine the properties of \( \mathcal{V}(z) \).

Then, \( \frac{d\mathcal{V}(x)}{dx} = (1 - x)^{-2}G(x) \), where

\[
G(x) = (1 - x)F^{-1}(x) + (b - 1)\Lambda(F^{-1}(x)).
\]  

(A4)
and
\[ \frac{dG(z)}{dz} = -bF^{-1}(z) + \frac{1 - \alpha}{f[F^{-1}(z)]}. \]  
(A5)

So, \( \frac{dG(z)}{dz} = \Psi(z) \) in Part 1. Thus, \( G(z) \) is also unimodular from the previous proof. Next, \( \lim_{z \to 0} G(z) = 0 \) and \( \lim_{z \to 1} G(z) = -(b-1)[z^\alpha - A(z^\alpha)] \). We find
\[ H(z) = z^{-\alpha}A(z^{-\alpha}) \geq 0 \text{ since } H(z^0) = 0 \text{ and } H'(z) = 1 - F(z) > 0. \]  
Thus, \( \lim_{z \to 1} G(z) < 0 \). Therefore, the optimal solution of the retailer’s markup is unique and \( x_R^* \in (0, 1) \), which is the solution of (A4). Furthermore, since \( z^* = F^{-1}(z^\ast) \) is a one-to-one correspondence, we can write (A4) as a function of \( z^\ast \) as \( G(z) = [1 - bF(z^\ast)]z^\ast + (b - 1)A(z^\ast) \), which is the same first order condition for \( z^\ast \) as in Theorem 1 of Wang et al. (2004). Thus, we obtain the same solution for optimal stocking factor \( z^\ast \) as under a price-only contract.

Part 3: From Part 1 and 2, both \( x^*_M \) and \( x^*_R \) are unique, and \( \frac{dG(z)}{dz} \big|_{z=x^*_R}<0 \). Thus, \( \Psi(x^*_R) \) is negative so that the optimal markup for the manufacturer is smaller than that for the retailer.

Proof of Corollary 2

Part 1: Substituting (12) into (17), we obtain that the retailer’s profit function under RFM is the same as that under price-only. So, if the retailer picks the ex-ante retail markup, she will select the \( x^*_R \) that leads to the same stocking factor, \( z^\ast \), as under a price-only contract due to the one-to-one correspondence between \( z \) and \( x \). Thus, \( \Pi^*_{RR}(x^*_R) = \Pi^*_{PG} \). From Lemma 1, we also know that retail price under RFM will be the same as under price-only if the retailer selects \( x^*_R \). From Theorem 3, wholesale price depends only on \( b \) and \( c \), thus, the order quantity under RFM and price-only will also be the same, so \( \Pi^*_{RM}(x^*_R) = \Pi^*_{PM} \).

Part 2: From Theorem 4 and Part 1 above, \( \Pi^*_{RR} \) is unimodular and for all \( x \), \( \Pi^*_{RR}(x) \leq \Pi^*_{RR}(x^*_R) = \Pi^*_{PG} \).

Part 3: Because \( x^*_M < x^*_R \) and \( \Pi_{RM} \) and \( \Pi_{RR} \) are unimodular, we can confine our analysis to \( x \in [x^*_M, x^*_R] \) since the manufacturer and retailer both prefer some \( x \in [x^*_M, x^*_R] \) to any \( x \notin [x^*_M, x^*_R] \). From Theorem 4, the manufacturer’s profit strictly decreases in \( x \) for \( x \in [x^*_M, x^*_R] \). Thus, \( \forall x \in [x^*_M, x^*_R], \Pi^*_{RM}(x) \geq \Pi^*_{RM}(x^*_R) = \Pi^*_{PM} \).

Proof of Proposition 2.

When the manufacturer is powerful and can choose the markup unilaterally,
\[ \Pi^*_M = \frac{3mM}{2(b-1)} \frac{(b+1)c}{b-1}^{-b} = \frac{3}{4}. \]

Thus, RFM achieves 75% of the profits available under the integrated scenario. We define the profit ratio between the RFM and price-only scenarios as
\[ \Sigma_{RP} = \frac{\Pi^*_M}{\Pi^*_P} = \frac{3(b-1)}{4(2b-1)} \left( \frac{b}{b-1} \right)^b. \]  
(A6)

Next, we have that
\[ \frac{d\Sigma_{RP}}{db} = \frac{3(b-1)}{4(2b-1)^2} \left( \frac{b}{b-1} \right)^b \left[ (b-1) \ln \left( \frac{b}{b-1} \right) - 2 \right]. \]

Now, because \( b \geq 2 \Rightarrow 1/b < 1 \), which satisfies the convergence radius of the Maclaurin series. So, we have
\[ \frac{(b-1) \ln \left( \frac{b}{b-1} \right) - 1}{b-1} = (1 - 2b) \ln \left( \frac{b}{b-1} \right) = \sum_{k=1}^{\infty} \frac{2b-1}{kb^2} + 2 > 2. \]

Thus, \( \Sigma_{RP} \) is increasing in \( b \). But \( \Sigma_{RP} = 1 \) at \( b = 2 \), so the supply chain performance of RFM is never worse than under price-only. However, \( \lim_{b \to 1} \Sigma_{RP} = 3e/8 \approx 1.0194 \), so the profit under RFM will never be more than \( \approx 1.94\% \) greater than under price-only. Therefore, the lower bound on the performance or price-only in relation to the integrated scenario is 0.75/0.375e = 73.6%. And the upper bound on price-only performance is the RFM profit, thus, price-only profits are bounded above at 75% of the integrated profits.

Proof of Proposition 3

Part 1: Rewriting the retailer’s profit as a function of the fixed markup, \( z \),
\[ \Pi^*_{RR} = \frac{AM}{2} \left( \frac{bc}{b-1} \right)^{1-b} z^2(1-z)^{b-1}. \]  
(A7)

Define the terms in (A7) that depend on \( z \) as \( R(z) = z^2(1-z)^{b-1} \).
\[ \frac{dR}{dz} = -z(1-z)^{b-2}(zb + z - 2). \]

Solving \( \frac{dR}{dz} = 0 \) yields three solutions for \( x \): \( x_1 = 0 \), \( x_2 = 2/(b+1) \), and \( x_3 = 1 \). However, \( x = 0 \) and \( x = 1 \) are minimizers. Therefore, \( \Pi^*_{RR} \) is unimodular, and \( x_2 = 2/(b+1) \) is the unique maximizer of \( \Pi^*_{RR} \) because \( \frac{dR}{dz} \big|_{z=x_2} = -2(2/(k+1))^{b-2} < 0 \) and \( \frac{dR}{dz} < 0 \) for all \( x > x_2 \) and \( \frac{dR}{dz} > 0 \) for all \( x < x_2 \).

Following a similar process, the profit of the decentralized supply chain under RFM is
\[ \Pi^*_{RD} = \frac{AcM}{2(b-1)} \left( \frac{bc}{b-1} \right)^{-b} z[(b-2)z + 2](1-z)^{b-1}. \]  
(A8)

Define \( S(z) = z[(b-2)z + 2](1-z)^{b-1} \). Thus, \( \frac{dS}{dz} = (1-z)^{b-2}[2 - 4x - (b-2)(b+1)x^2] \).

The single optimal solution of \( z \in (0, 1) \) is \( z^*_R = \frac{1}{\sqrt{2b^2-4b+2}} \).

Similar to above, it can be shown that
\[
\frac{d^2S}{dz^2} \bigg|_{z = z^*} = -2\sqrt{2(b-1)} \left[ \frac{\sqrt{b(b-1)}}{\sqrt{b(b-1)+\sqrt{2}}} \right]^{b-2} < 0
\]

and that \( \frac{df}{dz} < 0 \) for all \( z > z^*_3 \) and \( \frac{df}{dz} > 0 \) for all \( z < z^*_3 \) thus, \( \Pi_{RD} \) is unimodal and \( z^*_3 \) the unique maximizer between 0 and 1.

Part 2: First we prove \( x^*_3 < x^*_2 \).
\[
x^*_3 < x^*_2 \iff \frac{1}{b+1} < \frac{1}{\sqrt{b(b-1)/2}} \iff b > \sqrt{b(b-1)/2} \iff \nonumber
\]
\[
2b^2 > b(b-1) \iff b(b+1) > 0, \text{ which is obvious since } \nonumber
\]
\[
b \geq 2. \nonumber
\]
Next, we show that \( x^*_3 < x^*_R \).
\[
x^*_3 < x^*_R \iff \frac{1}{\sqrt{b(b-1)/2+1}} > \frac{b}{b+1} \iff \nonumber
\]
\[
\sqrt{b(b-1)/2} > \frac{b^2}{b+1} \iff \nonumber
\]
\[
2b(b-1) > (b-1)^2 \iff b^2 > 1, \text{ which follows from } \nonumber
\]
\[
b \geq 2. \text{ Therefore, } x^*_3 < x^*_R. \nonumber
\]

Proof of Proposition 4

Part 1: Because, \( z^* = F^{-1}(z), \frac{df}{dz} = \frac{1}{f[F^{-1}(z)]} \). Then from (23),
\[
\frac{dp^c}{dz} = \frac{c}{2(1-z)^2} + \frac{1}{2b^2F[F^{-1}(z)]} > 0. \nonumber
\]
Part 2: First,
\[
\frac{dp^c}{dz} = -\frac{bc}{2(1-z)^2} + \frac{1}{2b^2F[F^{-1}(z)]} = \frac{(1-z^2) - bc[f[F^{-1}(z)]]}{2b^2F[F^{-1}(z)]}. \nonumber
\]
Define \( \Gamma(z) = (1-z)^2 - bc[f[F^{-1}(z)]] \) and \( \Gamma'(z) = 0 \).

Then,
\[
\frac{d\Gamma(z)}{dz} = -2(1-z) - bc[f[F^{-1}(z)]]/f[F^{-1}(z)]. \quad (A9) \nonumber
\]
Thus,
\[
\frac{d\Gamma(z)}{dz} \bigg|_{z = z^*} = -\frac{1-z}{f[F^{-1}(z)]} \left[ 2f[F^{-1}(z)] + f[F^{-1}(z)](1-z) \right]. \nonumber
\]

Because \( z^* = F^{-1}(z) \) and \( \epsilon \) has a non-decreasing failure rate,
\[
2f^2[F^{-1}(z)] + f'[F^{-1}(z)](1-z) \nonumber
\]
\[
= f^2(z^*) + \left[ f^2(z^*) + f'(z^*)[1-F(z^*)] \right] \nonumber
\]
\[
> 0 \text{ from (A3)}. \nonumber
\]

Therefore, \( \frac{df}{dz} \bigg|_{z = z^*_3} \leq 0 \) and \( q'(z) \) is either unimodal or monotone decreasing in \( z \) and there is at most one solution to \( \frac{df}{dz} = 0 \). Note that \( \lim_{z \to 0} \Gamma(z) = 1 - bc[f(z^0)] \) and \( \lim_{z \to 1} \Gamma(z) = -bc[f(z^\infty)] < 0 \). Thus, if \( bc[f(z^0)] < 1, \frac{df}{dz} \bigg|_{z = 0} > 0 \) and \( \frac{df}{dz} \bigg|_{z = 1} < 0 \). Therefore, \( x^*_6 \in (0, 1) \) and \( q'(z) \) is unimodal. If \( bc[f(z^0)] > 1, \frac{df}{dz} \bigg|_{z = 0} < 0 \) for all \( z^* \in [0, 1] \). Thus \( q^*(z) \) is monotone and \( q^* \) decreases in \( z \).

Part 3: It is easily shown that \( \hat{z} = \frac{A+F^{-1}(\hat{z})-bc}{A+F^{-1}(\hat{z})} \) solves \( q^*(z) = 0 \).

If we can show that \( x^*_q < \hat{z} \) or \( \frac{df}{dz} \bigg|_{z = \hat{z}} < 0 \), then \( q \) decreases in \( z \) and we are done. It is equivalent to prove
\[
\Gamma(\hat{z}) = (1-\hat{z})^2 - bcf[F^{-1}(\hat{z})] < 0. \nonumber
\]
Because \( (1-\hat{z})[A+F^{-1}(\hat{z})] - bc = 0 \), it is sufficient to verify \( \Gamma(\hat{z}) = (1-\hat{z})^2 - (1-\hat{z})[A+F^{-1}(\hat{z})]f[F^{-1}(\hat{z})] < 0 \). Thus, we need\( (1-\hat{z}) - [A+F^{-1}(\hat{z})]f[F^{-1}(\hat{z})] < 0. \quad (A10) \nonumber\)

We know that \( z^* = F^{-1}(z) \), so \( \hat{z} = F(\hat{z}) \) where \( \hat{z} \) is the stocking factor leading to \( \hat{z} \). Then we rewrite \( (A10) \) as \( 1 - F(\hat{z}) - (A+\hat{z})f(\hat{z}) < 0 \iff (A+\hat{z})R(\hat{z}) > 1 \). It is easy to verify \( (A+\hat{z})R(\hat{z}) > 1 \) since \( (A+z^0)r(z^0) \geq Ar(z^0) \geq 1 \) and \( \frac{df}{dz} \bigg|_{z^0} = r(z) + Ar(z) > 0 \). Thus, \( (A10) \) is true for all \( z > 0 \). Thus, \( x^*_q < \hat{z} \) and \( q^* \) is decreasing at \( \hat{z} \). Therefore, \( q > 0 \) for \( z < \hat{z} \) and all rational selections of \( z \) are \( z \in (0, \hat{z}) \).

References


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