Design and Motion Planning of a Semi-Passive Mobile Platform

Amir Shapiro, Shraga Shoval Member, IEEE

Abstract

This paper presents a novel design and a motion planner for a semi-passive mobile robot. The robot consists of an upper circular body and three identical semi-passive driving mechanisms. Each mechanism consists of a passive wheel that can freely roll, a rotation actuator along the normal axes and a linear actuator for motion along the radial direction of the upper body center. The robot is equipped with an inclinometer to measure the surface slope. Each wheel is also equipped with a rotational encoder to measure roll. Using an odometric model, data from these encoders determines vehicle position. Kinematic analysis provides tools for designing a motion path that steers the robot to the desired location, and determines the singular configurations. Due to the passive roll, there is no longitudinal slippage, and lateral slippage is determined from the kinematic and odometric models. This enables accurate and reliable localization even with slippage. A gait pattern planer for downhill, as well as horizontal and uphill surfaces is presented. A prototype robot has been built and field tested. Experimental results verify the suggested models.

Index Terms

Passive motion planning, skid steering, slippage.

I. INTRODUCTION

Wheel slippage is one of the dominant features that affect the efficiency, reliability, feasibility and stability of mobile robot motion. Uncontrolled slippage causes undesired motions that result in erroneous position and orientation. The most common method for autonomous relative position estimate - odometry, is subject to unbounded errors due to slippage [1], and requires an additional positioning system (e.g. Map-Matching, GPS, Beacon-Based Triangulation). This problem becomes critical when no absolute positioning system is available (e.g. space, underground or indoor missions). Furthermore, additional tasks such as trajectory planning and obstacle avoidance cannot be reliably performed in the presence of uncontrolled slippage. Many researchers deal with robot-surface interaction, particularly on slippery terrains. Bidaud et. al. [2] deal with wheel-soil interaction models. Iagnemma et. al. [3] describe terrain estimation and sensing methodology using visual, tactile and vibrational feedback. Ferretti et. al. [4] exploits high resolution encoders to compensate for non linear friction terms. Physics based motion control that involves a model of traction mechanics with the consideration of force distribution among the wheels is discussed in [5]. In this approach the wheel-soil contact angle and the distribution of the load on each wheel are considered,
and a control system maximizes traction between the vehicle and the terrain. Yoshida and Hammano [6] investigate the tire-soil traction mechanics as well as the body-suspension-wheel dynamics of a mobile robot.

Conventional locomotion uses legs or powered wheels to generate motion. In contrast, our robot relies on relative motion of the joints to generate motion of the central body similar to the motion of a downhill skier on an icy surface, or locomotion on rollerblades. A novel robot design that has the ability to switch between skating and walking modes is the Roller-Walker [7]. This quadruped robot has the ability to switch between walking and skating modes. Passive wheels at the end of each leg fold flat to allow the robot to walk. In the skating mode, the wheels are rotated into place to allow the robot to carry out skating motion. Another example is the ROLLERBLADER [8]. This robot is different from the Roller Walker in its ability to raise the rollerblades off the ground. This allows the use of gaits that mimic those used by human rollerbladers. Shimizu [9] developed both a skiing robot and a snowboarding robot that can model how humans perform turns on skis or snowboard.

At the beginning of this paper we assume that the robot’s wheels does not slip. In the longitudinal direction this assumption is justified since the wheels are passive and can freely rotate. While in the lateral direction (perpendicular to the wheel) slippage can occur but is fully detectable through our slippage detection methodology. Our motion planning algorithm consists on the no-slippage assumption and uses slippage only for braking in downhill motion. In section V we provide the specific conditions that prevent slippage of the wheels under quasi-static motion assumption. The quasi-static assumption requires the inertial forces to be small relative to other forces applied on the system. This assumption holds true in the case of uphill motion.

The paper is organized as follows: In Section II we describe the robot design. Section III provides kinematic analysis and geometrical insights of singular configurations under no lateral slippage conditions. Section IV suggests an odometric model for localization and lateral slippage detection. Section VI describes motion patterns for downhill, uphill and horizontal locomotion. Section VII presents experimental results that verify the motion planner and the odometric model. Section VIII provides the conclusions.

II. ROBOT DESCRIPTION

The robot consists of an upper circular body and three identical semi-passive driving mechanisms shown in Fig. 1. Each driving mechanism consists of a passive wheel that rolls freely along its longitudinal direction. The mechanism has two actuators: an actuator that rotates the wheel along the normal to the central body, and a linear actuator that allows a lateral motion along the radial direction of the upper body center, shown in Fig. 2. Both actuators use Pittman DC servo motors. The linear actuators use a lead screw mechanism with two parallel slide guides and linear bearings. The rotational actuator uses timing belt mechanism to reduce the total robot height and lower the center of gravity. The robot is equipped with studded-like tires to increase traction and reduce slippage. The wheels can be easily replaced by ice skating blades or skis for motion on icy or snowy surfaces. The robot is equipped with an inclinometer to measure the surface slope.

The robot is equipped with six degrees of freedom, allowing for changes in the internal configuration which are required for various motion patterns (as described in the following sections). The rotation along
the normal to the central body determines the longitudinal rolling direction of the wheel. Since wheels are passive, we assume no longitudinal slippage (lateral slippage is permitted\(^1\)). Each of the passive wheels is equipped with a rotational encoder to measure rolling. Data from these encoders is used by the odometric model for relative position and orientation estimation of the robot. Furthermore, based on the kinematic and odometric models (discussed in Sections III and IV), the amount of lateral slippage on each wheel can be determined.

We now define the robot parameters used in the following sections. Figure 3 shows a top view of the robot.

\(\) Nomenclature:
\[ \begin{align*}
\mathbf{d}_b &= (x_b, y_b) : \text{the central base position} \\
\theta_b &= : \text{the central base orientation} \\
\mathbf{p}_i (i = 1, 2, 3) &= \text{position of the center of the } i^{th} \text{ wheel} \\
\theta_i (i = 1, 2, 3) &= \text{rotation of the } i^{th} \text{ wheel along the normal to the body} \\
d_i (i = 1, 2, 3) &= \text{the distance of the } i^{th} \text{ wheel from the base center} \\
\mathbf{r}_i (i = 1, 2, 3) &= \text{unit vector from the base center to the } i^{th} \text{ wheel} \\
\mathbf{c}_i (i = 1, 2, 3) &= \text{unit vector in the lateral direction of the } i^{th} \text{ wheel} \\
\phi_i (i = 1, 2, 3) &= \text{rolling angle of the } i^{th} \text{ wheel} \\
&\quad \text{: (measured by the encoders attached to the wheels)}
\end{align*} \]

**III. Kinematic Analysis**

Our goal in this section is to develop the kinematic system that governs the robot motion. Additionally, we wish to give some practical insight of this kinematic system. The robot’s c-space (configuration space) contains nine parameters, \(\mathbf{q} = (x_b, y_b, \theta_b, d_1, d_2, d_3, \theta_1, \theta_2, \theta_3) \in \mathbb{R}^9\), out of which only six are actuated. Hence the robot’s central base is un-actuated and the entire mechanism is said to be under-actuated. The goal is to design a motion path for the actuated joints such that it steers the entire robot to a desired location. To begin, we compute each wheel’s center point location using rigid body transformation:

\[
\mathbf{p}_i = \mathbf{d}_b + R_0 d_i \mathbf{r}_i \quad \text{for } i = 1, 2, 3
\]

\(^1\)This assumption is valid only for wheels with small inertia and for relatively small accelerations.
where $R_b$ is the rotation matrix of the central base angle $\theta_b$. The wheel’s center point velocity is the time derivative of (1):

$$\dot{\mathbf{p}}_i = \dot{\mathbf{d}}_b + R_b \dot{\mathbf{d}}_i \mathbf{r}_s - \dot{\theta}_b J R_b \dot{\mathbf{d}}_i \mathbf{r}_s \quad \text{for } i = 1, 2, 3$$

(2)

where $J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

Assuming no lateral slippage, the wheel’s center point velocity has no component along the lateral direction. Therefore:

$$c_i^T \dot{\mathbf{p}}_i = 0 \quad \text{for } i = 1, 2, 3$$

(3)
\[ \mathbf{c}_i = R_b R_i (1, 0)^T \] is a unit vector in lateral direction of the \( i^{th} \) wheel, and \( R_i \) is the wheel’s rotation matrix of angle \( \theta_i \). Let us make the following definitions:

\[
V(q) = -\text{diag}(\mathbf{c}_1^T R_b \dot{\mathbf{r}}_1, \mathbf{c}_2^T R_b \dot{\mathbf{r}}_2, \mathbf{c}_3^T R_b \dot{\mathbf{r}}_3) \in \mathbb{R}^{3 \times 3}
\] (4)

and

\[
K(q) = \begin{bmatrix}
\mathbf{c}_1^T & -d_1 \mathbf{c}_1^T J R_b \dot{\mathbf{r}}_1 \\
\mathbf{c}_2^T & -d_2 \mathbf{c}_2^T J R_b \dot{\mathbf{r}}_2 \\
\mathbf{c}_3^T & -d_3 \mathbf{c}_3^T J R_b \dot{\mathbf{r}}_3
\end{bmatrix} \in \mathbb{R}^{3 \times 3}.
\]

After substituting the expression of \( \dot{p}_i \) into (3) the no-slippage constraint can be written in matrix form as follows:

\[
\begin{bmatrix}
\dot{\theta}_b \\
\dot{d}_1 \\
\dot{d}_2 \\
\dot{d}_3
\end{bmatrix} = G(q) \begin{bmatrix}
\dot{d}_1 \\
\dot{d}_2 \\
\dot{d}_3
\end{bmatrix} \quad \text{where} \quad G(q) = K^{-1}(q) V(q) \in \mathbb{R}^{3 \times 3}.
\] (5)

This constraint depends on velocities as well as on the configuration. Therefore, (5) introduces three non-holonomic constraints, and the robot is said to be a non-holonomic, under-actuated system.

We are now ready to introduce the robot’s kinematic system.

**Theorem 1:** Let \( u = (u_d, u_\theta)^T \in \mathbb{R}^6 \) be vector of control inputs, where \( u_d = (\dot{d}_1, \dot{d}_2, \dot{d}_3)^T \) and \( u_\theta = (\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3)^T \). Then the robot’s kinematic system is:

\[
\dot{q} = \begin{bmatrix}
G(q) u_d \\
u_\theta
\end{bmatrix}.
\] (6)

**Proof:** The proof consists of utilizing the constraints equation (5) which expresses the central body velocities in terms of the robot’s configuration and the linear joints velocities. This together with the definition of the actuated joints’ velocities as control inputs yields the robot’s kinematic system of the theorem.

The central base velocity is uniquely determined by the linear actuators’ velocities only if \( G(q) \) is of full rank. Moreover, if \( \text{rank}(G(q)) < 3 \) existence and uniqueness of solution to the robot’s kinematic system is not assured.

Thus in the next Proposition we analyze when \( G(q) \) lose it’s full rank, and we show the geometrical implications of the configurations where \( G(q) \) is singular.

**Proposition 3.1:** Let us define three lines along the axes of the robot’s wheels as follows: \( l_i = p_i + t_i \mathbf{c}_i \) for \( i = 1, 2, 3 \), where \( t_i \) is a length parameter along the \( i^{th} \) line (Fig. 3). Matrix \( G(q) \) is of full rank if, and only if, the three lines \( l_1, l_2, \) and \( l_3 \) do not intersect in a single point and are not mutually parallel and all wheels are not in radial direction.

The proof of this proposition appears in Appendix I.

If the robot is not in a singular configuration (i.e. \( \text{rank}(G(q)) = 3 \)), the central base velocity is fully controllable using the linear actuators velocities. Later on we use this fact to conduct uphill motion. However, singular configuration can be used for free slide in downhill motion.
IV. ODOMETRIC MODEL AND SLIPPAGE DETECTION

In this section we describe the odometric model of the robot and a method for slippage detection. As previously discussed, each wheel is equipped with a rotational encoder to measure the passive roll - \( \phi_i \), (Fig. 3). Given the central body velocity, the wheels’ center point velocity is computed in (1). Taking the derivative of \( \phi_i \) and multiplying by the wheel radius - \( W_r \), gives the \( i^{th} \) wheel center point longitudinal velocity. Equating the latter term with the \( i^{th} \) wheel center point velocity projected on the longitudinal direction, denoted \( \hat{c}_i = -Jc_i \), results in:

\[
\hat{c}_i^T \dot{p}_i = \dot{\phi}_i W_r \quad \text{for } i = 1, 2, 3.
\]  

(7)

Based on (7) it is possible to evaluate the central base velocity while measuring the passive wheels’ rotation velocities and the actuators’ positions and velocities. The following theorem represents this idea.

**Theorem 2**: Let us define the \( 3 \times 3 \) matrix \( \hat{K}(q) \) as follows:

\[
\hat{K}(q) = \begin{bmatrix}
\hat{c}_1^T & -d_1 \hat{c}_1^T J R_0 r_1 \\
\hat{c}_2^T & -d_2 \hat{c}_2^T J R_0 r_2 \\
\hat{c}_3^T & -d_3 \hat{c}_3^T J R_0 r_3 
\end{bmatrix} \in \mathbb{R}^{3 \times 3}.
\]  

(8)

While measuring the rotation velocity of the robot’s passive wheels and the state of the actuated joints, the central base velocity is determined by:

\[
\begin{pmatrix}
\dot{x}_b \\
\dot{y}_b \\
\dot{\theta}_b
\end{pmatrix} = \hat{K}^{-1}(q) \begin{pmatrix}
\dot{\phi}_1 W_r - \dot{d}_1 \hat{c}_1^T R_0 \hat{r}_1 \\
\dot{\phi}_2 W_r - \dot{d}_2 \hat{c}_2^T R_0 \hat{r}_2 \\
\dot{\phi}_3 W_r - \dot{d}_3 \hat{c}_3^T R_0 \hat{r}_3
\end{pmatrix}.
\]  

(9)

**Proof**: The proof consists of substituting \( \dot{p}_i \) from (2) into (7) and rewriting the set of three equation in matrix form. Substitution of \( \dot{p}_i \) from (2) into (7) results with:

\[
\hat{c}_i^T d_0 - \hat{c}_i^T \dot{\theta}_b J R_0 d_i \hat{r}_i = \dot{\phi}_i W_r - \dot{d}_i \hat{c}_i^T R_0 \hat{r}_i \quad \text{for } i = 1, 2, 3.
\]  

(10)

In order to solve (10) to obtain the base velocity we write (10) in matrix form as follows:

\[
\begin{pmatrix}
\dot{x}_b \\
\dot{y}_b \\
\dot{\theta}_b
\end{pmatrix} \hat{K}(q) = \begin{pmatrix}
\dot{\phi}_1 W_r - \dot{d}_1 \hat{c}_1^T R_0 \hat{r}_1 \\
\dot{\phi}_2 W_r - \dot{d}_2 \hat{c}_2^T R_0 \hat{r}_2 \\
\dot{\phi}_3 W_r - \dot{d}_3 \hat{c}_3^T R_0 \hat{r}_3
\end{pmatrix}.
\]  

(11)

The central base velocity (9) is simply the solution of (11).

Numerical integration of the central base velocity along the motion path determines the robot’s central base position. Note that as long as \( \hat{K}(q) \) is not singular it is possible to calculate the base position even under lateral slippage.

The following proposition asserts the condition for \( \hat{K}(q) \) to be of full rank.

**Proposition 4.1**: Let us define three lines along the axes of the robot’s wheels as follows: \( \tilde{l}_i = p_i + \hat{c}_i \hat{c}_i \) for \( i = 1, 2, 3 \), where \( \hat{c}_i \) is a length parameter along the \( i^{th} \) line (Fig. 3). Then matrix \( \hat{K}(q) \) is of full rank if, and only if, the three lines \( \tilde{l}_1, \tilde{l}_2, \) and \( \tilde{l}_3 \) do not intersect at a single point and are not mutually parallel (in this case it said to be intersecting at infinity).

**Proof**: We observe that \( \hat{K}(q) \) has exactly the same structure as \( K(q) \) except of replacing \( c_i \) with \( \hat{c}_i \).
Thus if the lines $\tilde{l}_i$ will be in the direction of $\tilde{c}_i$ instead of $c_i$ the condition for $\tilde{K}(q)$ to become singular is that the three lines intersect at a single point (or be mutually parallel). The latter fact is based on the proof of proposition 3.1.

**Slippage detection method:** After evaluating the central body velocities, it is possible to compute each wheels’ center point velocity according to (1). This velocity vector, $\tilde{p}_i$, can be divided into two components: the longitudinal component, $\tilde{c}_i^T \tilde{p}_i$, and the radial component,

$$\dot{\tilde{s}}_i = \tilde{c}_i^T \tilde{p}_i \text{ for } i = 1, 2, 3.$$  

Note that $\dot{\tilde{s}}_i$ is the lateral velocity of the wheel center point. If the three lines $\tilde{l}_1, \tilde{l}_2$ and $\tilde{l}_3$ (Fig. 1(c)) do not intersect at a single point and are not mutually parallel, then we can explicitly compute the amount of lateral slippage of each wheel.

V. Quasi-static Forces Analysis

In this section we analyze the contact forces between each wheel and the ground. We develop certain bounds that assure no slippage of the wheel. The analysis presented in this section assumes quasi-static force constraints. In quasi-static motion the inertial forces are kept small relative to other forces in the system.

First we have to compute the normal (i.e. normal to the ground surface) reaction force at each wheel contact point. Since the gravitational force act at the robot’s center of mass we need to compute the exact location of the center of mass denoted $r_c$. The robot’s wheel mechanism has considerable weight that changes it position due to changes in the actuators positions. The weight of all the moving parts of each wheel mechanism is denoted $m_i$ for $i = 1, 2, 3$, while the weight of the central base is denoted $M$. Therefore the location of the robot’s center of mass is given by:

$$r_c = \frac{\sum_{i=1}^{3} m_i p_i}{M + \sum_{i=1}^{3} m_i},$$  \hspace{1cm} (12)

where $p_i$ is the $i^{th}$ wheel’s center point location. Let us denote $i, j, k$ the wheels indices and $N_i, N_j, N_k$, the associated normal forces at the contacts. We identify the fact that the normal forces $N_j$ and $N_k$ does not apply torque around the line connecting $p_j$ with $p_k$. The torque applied by $N_i$ around this line is

$$T_{N_i} = \left( \frac{p_k - p_j}{\|p_k - p_j\|} \times (p_i - p_j) \right) N_i,$$  \hspace{1cm} (13)

where the cross of 2D vectors is the scalar product of $a \times b = det[a, b]$. The term $\left( \frac{p_i - p_j}{\|p_k - p_j\|} \times (p_i - p_j) \right)$ is the minimal distance between $p_i$ and the line connecting $p_j$ with $p_k$, since it equals the length of $\frac{p_k - p_j}{\|p_k - p_j\|}$ (which is one unit) times the length of the vector $p_i - p_j$ times $\sin$(the angle between the two vectors). The torque that the gravity applied around the same line is

$$T_g = \left( \frac{p_k - p_j}{\|p_k - p_j\|} \times (r_c - p_j) \right) (M + \sum_{i=1}^{3} m_i) g \cos(\alpha),$$  \hspace{1cm} (14)

where $g$ is the gravity acceleration, and $\alpha$ is the surface inclination (od declination) angle. Note that since the gravity force is applied at the center of mass the term $r_c$ that was computed in (12) appears in (14).
Since $T_g$ and $T_{Ni}$ are the only external torques applied around the line connecting $p_j$ with $p_k$ the two torques must be of the same magnitude and in opposite directions, that is $T_{Ni} = T_g$. From the latter equation of torques we can find out that

$$N_i = (M + \sum_{i=1}^{3} m_i) g \cos(\alpha) \left( \frac{p_k - p_j}{\|p_k - p_j\|} \times (r_c - p_j) \right) / \left( \frac{p_k - p_j}{\|p_k - p_j\|} \times (p_i - p_j) \right),$$

where the indices $i, j$, and $k$ can be changed in cyclic manner over the robot three wheels (e.g. 1, 2, 3 or 3, 1, 2 or 2, 3, 1) to find out all the three normal reaction forces at the contacts between the wheels and the ground.

To prevent the $i^{th}$ wheel from sliding we introduce the friction constraint as follows:

$$F_i < \mu N_i,$$

where $F_i$ is the lateral contact force and $\mu$ is the coefficient of friction between the wheel and the ground. Note that when the wheel is sliding $F_i = \mu N_i$. We use these friction forces for braking in downward motion, in that case we allow sliding of wheels in their lateral direction. In uphill motion we want to prevent sliding, thus we need inequality (16) to be satisfied for $i = 1, 2, 3$.

VI. MOTION PATTERNS

In this section we describe motion planning paradigms for the robot. Since motion is based on a semi-passive mechanism, there are two cases: Uphill and horizontal locomotion, and downhill motion. For both cases we describe two different motion planners.

A. Uphill and Horizontal Locomotion

In horizontal or uphill locomotion, gravitational force cannot be used to conduct motion. Rather, the robot actuators produce the required central body velocity. The motion planning problem is as follows: For a given path, $\alpha(t)$ of the robot’s central body, what should be the actuators’ velocities. The Lafferriere and Sussmann method [10] is an example of such a motion planning method for under-actuated non-holonomic systems. The Lafferriere and Sussmann method requires the system to be nilpotent (i.e. high order of Lie products vanish). However, our system contains trigonometric function whose derivatives never vanish and therefore is not nilpotent.

1) Uphill and Horizontal Analytical Locomotion Paradigm: Our first motion planning paradigm for uphill and horizontal locomotion is a general control algorithm designed to follow a given path. It is based on the analytical kinematic system of the robot. Equation (5) shows that, in order to provide the robot’s central body with any desired velocity, the linear joints should supply the joint’s velocities

$$u_d = V^{-1}(q)K(q)\begin{pmatrix} \dot{d}_b \\ \dot{\theta}_b \end{pmatrix}_{desired},$$

where $(\dot{d}_b, \dot{\theta}_b)_{desired}$ is computed based on the geometry of $\alpha(t)$. Denote $\alpha(t) = (\alpha_x(t), \alpha_y(t))^T$ then $\dot{\alpha}(t) = (\alpha'_x(t), \alpha'_y(t))^T = \dot{\alpha}(t)$ is the central base linear velocity tangential to $\alpha(t)$. Next, $\theta_b$ is the direction the robot is heading to, and it should be directed along the $\dot{\alpha}(t)$ vector. The time derivative of the heading
direction (angle) results with \( \dot{\alpha}_b = \frac{-\alpha'_x(t)\alpha'_y(t)+\alpha'_y(t)\alpha'_z(t)}{\alpha'_x(t)^2+\alpha'_y(t)^2} \) which is related to the curvature of \( \alpha(t) \). Now we can write the desired central body velocity of (17) as follows:

\[
\begin{pmatrix}
\dot{d}_b \\
\dot{\theta}_b
\end{pmatrix}_{\text{desired}} = \begin{pmatrix}
\alpha'_x(t) \\
\alpha'_y(t) \\
\frac{-\alpha'_x(t)\alpha'_y(t)+\alpha'_y(t)\alpha'_z(t)}{\alpha'_x(t)^2+\alpha'_y(t)^2}
\end{pmatrix}.
\]  

(18)

Applying the velocities described in (17) and (18) to the linear actuators provides the central body with the desired velocity and it precisely follows the \( \alpha(t) \) path. This motion is limited by the linear actuators’ stroke. When one of the linear actuators reaches its limit, all actuators stop. Next, the linear actuators return to their initial configuration without causing the robot’s central body to move. From (4) we notice that if the matrix \( V(q) \) is the \( 3 \times 3 \) zero matrix, motion of the linear actuators will not affect motion of the robot’s central body. \( V(q) \) is a diagonal matrix with the terms \( \hat{c}_i^T R_b \hat{r}_i \) on the diagonal. The \( \hat{c}_i^T R_b \hat{r}_i \) terms vanish if each \( \hat{c}_i \) is perpendicular to \( R_b \hat{r}_i \). This happens only when the wheels are in the radial directions.

2) Uphill and Horizontal Motion Primitives: Our second motion planning paradigm is based on using motion primitives. We describe two motion primitives one for linear motion and one for rotation. The two motion primitives can be combined in various manners to produce different motion paths. Figures 4 and 6 illustrate the principle of our motion patterns for linear motion and for circular motion around the robot’s center. Motion consists of four steps, in which some or all wheels change their angular and/or linear configuration resulting in the desired path for the robot’s central body. Other trajectories can be generated using similar patterns.

**Linear motion pattern** consists on four phase motion: First the front two wheels (wheels 2 and 3) rotate \( \Delta \theta_2 \) and \( \Delta \theta_3 \) to the required configuration (Step I). Next, the linear actuators of wheels 2 and 3 move to provide the desired velocity to the central body (step II). The actuators move \( \Delta d_2 \) and \( \Delta d_3 \), resulting in a longitudinal motion of wheels 2 and 3 of \( \Delta p_2 \) and \( \Delta p_3 \), and a central body linear motion of \( \Delta d_b \). It should be noted that \( \Delta p_2 = \Delta p_3 = \Delta d_2 \cos \Delta \theta_3 \), and \( \Delta d_b = \Delta d_2 \sin \Delta \theta_3 \). Once the linear actuators reach their maximum stroke, the wheels rotate such that their longitudinal axes coincide with the radial direction to the base center (in our case \( -\Delta \theta_2 \) and \( -\Delta \theta_3 \)). Finally, the linear actuators return to their initial configuration.

The net force that produce the linear motion is due to the radial reactional friction forces applied on wheels 2 and 3, \( F_2, F_3 \). Figure 5 shows the forces applied on the wheels. The control forces, \( F_{C_2} \) and \( F_{C_3} \), applied by the linear actuators are internal forces that do not produce motion of the overall center of mass. But since in quasi-static analysis all parts must be at equilibrium, and especially the radial forces applied on each wheel must be balanced. Therefore the equation of the balanced radial forces applied on the wheels are:

\[
F_2 = F_{C_2} \cos(\theta_2 - \beta), \quad \text{and} \quad F_3 = F_{C_3} \cos(\theta_3 - \beta).
\]  

(19)

Hence the net radial force applied on the robot is the vector sum of the friction forces \( F_2 \) and \( F_3 \). From symmetry the net forward force is simply

\[
\text{net forward force during linear motion} = F_2 \sin(\theta_2) + F_3 \sin(\theta_3),
\]
where $F_2$ and $F_3$ can be computed from (19). Since radial slippage is undesired then the frictional constraint is introduced $F_2 \leq \mu N_2$ and $F_3 \leq \mu N_3$, where $N_2$ and $N_3$ are computed in (15).

**Circular motion pattern:** In the first step all wheels simultaneously rotate $\Delta \theta$ at the same direction. Next, all linear actuators move simultaneously the same distance $\Delta d$. This linear motion generates tangential forces that rotate the robot’s body $\Delta \theta_b$ around its center. Once the linear actuators reach their limit, the wheels rotate such that their longitudinal axes coincide with the radial direction to the base center and the linear actuators return to their initial configuration. The rotation around the robot center
\( \Delta \theta_b \) is given by
\[
\Delta \theta_b = \frac{\Delta d}{d} \tan(\Delta \theta)
\]
where \( d \) is the distance between the robot center and the wheels. According to this equation, larger rotation angle of the wheels \( \Delta \theta \) in step I increases the rotation of the robot’s body in step II. However, \( \Delta \theta = 90^\circ \) is a singular configuration in which the body can rotate freely with around its center. It also should be noted that as the wheels approach the robot’s center, the rotation rate increases for the same \( \Delta \theta \). However, the friction forces required for this rotation increase, and eventually break the static friction constraint, resulting in a lateral slippage of the wheels.

In the circular motion the magnitude of the reaction friction forces are identical for all wheels. However it is the direction of the forces and the location of the point of application that make the difference and generating the rotating torque. The magnitude of the contact forces is:
\[
F_i = F_{c_i} \sin(\Delta \theta) \text{ for } i = 1, 2, 3.
\]
In our case of pure rotation we choose all the control forces, \( F_{c_i} \), applied by the linear actuator to to be identical. Thus the magnitudes of all \( F_i \) are identical. The force \( F_i \) composed of radial and tangential components. Because of the symmetry the vector sum of all the radial components vanish. However the tangential part of \( F_i \) generates a rotating torque on the entire mechanism in the amount of:
\[
\tau = \sum_{i=1}^{3} d_i F_{c_i} \sin(\Delta \theta) \cos(\Delta \theta).
\]
It can be shown that \( \tau \) is maximal for \( \Delta \theta = \pm 45^\circ \) or \( \Delta \theta = \pm 135^\circ \).

Other trajectories can be generated using similar patterns. For example, a rotation around one of the robot’s wheels is shown in Figure 8 (in this figure around wheel 3). In step I wheel 2 rotate \( 60^\circ \) and
wheel 3 rotate $90^\circ$ to the configuration shown. Next, the linear actuator of wheel 1 generates the rotation of the body by moving $d_1$. In step III the wheels rotate back to the radial configuration and in step IV the linear actuator of wheel 1 returns to the initial configuration.

**B. Downhill Locomotion**

In downhill motion the gravitational force is used for dragging the robot downward. Motion patterns are similar to the skiing patterns used in downhill skiing. Each of the various downhill skiing methods can be implemented in our mechanism, using the three semi passive driving wheels. We start with a robot configuration in which the lines $l_i$’s intersect in a single point and the matrix $K(q)$ is singular.
In this case the robot is constrained to move along an arc shaped path. The center of the arc is in the intersection point of the l₁, l₂ and l₃ lines. Since the robot is an Euler-Lagrange system and since there is friction in the wheels' bearings, the system is passive and governed by gravitational potential energy. According to Koditschek [11] the configuration in which the system’s potential energy is minimal is an asymptotically stable equilibrium point of the system. According to this observation, we find the radius and center of curvatures at each point of the desired motion path. Then we continuously set the l₁’s intersection point at the center of curvature of the desired path by changing the robot’s configuration. This way, the robot passively glides along the desired path, similar to the way a slalom skier turns. Figure 9 shows a downhill motion pattern of the robot in this mode. From the initial static configuration, each wheel is rotated Δθᵢ such that all ēᵢ (unit vectors in the lateral direction to wheel i) intersect in a single point. The robot can move to the minimal potential energy position (where it eventually comes to a halt), or it can change its configuration towards a new target position. The robot can also stop at any point along the path by changing its internal configuration to the radial configuration (all three wheels are radial to the robot center). A specific configuration of this mode is when the center of curvature is at infinity, resulting in a configuration where all wheels are mutually parallel. This is similar to a parallel downhill skiing or gliding. Again, this is a singular configuration, but unlike the previous configuration, there is no asymptotical stable equilibrium point, the robot’s speed is limited only by the friction in the bearings and the air drag. The robot can come to a halt only by changing the wheels configuration either to the radial configuration (total stop) or to minimal potential energy point.

In the snow plough motion pattern (similar to the snowplough skiing method) two wheels are rotated in a ”snow plough” configuration, while the third wheel is used for steering. In this mode, speed is controlled according to the slide angle of the wheels relative to the motion direction, as shown in Figure 10. The gravitational downhill drag force is in the magnitude of:

$$\text{gravitational downhill drag force} = g(M + \sum_{i=1}^{3} m_i \sin(\gamma)),$$

where g is the gravity acceleration, and γ is the slope of the surface on which the robot moves. Since
wheels 2 and 3 slides on their radial direction their radial friction force is:

\[ F_i = \mu N_i \cos(\Delta \theta_i + 30^\circ) \quad \text{for} \ i = 1, 2. \]

Thus the net downhill forward force applied on the robot is given by:

\[
\text{net downhill forward force} = g\left(M + \sum_{i=1}^{3} m_i \sin(\gamma) - \sum_{i=1}^{2} \mu N_i \cos(\Delta \theta_i + 30^\circ)\right).
\]

Maintaining a constant speed and controlling motion direction requires continuous upgrade of the odometric model (discussed in Section IV), as the lateral slippage varies according to the surface and wheels interaction, slope angle, as well as robot load distribution among the wheels.

VII. EXPERIMENTAL RESULTS

In this section we describe the experiments conducted with our autonomous robot, shown in Figure 11. In the first experiment we examine the linear motion pattern on a horizontal surface. Figure 12 shows the robot configuration (rotation angle and linear actuator of all wheels) during motion. Wheels 2 and 3
perform the required rotation and translation as shown in figure 4, while wheel 1 remains passive. The second part of figure 12 shows the actual wheels locations during motion as determined by our odometric model. Although the nominal path of the robot center is linear, actual path is not linear and bends to the right. This is expected as the experiment is conducted on a non-homogenous surface, and lateral slippage occurs, especially during stage II. This is also the reason for the oscillated motion of wheel 1, which nominally remains passive during that motion. However, the slippage is clearly detected by the odometric model.

Figure 13 shows the robot configuration (rotation angle and linear actuator of all wheels) during circular motion around the center of the robot. Instead of returning to initial configuration (Step III in Figure 6) all wheels are rotated $-2\Delta\theta$ before the linear actuators return. This way rotation of the central body continues during Step IV, resulting in a double rotation angle for a full motion period.

Figure 14 shows a downhill motion using the "snow-plough" method. In this motion all linear actuators remains stationary, and wheels 2 and 3 are rotated until motion starts. Returning to the initial radial configuration stops the motion. The odometric model shows identical, parallel and near-linear motion of
all wheels and robot’s body. The non-linearity of the path occurs at the beginning and end of motion due to rotation of the wheels.

Finally we show an experiment for downhill rotation. In the experiment shown in figure 15, the robot rotates around wheel 3 according to the pattern shown in figure 9. In this pattern wheels 1 and 2 rotate $\Delta \theta_i$ such that $l_1$ and $l_2$ (lines through wheels 1 and 2 in the lateral direction) intersect at the contact point of wheel 3. The robot rotates about wheel 3 and stops when in reaches a minimal potential energy position.

VIII. CONCLUSIONS

In this paper we present a mobile robot, designed for motion on slippery surfaces. The robot consists of a base and three driving mechanisms. Each mechanism can rotate around the normal to the central base and translate along the radial direction to the center of the base. Motion is performed by changes in the internal configuration of the robot, using passive rolling studded-like wheels. Such a mechanism significantly reduces longitudinal slippage, and minimizes lateral slippage. Encoders measure the roll of each wheel. Kinematic model determines the required joints’ velocities that steer the robot to a target
position. Odometric model accurately determines the robot’s position even in the presence of slippage. A method for evaluating the lateral slippage based on the odometric and kinematic models is presented. Gait patterns for motion up and down hills, as well as on horizontal surface are presented. Experimental results verify our models and slippage estimate, and show the reliability and accuracy of motion on slippery surfaces. Field experiments for the suggested gait patterns on various slopes and terrains have been carried out using our prototype model. In future work we intend to develop a dynamic model and investigate the effect of various terrain types on the suggested gait patterns.

APPENDIX I

PROOF OF PROPOSITION 3.1

In this section we provide a detailed proof of Proposition 3.1 as follows:

Proposition 3.2: Let us define three lines along the axes of the robot’s wheels as follows:

\[ l_i = p_i + t_i \hat{c}_i \quad \text{for} \quad i = 1, 2, 3, \]

where \( t_i \) is a length parameter along the \( i^{th} \) line (Fig. 3). Matrix \( G(q) \) is of full rank if, and only if, the three lines \( l_1, l_2 \) and \( l_3 \) do not intersect in a single point and are not mutually parallel and all wheels are not in radial direction.
**Proof:** Since \( G(q) = K^{-1}(q)V(q) \) matrix \( G(q) \) is of full rank if, and only if, matrices \( K(q) \) and \( V(q) \) are of full rank.

Let us begin with the conditions for \( \text{rank}(K(q)) = 3 \). We will show that the condition for \( l_1, l_2 \) and \( l_3 \) to intersect at a single point and the condition for \( \text{rank}(K(q)) < 3 \) are identical. We assume that lines \( l_1, l_2 \) and \( l_3 \) are not mutually parallel (we discuss this case later on). Now we derive the condition for the three lines to intersect at a single point. We find the intersection point of \( l_1 \) and \( l_2 \), then we obtain the condition for \( l_3 \) to pass through this intersection point. To find the intersection point of \( l_1 \) and \( l_2 \) we need to find \( t_1^* \) and \( t_2^* \) such that \( l_1(t_1^*) = l_2(t_2^*) \), where \( t_1^* \) and \( t_2^* \) are the values of \( t_1 \) and \( t_2 \) at the intersection point. Substituting the line definition from (20) into this condition yields:

\[
p_1 + t_1^* \hat{c}_1 = p_2 + t_2^* \hat{c}_2 \tag{21}
\]

Next we substitute \( p_i = d_i + d_iR_i \hat{r}_i \) and \( \hat{c}_i = R_iR_i(1, 0)^T \) into (21), we subtract \( d_i \), multiply by \( R_i^{-1} \), and we end up with:

\[
d_1 \hat{r}_1 - d_2 \hat{r}_2 = (t_2^*R_2 - t_1^*R_1)(1, 0)^T \tag{22}
\]
This is a linear vector equation which contains two scalar equations with two unknowns: \( t_1^* \) and \( t_2^* \). Solving (22) for \( t_1^* \) yields
\[
t_1^* = (1, 0) R_2^T J^T (d_1 \, \hat{r}_1 - d_2 \hat{r}_2) / (1, 0) R_2^T J R_1 (1, 0)^T ,
\]
(23)
where \((1, 0) R_2^T J R_1 (1, 0)^T = \sin(\theta_1 - \theta_2)\). The term \(\sin(\theta_1 - \theta_2)\) vanishes only when \( I_1 \) is parallel to \( I_2 \) which is not the case we are dealing with now. In order for \( I_3 \) to pass through the intersection point of \( I_1 \) and \( I_2 \) it must be directed from \( p_3 \) toward the intersection point. In other words the line connecting the intersection point with \( p_3 \) must not have a component in the direction perpendicular to \( c_3 \) (i.e. the longitudinal direction) as follows:
\[
\hat{c}_3 (l_1 (t_1^*) - (d_b + d_3 R_b \hat{r}_3)) = 0 .
\]
(24)
Next we substitute \( t_1^* \) from (23) into \( l_1 (t_1^*) \) of (24) and evaluate (24) to get the following condition for \( l_1, l_2 \) and \( l_3 \) to intersect at a single point:
\[
d_1 \begin{pmatrix} -\sin(\theta_1) \\ \cos(\theta_1) \end{pmatrix} \sin(\theta_2 - \theta_3) \hat{r}_1 + d_2 \begin{pmatrix} \sin(\theta_2) \\ -\cos(\theta_2) \end{pmatrix} \sin(\theta_1 - \theta_3) \hat{r}_2 + d_3 \begin{pmatrix} -\sin(\theta_3) \\ \cos(\theta_3) \end{pmatrix} \sin(\theta_1 - \theta_2) \hat{r}_3 = 0
\]
(25)
Now we consider the condition for \( \text{rank}(K(q)) < 3 \). Since \( \text{det}(K(q)) = 0 \) implies that \( \text{rank}(K(q)) < 3 \), we compute \( \text{det}(K(q)) = 0 \) and get the second condition as follows:
\[
-d_1 c_1^T JR_b \hat{r}_1 c_2^T J \hat{c}_3 + d_2 c_2^T JR_b \hat{r}_2 c_1^T J \hat{c}_3 - d_3 c_3^T JR_b \hat{r}_3 c_1^T J \hat{c}_2 = 0
\]
(26)
Substitution of \( \hat{c}_i = R_b \hat{r}_i \) into (26) and evaluation of (26) gives the condition for \( \text{rank}(K(q)) < 3 \). It can be shown that the latter condition is identical to (25). Therefore the two conditions must be satisfied simultaneously. Next we consider the case where \( I_1, I_2 \) and \( I_3 \) are mutually parallel. In this case \( \theta_1 = \theta_2 = \theta_3 \) and the two condition are satisfied. We conclude that \( \text{rank}(K(q)) = 3 \) if, and only if, the three lines \( I_1, I_2 \) and \( I_3 \) do not intersect in a single point and are not mutually parallel.

Next we show that \( \text{rank}(V(q)) = 3 \) if, and only if all wheels are not in radial direction. \( V(q) \) is a diagonal matrix with the terms \( c_1^T R_b \hat{r}_1 \) on its’ diagonal. Note that \( \hat{c}_i \) is the wheel’s lateral direction while \( R_b \hat{r}_i \) is the radial direction. Therefore if a wheel is in radial direction (i.e. the wheel’s longitudinal direction is along the radial direction) the lateral direction the wheel’s is perpendicular to the radial direction and \( c_1^T R_b \hat{r}_i = 0 \). Since \( V(q) \) is a diagonal matrix even if one wheel is in radial direction \( V(q) \) becomes singular.

**References**


