Modeling of CFRP strengthened RCC beam using the nonlinear finite element method

Umesh Basappa* and Amirtham Rajagopal*,

*Email: rajagopal@iith.ac.in

*Department of Civil engineering, Indian institute of Technology Hyderabad, 502 205, INDIA.

Received: 31 August 2012; Accepted: 29 October 2012

Computational modeling of fracture in reinforced cement concrete (RCC) beam considering various phenomena has been a challenging task over the years. This paper presents a crack modeling methodology in three dimensions for carbon fiber reinforced polymer (CFRP) strengthened RCC beam by performing a three dimensional nonlinear finite element analysis of the beam subjected to four point loading. The concrete is modeled as inelastic material. Various concrete failure parameters such as shear transfer coefficients, uniaxial tensile and compressive strengths, biaxial compressive and crushing strengths and stiffness reduction in cracked concrete in tensile region are considered. In numerical studies an unstrengthened beam with and without hanger bars are considered for the analysis and results are compared with experimental results. In the next case, a CFRP strengthened RCC beam is considered for analysis. A parametric study is performed considering different length of CFRP used for flexural strengthening of beams, modeling CFRP as Isotropic and orthotropic and varying the area of steel reinforcement in tension region. The study indicates that the proposed method is able to accurately predict the behavior, crack patterns and load carrying capacity. The results are comparable with the experimental results available in the literature.

KEYWORDS: Nonlinear FE analysis; crack in RC beams; flexural strengthening; CFRP.

Reinforced cement concrete (RCC) structures has found a wide range of applications like for instance in high rise buildings, bridges, pre-cast structures, dams, tunnels are some amongst others. These structures are designed to satisfy serviceability criteria (prediction of cracks and deflection under service load) and safety criteria (prediction of load deformation behavior of RCC structures and estimation of ultimate load). In the present scenario, the construction of modern structures and loading histories are more complex together with an increase in the cost of construction seeking innovative design without negotiation for safety of structure because any structural failure would include loss in the human life and assets. Thus there is a need for accurate and reliable method to assess the safety and serviceability of RCC structure.

Experiments are extensively conducted to predict the response of RCC structures providing a real life response. However experiments are time consuming, costly and include improper simulation of loading and support conditions of the actual structure. With the advent of digital computers and analysis method such as finite element method (FEM), it is possible to predict the nonlinear behavior of RCC structures numerically.

Classically two methods are available for numerical modeling of cracking in concrete namely discrete
crack model and smeared crack model\textsuperscript{1,2}. Among these methods smeared model has become more popular and researchers started to develop constitutive models for concrete under the triaxial behavior\textsuperscript{3}. The outcome of these studies include two major parameters with respect to cracking phenomenon namely shear retention factor and tension stiffening\textsuperscript{4}. It was observed in some of these studies that tension stiffness (brittle crack model) must be replaced with tension softening behavior (linear or gradual weakening behavior), to match with the experimental data. However the tension softening behavior was found to be prone to mesh sensitivity\textsuperscript{5} and does not consider some concepts from fracture mechanics\textsuperscript{6}.

Later models proposed included a tension softening behavior that was mesh independent\textsuperscript{5}. Discrete crack modeling assumes a predefined crack in the structure and uses concepts from linear elastic fracture mechanics (LEFM), together with adaptive mesh refinement and or interface elements to predict the crack propagation. The advantage of this method is that it gives a near correct prediction of crack propagation path. However, the method is applicable only when a single dominant crack in the structure leads to complete structural failure. Smeared crack models predict the cracking behavior by making changes in the constitutive relation. In this work we present a smeared crack model to study the cracks in reinforced cement concrete.

Classical approaches are more related to global stress-strain relation in defining fracture mechanisms of concrete materials. These techniques do not include material structure at different length scale (nanometer to millimeter). Bazant and Schlangen studied 2D fracture mechanisms in heterogeneous materials like concrete, using random particle or lattice model\textsuperscript{7,8}. The method consists discretization of material with the use of truss or beam element as lattice and assigning the different properties to map the material microstructure. The fracture mechanism is captured by deleting lattice elements based on the linear elastic approach. It is observed that, the microstructure is the one which extensively leads to fracture mechanisms in heterogeneous material like concrete\textsuperscript{9-10}.

Concrete structures have very low tensile strength, fatigue resistance, fracture toughness etc. In earlier days, researchers improved the mechanical behavior of concrete by adding fibers to matrix called as steel fiber reinforced cementations composites (SFRCC’s). Recently, computational modeling of SFRCC’s gained interest in research. Discrete or lattice crack model is extensively used to study the behavior of matrix-fiber and matrix-aggregate interface and both meso/macro level of approach are used for the analysis\textsuperscript{11-13}.

Many studies have been carried out in developing efficient renovation or retrofitting techniques for post strengthening of concrete structures in bridges. Because of deterioration of strength concrete structures due to corrosion of steel reinforcement and continuous increase in the traffic volume. Even, structures like beam, column and slab amongst others require post strengthening. Traditional approach of strengthening concrete structures was by external post tensioning and steel plate bonding. However bonded steel plates are also found to exhibit corrosion due to continuous exposure to weathering and it was recommended that partial substitution of steel plates with polymer matrix/fiber composites. A comparative study earlier made concluded that carbon fiber reinforced polymer (CFRP) are best suitable for the post strengthening of concrete structures\textsuperscript{14-15}.

Effective application of CFRP to post strengthen the concrete is not possible until the different failure mechanisms of CFRP reinforced concrete are studied. In one of the earlier works various possible modes of failure of the retrofitted concrete beam has been identified as flexural compression failure, shear failure in concrete beam, delamination of CFRP and debonding of a layer of concrete at the flexural steel level\textsuperscript{16}.

In the present work a three dimensional nonlinear finite element analysis is performed to predict the flexural cracking behavior of CFRP strengthened RCC beams. A smeared crack approach was assumed and analysis is performed using commercially available software ANSYS 2010. The behavior RCC beam with and without hangers is compared with experimental results. Later, the crack patterns for the various area of steel reinforcement were simulated to judge the failure pattern of RCC beam with hanger bars. A parametric study is made for various lengths of carbon reinforced fiber polymer (CFRP) used for post strengthening of the flexural capacity of RCC beam with and without hanger bars.
MATERIAL BEHAVIOR

Modeling of Concrete

Concrete is a heterogeneous material (composed of fine and coarse aggregates) characterized by highly nonlinear and ductile stress-strain relationship. The nonlinear behavior is attributed to the formation and gradual growth of microcracks under loading. The microcracks can be categorized as bond cracks and mortar cracks. Bond cracks occur along the interface between the mortar and coarse aggregate and exist due to the improper bonding or due to the differences in the stiffness between the coarse aggregates and mortar. These cracks also occur due to shrinkage of concrete and at low load levels. Mortar cracks are present in the mortar between the pieces of aggregate and develop at high load levels or stresses.

The uniaxial stress-strain behavior of concrete in compression is shown in Fig. 1a. Various mathematical models are available to approximate this nonlinear behavior namely linearly elastic-perfectly plastic model, inelastic-perfectly plastic, Hognestad and piecewise linear model. In the present study a modified Hognestad mathematical model (see Fig. 1b) has been used for the approximation of the stress-strain behavior of concrete.

1. Initial tangent modulus of elasticity increases with an increase in compressive strength. So, the elastic modulus ($E_C$) is given by

$$E_C = 57000 \sqrt{f_C} \quad (1)$$

where, $f_C$ compressive strength of concrete at 28 days (Psi)

The stress-strain relation initially must satisfy the Hooke’s law.

2. The strain at maximum stress increases as the compressive strength increases.

$$\varepsilon_0 = \frac{2f_C}{E_C} \quad (2)$$

3. The raising portion of the stress strain curve resembles a parabola with vertex at the maximum stress.

$$f = \frac{E_C \varepsilon}{1 + \left(\frac{\varepsilon}{\varepsilon_0}\right)^2} \quad (3)$$

Where, $f$ is stress for an value of strain in stress-strain relationship of concrete.

The stress-strain behavior of concrete under tension includes raising part and descending part. The raising part is slightly curved, approximated either as straight lines or parabola. The descending part drops rapidly with increased elongation after the maximum stress is crossed. The tensile strength of concrete varies 8-15% of the compressive strength.

The various tensile crack models are available in the literature for crack modeling in concrete: Brittle, linear and nonlinear models. In case of brittle crack model crack occurs when maximum principal stress exceeds the tensile strength of concrete. In linear and nonlinear crack models, crack occurs when the principle stress exceeds the minimum tensile strength and residual tensile strength as given in Eq. (4) through Eq. (6).

- Brittle tensile crack model

$$\frac{f(\varepsilon)}{f_t} = \begin{cases} 1 & 0 \leq \varepsilon \leq \varepsilon_1 \\ 0 & \varepsilon_1 < \varepsilon < \infty \end{cases} \quad (4)$$

- Linear tensile softening crack model

$$\frac{f(\varepsilon)}{f_t} = \begin{cases} 1 \frac{\varepsilon}{\varepsilon_{ult}} & \varepsilon_1 < \varepsilon \leq \varepsilon_{ult} \\ 0 & \varepsilon_{ult} < \varepsilon < \infty \end{cases} \quad (5)$$
Nonlinear tensile softening crack model

\[
\frac{f(\varepsilon)}{f_t} = \begin{cases} 
1 - \left( \frac{\varepsilon}{\varepsilon_{ult}} \right)^{\frac{1}{10}} & \varepsilon_{ult} < \varepsilon \leq \varepsilon_{ult} \\
0 & \varepsilon > \varepsilon_{ult}
\end{cases}
\]  

(6)

where, \(f_t\) is tensile strength of the concrete, \(\varepsilon_{ult}\) is strain at failure of concrete, \(f\) is the stress at an value of strain \(\varepsilon\) in the stress strain relation of concrete.

The idealization of finite elements and size of mesh chosen clearly indicates the continuum level of approach to modeling concrete material. Willam and Warnke derived a mathematical model to plot the failure surface under triaxial behavior of concrete material (see Fig. 3). Concrete is assumed as isotropic material, as a result the failure surface are expressed in principle space.

The four aspects considered in deriving the mathematical model of failure surface:

1. The close fit of experimental data in the operating range is considered. The principle stresses are ordered as \(\sigma_1 \geq \sigma_2 \geq \sigma_3\), and then the failure surface can be expressed as a function of hydrostatic and deviatoric stresses. The hydrostatic section contains equisectrix \(\sigma_1 = \sigma_2 = \sigma_3\) as an axis of revolution. The deviatoric section lies in a plane normal to the equisectrix.

2. Simple identification of model parameters from the standard test data. The parameters are included in the mathematical model such that they are easily identified through the standard test (Uniaxial compression, tension and biaxial compression).

3. Smoothness in failure surface i.e., continuous surface with continuously varying tangent planes.

4. Convexity in failure surface i.e., monotonically curved surface without inflection points.

The condition for the failure of plane concrete member under the triaxial stresses is given by,

\[
\frac{F}{f_c} - S \geq 0
\]

(7)

where, \(F\) is a function of principle stresses (\(\sigma_{xp}\), \(\sigma_{yp}\), \(\sigma_{zp}\)), \(S\) is the failure surface expressed in terms of principal stresses and five input parameters, \(f_C\) is the uniaxial crushing strength and \(\sigma_{xp}\), \(\sigma_{yp}\), \(\sigma_{zp}\) are the principal stresses in the principal directions.

Failure surface \(S\) is plotted using five input parameters namely: Ultimate uniaxial tensile strength \((f_t)\), Ultimate uniaxial compressive strength \((f_c)\), Ultimate biaxial compressive strength \((f_{cb})\), Ambient hydrostatic stress state \((\sigma_h)\), Ultimate compressive strength for state of biaxial compression superimposed on hydrostatic stress state \((f_1)\), The ultimate compressive strength of a state of uniaxial compression superimposed on hydrostatic stress state \((f_2)\). The failure surface can also be specified with two parameters \(f_t\) and \(f_c\) keeping other parameters to default values.

\[
f_{cb} = 1.2 \ f_c
\]

(8)

\[
f_t = 1.45 \ f_c
\]

(9)

\[
f_2 = 1.725 \ f_c
\]

(10)

These values are valid only for the condition, \(\sigma_h \leq \sqrt{3} f_c\).
2.2 Modeling of Reinforcement steel

The mechanical behavior of reinforcing steel bar is obtained by testing the bar under monotonic tension loading. The steel bar initially exhibits linear elastic portion followed by a yield plateau, strain hardening and then stress drops till fracture occurs. The behavior of steel bar remains same in compression and tension loading.

The stress-strain behavior of steel bar is independent of environmental conditions and time and also the reinforcements are used as bars in concrete. Hence, three dimensional mechanical behavior of the reinforcement steel bar is ignored unlike the triaxial behavior of concrete considered in the RCC beam\(^25\).

The model adopted to represent the reinforcement is elastic-perfectly plastic as shown in Fig. 4. The stress for any value of strain is given as follows\(^6\)

\[
\begin{align*}
\text{Liner elastic-perfectly plastic model:} & \\
& f_s = \pm E_s \varepsilon_s \quad \text{for} \ -\varepsilon_y \leq \varepsilon \leq +\varepsilon_y \quad (11) \\
& f_s = \pm f_y \quad \text{for} \ \pm\varepsilon_y \leq \varepsilon \leq \pm \varepsilon_{ult} \quad (12)
\end{align*}
\]

where, \(f_s\) is the stress for any value of strain \(\varepsilon_s\) in stress strain relation of steel reinforcement. \(E_s\) and \(f_y\) is the modulus and yield value of the steel reinforcement.

Initially, the strains are very small at the bottom fiber resulting in uncracked elastic stage. As the stress reaches the tensile strength of concrete, primary cracks are initiated. At discrete cracked locations the tensile stresses are entirely carried by the steels and also less concrete sections are effective in resisting the load, resulting reduction in the stiffness of the beam. The decrease in the stiffness of beams increases with increase in loading. Thus, the slope of the load-deformation behavior also decreases as shown by the crack propagation stage.

When the stresses in reinforcement reach the yield value deformation increases quickly with little increase in loading. Finally, beam fails due to crushing of concrete as shown by the plastic stage\(^21\).

Material Property of CFRP

Carbon fiber reinforced polymer (CFRP) consists of carbon fiber embedded in a matrix of polymer resin. Carbon fiber provides strength and stiffness to composite and matrix protects the fiber from environmental impact and ensures proper sharing of the load among fiber. Typical stress-strain behavior of carbon fiber, matrix and CFRP is shown in Fig. 6b. The strain of the carbon fiber is less than the matrix\(^26-27\).

The material property of CFRP is either considered as linear isotropic or linear orthotropic property. The thickness of CFRP is taken as 1.2mm. The material properties for the CFRP are shown in Table 1.
TABLE 1
MATERIAL PROPERTIES OF CFRP

<table>
<thead>
<tr>
<th>Isotropic property</th>
<th></th>
<th>Linear orthotropic property</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>Values</td>
<td>Parameter (MPa)</td>
<td>Values</td>
</tr>
<tr>
<td>Exx (MPa)</td>
<td>165000</td>
<td>Exy</td>
<td>165000</td>
</tr>
<tr>
<td>Pxx</td>
<td>0.3</td>
<td>Exz</td>
<td>9650</td>
</tr>
<tr>
<td>Eyx</td>
<td>9650</td>
<td>Pyz</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Fig. 6 (a) Schematic diagram of CFRP (b) stress strain plot for CFRP

FINITE ELEMENT MODELING OF RCC BEAM

Finite element modeling comprises of using an idealized element and meshing of elements to replicate the RCC beam. In this study discrete method is used to model the RCC beam (See Fig. 7). A perfect bond is considered between the concrete and steel reinforcement.

A 3D finite element was used to model the concrete. The element contains 8 nodes with 3 degrees of freedom at each node: translation in X, Y and Z directions (see Fig. 8). The element is capable of cracking in three orthogonal directions in tension, crushing in compression and plastic deformation.

Linear isotropic and nonlinear inelastic multi-linear isotropic based on Von Mises failure criterion material properties were used for the solid element. The cracking and crushing behavior of concrete is based on William and Warnke model. The inputs required to implement these behaviors are listed in Table 2, Table 3 and Table 4. The value of the shear transfer coefficient as suggested by Wolanski, 0.3 and 1 for open and closed crack were considered to avoid the convergence problem. The element is also used for modeling of loading and support plates. Linear isotropic material model was considered for the loading and supporting plates. The input parameters required are modulus of elasticity equal to modulus elasticity of steels and Poisson’s ratio (0.3).
TABLE 2
CONCRETE ELASTIC PROPERTIES

<table>
<thead>
<tr>
<th>Linear Isotropic</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>30000 N/mm²</td>
</tr>
<tr>
<td>$\emptyset$</td>
<td>0.2</td>
</tr>
</tbody>
</table>

TABLE 3
CONCRETE INELASTIC PROPERTIES

<table>
<thead>
<tr>
<th>Nonlinear inelastic- Multi-linear isotropic</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Strain</td>
<td>Stress</td>
</tr>
<tr>
<td>Pt. 1</td>
<td>0.0003</td>
</tr>
<tr>
<td>Pt. 2</td>
<td>0.0006</td>
</tr>
<tr>
<td>Pt. 3</td>
<td>0.0009</td>
</tr>
<tr>
<td>Pt. 4</td>
<td>0.0012</td>
</tr>
<tr>
<td>Pt. 5</td>
<td>0.0015</td>
</tr>
<tr>
<td>Pt. 6</td>
<td>0.0020</td>
</tr>
<tr>
<td>Pt. 7</td>
<td>0.0022</td>
</tr>
</tbody>
</table>

TABLE 4
CONCRETE FAILURE PROPERTIES

<table>
<thead>
<tr>
<th>Constants</th>
<th>-</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear transfer coefficient for open crack $\beta_i$</td>
<td>0.35</td>
<td></td>
</tr>
<tr>
<td>Shear transfer coefficient for closed crack $\beta_c$</td>
<td>0.95</td>
<td></td>
</tr>
<tr>
<td>Uniaxial tensile strength of concrete $\sigma_t$</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Uniaxial compressive strength of concrete $\sigma_c$</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>Biaxial compressive strength of concrete $\sigma_{bc}$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Ambient hydrostatic stress state $\sigma_h$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Biaxial crushing stress under the ambient hydrostatic stress state $\sigma_1$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Uniaxial crushing stress under the ambient hydrostatic stress state $\sigma_2$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Stiffness multiplier for cracked tensile condition</td>
<td>-</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Whenever RCC beam cracks at discrete locations as load exceeds the tensile strength, stiffness reduces at this discrete location compared to surrounding concrete elements. In FEA, this stiffness is suddenly reduced to zero at cracking creating convergence problem. A stress relaxation technique was used to gradually reduce the stiffness surrounding the cracked element and solve the convergence problem. The value of the stiffness multiplier for cracked tensile condition was taken as 0.6²⁹.

In a pure compression failure of concrete secondary tensile strains are induced by Poisson’s effect perpendicular to the load. Because of concrete weak in tension these cracks eventually lead to failure. Therefore, in this study, the crushing capability was turned off³⁰.

3D spar element was used to model the reinforcement. The element has 3 degrees of freedom at each node: translation in X, Y and Z directions. The element is capable experiencing uniaxial tension, compression and plastic deformation. The geometry and dofs at each node are shown in Fig. 9. Two real constant was used for the longitudinal reinforcement and stirrups.

Another two real constant are required because of the symmetry considered along longitudinal and transverse direction, which reduces the reinforcement area by half. The material model required to replicate the reinforcement behavior is nonlinear inelastic bilinear isotropic based Von Mises failure criterion. This material model requires specification of modulus of elasticity ($2 \times 10^5$ MPa), poisons ratio (0.3) and yield stress (420 MPa).

A 3D shell element having in plane membrane stiffness but no out plane bending stiffness has been used for modeling the CFRP. The element has 4 nodes with 3 dof at each node: translation in the nodal X, Y and Z directions (see Fig. 10). The element is capable large deflection, stress stiffening, variable thickness and cloth option.

Fig. 9 3D Spar Reinforcement Element

Fig. 10 3D Shell CFRP Element
NONLINEAR FINITE ELEMENT FORMULATION

The strong form of the differential equation for continua is given by

\[ \nabla \cdot [ a(U) \nabla U ] + f = 0 = \text{Residual} \quad (13) \]

The residuals are minimized using one of the weighted residual approaches as follows:

\[ \int_{\Omega} \{ \nabla \cdot [ a(U) \nabla U ] \} W + \int_{\Omega} f W = 0 \quad (14) \]

where \( W \) is the weighting function and \( U \) is the trail function. After simplifying the integral form results in a weak form.

\[ \int_{\Omega} a(U) \nabla U \nabla W = \int_{\Omega} f W + \int_{\Gamma} R W \quad (15) \]

Expressing in terms of Bilinear and Linear functions,

\[ B(U, W) = L(W) \quad (16) \]

The material is discretized as,

\[ U(x, y, z) = U^h(x, y, z) = \sum_j U_j N_j(x, y, z) \]
\[ W(x, y, z) = \sum_i W_i N_i(x, y, z) \quad (17) \]

Including the discretization process in weak formulation,

\[ \int_{\Omega} a(U) \left( \sum_j U_j \frac{dN_i}{dx} \right) \frac{dN_j}{dx} = \int_{\Omega} f N_i + \int_{\Gamma} R N_i \quad (18) \]

Writing in matrix form:

\[ K_{ij} U_j = f_i \quad (19) \]

Where,

\[ K_{ij} = \int_{\Omega} a(U) \frac{dN_i}{dx} \frac{dN_j}{dx} \quad (20) \]
\[ f_i = \int_{\Omega} f N_i + \int_{\Gamma} R N_i \quad (21) \]

Material Nonlinearity: Expressing modulus as a function of displacement.

\[ a(U) = C_{ijkl} \left( 1 - \sum_k U_k \frac{dN_k}{dx} \right) \quad (22) \]

Material non-linearity including discretization.

\[ a(U) = C_{ijkl} \left( 1 - \sum_k U_k \frac{dN_k}{dx} \right) \quad (23) \]

Stiffness matrix modified to include the material nonlinearity.

\[ K_{ij} = C_{ijkl} \int_{\Omega} \left( 1 - \sum_k U_k \frac{dN_k}{dx} \right) \frac{dN_i}{dx} \frac{dN_j}{dx} \quad (24) \]

Writing in matrix form,

\[ K(U) U = f \quad (25) \]

The Newton Raphson method used to trace the nonlinear material behavior.

\[ r(U) = K(U) U - f \]
\[ r(U) = 0 \]
\[ U_{r+1} = U_r - T^{-1}(U_r) r(U) \quad (27) \]

where,

\[ T(U_r) = \frac{du}{dr(U_r)} = \frac{d[K(U)U - f]}{du} \quad (28) \]
\[ T(U_r) = K(U_r) = \frac{d[K(U_r)]}{du} U_r \quad (29) \]

Writing in matrix form,

\[ T_{ij} = K_{ij} + \sum_{m=1}^n \frac{\partial K_{im}}{\partial u} U_m \quad (30) \]

Here, the iteration process is terminated when the residual \( r(U) \) is very small or the successive displacement is small compared to user specified tolerance. An \( L_2 \) norm of residual or successive displacement was considered in the comparison with the tolerance values.

The \( L_2 \) norm expression for the residual or successive displacement are given as,

\[ L_2 \text{ norm for the residual } ||r||_2 \leq \text{Tolerance} \quad (31) \]
\[ L_2 \text{ norm for the successive displacements, } \frac{||U_{r+1}-U_r||_2}{U_{r+1}} \leq \text{Tolerance} \quad (32) \]

In Newton-Raphson (N-R) method, end load step are divided into a number of small increments called substep to predict the nonlinear changes in the structural behavior. N-R method provides convergence requirement at the end of each load step with user specified tolerance. To achieve the convergence of a solution tolerance was increased by 5 times to the default values due to the nonlinear behavior of RCC beam.
NUMERICAL STUDIES

In the present work, three dimensional nonlinear finite element analysis of RCC beam has been made to capture the crack formation and growth in flexure\textsuperscript{32}. A four point loaded beam is considered for nonlinear analysis. The geometry and loading details of the beam considered in the present work are taken from earlier experimental works done by the Blockhouse and Foley and are available in the literature\textsuperscript{32}.

Nonlinear analysis is performed in two cases namely with and without hanger bars to simulate the crack pattern. Considering symmetry in both longitudinal (loading) and transverse direction (geometry), only one fourth of the beams were considered in making the FE model. Both 3D solid concrete and 3D spar element formulations are used in the concrete and reinforcement for the discretization of the beam. Three numerical examples have been considered. In the first example a simple reinforced concrete beam with and without hanger bars subjected to four point load is considered for analysis. In the second numerical example a reinforced concrete beam without hanger bars and strengthened with a layer of CFRP is considered for analysis. In the third example a parametric study is performed by varying the amount of reinforcement steel and considering the beam to be strengthened with CFP. The loads are gradually applied in small increments till the failure of the beam and crack pattern observed at the end of each load increment are recorded. The numerical examples are discussed in detail in the following sections.

Numerical Example 1

A flexural beam with four point load was considered for the FE analysis. The geometry and reinforcement details of beam are shown in Fig. 11. Here, the beam RB1 and RB2 represents the beam without and with hanger bars considered for the study. The shear reinforcement provided is sufficient enough for the shear strength of the beam. The longitudinal reinforcement is sufficient enough to show sufficient cracks or warning before the failure of the beam.

![Diagram of beam with reinforcement details](image)

**Fig. 11** Geometrical details of RCC beam (a) Longitudinal section details (b) Cross section details Foley and Buckhouse Experiment 1998
Cracking patterns in RB1

The entire behavior of RCC beam is divided into three stages: linear elastic stage, crack propagation and plastic stage. The linear elastic stage depends on the tensile strength of concrete. Once applied load exceeds tensile strength cracks are initiated at the bottom fiber of the RCC beam. In this study, the initial cracks are identified at load 18.735 kN as shown in Fig. 12.

Fig. 12 first crack at load 18.735kN for RB1

The plot of stress distribution at first cracking is shown in Fig. 13. Experimentally it is difficult to find the load corresponding to first crack. The first crack is identified at a distance of 2.075m from the left support (Fig. 12). As the load crosses 18.735kN flexural cracks start propagating in the vertical direction as shown by the red circle outline in Fig. 14. At this stage the loads are carried by the reinforcement at the cracked element.

Flexural diagonal cracks are identified at 35kN load. In general, these cracks are identified between the loading and support plate as shown by an incline red circle outlines in Fig. 15. At this stage entire load in tension region is carried by the reinforcement. The yielding of steel is identified at 57.0 kN load.

The second and third cracks, green and blue circle outline in tension region indicating the yielding of steel reinforcement as shown in Fig. 16. The deflection of the beam increases at a faster rate after the yielding of reinforcement showing large numbers of cracks in the constant moment region. These cracks are shown in Fig. 17 by a large numbers of green and blue cracks. RCC beam reaches ultimate failure at 84kN load shown in Fig. 18.

Fig. 13 Stress plot at first cracking for RB1

Fig. 14 Flexural diagonal cracks for RB1 at load (a) 18.84kN (b) 35.76 kN

Fig. 15 Flexural diagonal cracks for RB1 at load (a) 44.64kN (b) 54.00 kN

Fig. 16 RB1 - Cracks during yielding of steel reinforcement at load (a) 57.60 kN (b) 64.48 kN (c) 68.52 kN
Cracking at 69.96 kN                 a)
Cracking at 78.24 kN (b)
Crack propagation in to compression

Fig. 17 RB1 - Cracks after yielding of steel reinforcement at load
(a) 69.96 kN (b) 78.24 kN

Cracking at 78.24 kN (b)
Cracking at 84.00 kN (a)

Fig. 18 Cracks at ultimate failure (a) RB1 at load 84.00 kN (b) RB2 at load 77.00 kN

The load-deformation behavior of RCC beam is as shown in Fig. 19. All the three stages: linear elastic, crack propagation and plastic stage are simulated exactly. The point at which the RCC beam collapse is identified by insoluble convergence failure of the solution.

The outcome of the analysis is compared with the experimental results as shown in Fig. 19. It is observed that 3D nonlinear FE analysis of RCC beam almost matches with the experimental results, showing stiff and ductile response.

Table 5 gives a numerical comparison between RCC and Experimental results. The result also indicates that RCC beam losses it’s ductility provision of hanger bars. In the next numerical example, RCC beam RB1 is post strengthened with CFRP attached in tension region. Also, a parametric study is done for various lengths of CFRP. The behavior of RCC beam RB2 is studied by varying the area of steel reinforcement in the tension region. Also, beam RB2 are strengthened with CFRP for different cases of area steel reinforcement.

The study of crack patterns reveals the load at which initial crack, flexural crack, flexural diagonal cracks is started. Finite element software is evaluated by comparing the mechanical behavior of RCC beam with and without hanger with the Buckhouse experiment. The crack pattern depends on the area of longitudinal reinforcement provided. The larger the area of reinforcement beam fails to show larger cracks (more warning) and smaller the area of reinforcement beam fails showing a few cracks (less warning). Hence, beam with hanger bar was considered to study the crack pattern for various area of reinforcement and at the same time under, balanced and over sections are identified by comparing the mechanical behavior of the beam. Later, the flexural capacity of RCC beam is post strengthened using CFRP. A parametric study was also made considering various lengths CFRP.

A 3D shell element was used for the modeling of CFRP. Beam with and without hanger bar considered for the post strengthening. The interface between beam and CFRP is considered as perfect bond. The crack
pattern simulated and mechanical behavior of beam post strengthened with CFRP are compared.

**Numerical Example 2**

In the second study, RCC beam RB1 are strengthened using CFRP attached in tension region. Using same geometry and reinforcement shown in RB1, details of the CFRP are shown in Fig. 20. Two different material properties were considered for the CFRP: isotropic and orthotropic. It is assumed that a perfect bonding exists between the RCC beam and CFRP.

Three different lengths of CFRP are considered: Effective length, half of the clear span and three fourth of the clear span. CFRP are attached at the bottom corner of the RCC beam, as it is observed that initiation of cracks starts in this region. Only one CFRP was used FE modeling taking advantage of symmetry.

The behavior of RCC beam strengthened with CFRP is shown in Fig. 21 and Fig. 22. It is obvious that, as the length of CFRP increases ultimate strength of the RCC beam increases. In case of CFRP with isotropic property (Unidirectional property), ultimate strength is higher for the RBC2 compared to RBC1 indicating CFRP losses stiffness as the length exceeds beyond the critical length. Also, the beam becomes less ductile by provision of CFRP with isotropic property compared with orthotropic property.

**CFRP with Orthotropic property** (Fig. 23a, c and e): Cracks in RCC beam is directly depends on the length of the CFRP, larger the length of CFRP more the cracks in the beam showing larger load carrying capacity and ductility. But, beam losses its ductility with decrease in length of CFRP.

**CFRP with Isotropic property** (Fig. 23b, d and f): Cracks in RCC beam is not directly depends on the length of the CFRP. More cracks are identified for CFRP length equal to half the clear span of the beam. Indicating beam loses its stiffness beyond the critical length.

Comparing both isotropic and orthotropic property of CFRP it is observed that, for smaller length of CFRP with isotropic property shows higher load carrying capacity and ductility than the orthotropic property.

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**Fig. 20** RCC beam strengthened with CFRP at tension region (a) Longitudinal details of RBC1 (b) cross sectional details for various lengths of CFRP
The objective in this numerical example is identifying the under, balanced and over reinforced section by increasing the area of steel reinforcement for the beam RB2. Later the flexural capacity of typical under, balanced and over section of RCC beam RB2 is post strengthened using CFRP in the tension region to compare the material behavior.

Hence, geometry and reinforcement details of RB2 are used in this study as shown in Fig. 26. In the earlier study it is observed that CFRP with orthotropic property is more effective than isotropic property. The length of CFRP is taken as the effective length of the RCC beam comparing the earlier studies.

The load deformation behavior of RCC beam with increase area of steel reinforcement is shown in Fig. 24. Load deformation behavior shows increased yield, ultimate and ductile with the increase in the area steel reinforcement up to a critical value. Beyond this value the load deformation behavior becomes brittle with an increase in the yield and ultimate strength value. Hence, area of reinforcement equal to the critical value is termed as balanced reinforced section, below termed as under reinforced section and above termed as over reinforced section. Area steel reinforcement considered in the case RB1 and RB2 indicates an under reinforced section.
The crack pattern for the under, balanced and over reinforcement are shown in Fig. 27a through Fig. 27d and Fig. 28e through Fig. 28h. It is observed that, failure of the beam in under reinforced section is due to flexure dominated cracks, tensile failure. It is shown by a large numbers of green and red circle outlines in the constant moment region. In balanced reinforced section the failure of the beam is due to compression failure of the concrete. It is shown by a red circle outline in the compression region. In over reinforced section the failure of the beam is due to the shear failure of concrete. It is shown by a large numbers of inclined red circle outline between the support and loading points.
SUMMARY

In this present work a crack modeling methodology in three dimensions for carbon fiber reinforced polymer (CFRP) strengthened RCC beam is made by performing a three dimensional nonlinear finite element analysis of the beam subjected to four point loading. The results are comparable with the experimental results available in the literature. FE analysis of RCC beams without (RB1) and with (RB2) hanger bars are carried out and the results are compared with the experiments.

REFERENCES

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(Discussion on this article must reach the editor before September 30, 2013)