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Optimal Node Placement in Hybrid Solar Powered WLAN Mesh Networks

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***Abstract*—Hybrid WLAN mesh networks use a combination of nodes that are continuously powered and those that are powered using an energy sustainable source such as solar power. In this paper we consider the problem of cost-optimal placement of the energy sustainable nodes in these types of hybrid networks. We first introduce a cost model that takes into account the provisioning required to operate the solar/wind powered nodes subject to a desired node outage criterion. We then formulate the design problem as a Mixed Integer Quadratic Problem (MIQP). A branch and bound approach is used to obtain node positioning solutions and is compared with a proposed algorithm that uses optimum shortest path routes. Our results show that there is a significant improvement in cost that can be obtained using the proposed methodology and that the branch and bound approach achieves the optimum assignment for a variety of network examples.**

I. Introduction

Wireless LAN mesh networks based on IEEE 802.11 are being rapidly installed in many metropolitan areas. These networks provide end-user Wi-Fi coverage and reduce deployment costs by performing backhaul traffic relaying between mesh nodes. One of the major costs of many WLAN mesh network deployments is that of providing certain mesh nodes with electrical power. An alternative to continuous power connections is to operate some of the mesh nodes using a sustainable energy source such as solar or wind power. The resulting system is a *hybrid network* consisting of a mixture of nodes powered continuously, and by those powered using renewable energy sources.

In this paper we consider cost-optimal placement of energy sustainable nodes in a hybrid WLAN mesh network. A cost model is first introduced that takes into account the solar panel and battery provisioning required to operate the solar powered nodes subject to a desired node outage criterion. A complicating factor is that the cost of renewable energy powered nodes is dependent on the amount of traffic for which a given node is provisioned, and therefore the node placement and traffic routing must be considered jointly. The design problem is formulated as a Mixed Integer Quadratic Problem (MIQP) and is shown to be NP-complete. A branch and bound approach is used to obtain node positioning solutions, and this is compared with a proposed algorithm that uses optimum shortest path routes. The proposed algorithms result in significant improvements in cost and the branch and bound approach achieves the optimum assignment for various network examples which can be computed exhaustively.

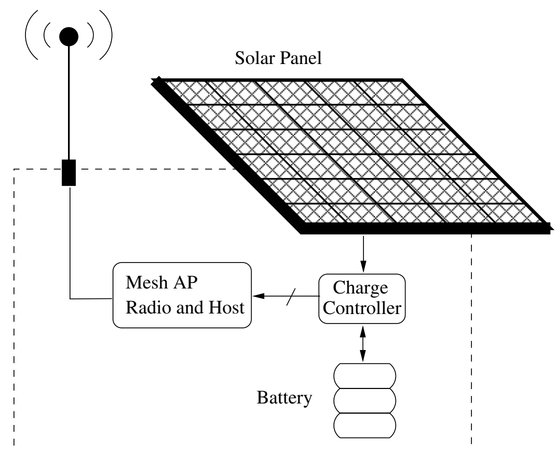

Fig. 1. Solar Powered WLAN Mesh Node

II. Background

To the best of our knowledge, optimum cost node assignment for hybrid sustainable-energy infrastructure deployments has not been dealt with in the context of wireless mesh networks. However, there is previous work that deals with similar problems within the context of sensor and cellular networks.

In [1], a Base Station Positioning (BSP) problem is considered for sensor networks. It is shown that the problem is NP-complete, and the paper describes results for greedy and local search algorithms. In [2] a methodology is presented for approximating the base station placement solution. The proposed technique is guaranteed to find a solution within a specified error bound. In [3] a two-tier sensor network is considered which is composed of sensors and aggregating nodes. The problem of maximizing the network lifetime of the forwarding nodes is considered by adding energy to each node or by adding relay nodes at new locations. In [4] a hybrid sensor network is considered, containing resource-rich microservers and resource-impoverished sensors. An investigation of the maximum lifetime of the network and the optimal microserver placement is performed using a tree-based anycast routing algorithm. Reference [5] describes an optimal access

Load Power		Phoenix, AZ.	Toronto, ONT.	Yellowknife, NWT.
4W	Battery, Ah.	28.25	62.45	185.12
	Panel, W.	30	79	134
3W	Battery, Ah.	22.12	47.5	148
	Panel, W.	22	59	96
2W	Battery, Ah.	14.2	30.5	92.6
	Panel, W.	15	40	67
1W	Battery, Ah.	6.6	15.3	53.2
	Panel, W.	8	20	30
0.5W	Battery, Ah.	3.4	9.8	24
	Panel, W.	4	10	16
0.25W	Battery, Ah.	1.9	4.9	12
	Panel, W.	2	5	8

TABLE I
MINIMUM COST SOLAR PANEL AND BATTERY SIZES

point and traffic allocation algorithm for WLANs and in [6] a base station location optimization problem is described for UMTS networks in which the demand node maps and the target base station sites are known beforehand.

In [7] the cost tradeoff is considered between installing a photo-voltaic (PV) system versus powering from the AC power mains for Optical Network Units (ONU). The authors show that there is a "solar cost effective distance" where the cost of using the AC mains is higher than deploying a PV system.

III. ENERGY FLOW MODEL

A block diagram of a sustainable energy mesh node is given in Figure 1. The solar panel and battery are connected to the system using a charge controller which protects the battery from under-charging and over-charging. A standard discrete-time flow model describes the energy input/output for this system [8]. $\mathcal{E}_{source}(k)$ is defined to be the total energy produced by the energy source(s) over the time increment $[(k-1)\Delta, k\Delta]$. In photo-voltaic (PV) systems, data collection and modeling is done in discrete time, often using hourly Δ increments. We also define $\mathcal{B}(k)$ to be the residual battery energy stored at time $k\Delta$, and $\mathbf{B}_{max}$ is defined to be the total battery capacity. If we assume that $\mathcal{L}(k)$ is the load energy demand over the time duration $[(k-1)\Delta, k\Delta]$, then we can write that,

$$\mathcal{B}(k) = \min\{\max[\mathcal{B}(k-1)+ \mathcal{E}_{source}(k) - \mathcal{L}(k), \mathbf{B}_{outage}], \mathbf{B}_{max}\}. \quad (1)$$

This equation simply states that the battery energy at time k can be found by taking the energy at time $k-1$, adding in the energy supplied by the energy source(s), and subtracting that which what was consumed by the load over that period. In this equation $\mathbf{B}_{outage}$ is the maximum allowed depth of battery discharge [9]. When $\mathcal{B}(k) < \mathbf{B}_{outage}$, the charge controller disconnects the load and the node will experience a radio outage. It is also important to incorporate temperature effects into the energy flow model since any reduction in temperature leads to a reduced charge storage capability in the battery.

Using public solar insolation or wind data for a particular geographic location[1] and the energy flow model, the performance of the system can be determined by simulating its behavior over many years of meteorological history. This is currently the most accurate way to perform system provisioning [10].

IV. SOLAR POWERED PROVISIONING COST MODEL

For a given geographic location, and using the procedures described above, solar panel versus battery size contours can be generated for an assumed traffic profile and a given probability of node outage, P_{Out}. From these curves, the costs of the battery and panel can be used to compute the minimum total provisioning cost. In this paper it is assumed that the node cost function is made up of the cost of the battery and the solar panel only, since the rest of the node consists of (typically much smaller) fixed costs. In Table I for example, we show the resources required to deploy one such node at different locations in North America for $P_{Out} = 10^{-4}$ for the power consumption cases shown.

Many of these types of node configurations have been performed, and our simulations have shown that the solar provisioning cost C_{SP} can be approximated as a linear function of the normalized traffic flow, L, for which the node is designed, i.e.,

$$C_{SP}(P_{Out}) \simeq K_1 L + K_2, \quad (2)$$

where K_1 and K_2 are dependent on P_{out}. For example, in Toronto Canada, for the case where the target outage probability P_{out} is 0.1, $K_1 = 110$ and $K_2 = 37$.

V. PROBLEM DEFINITION

It is assumed that a target end-to-end traffic profile is known. Since the traffic profile is long-term, we also assume that all flows are splittable. In addition, the positions of the mesh nodes has been determined in advance based on WLAN coverage considerations. The problem is to decide which

[1]This can be obtained in the USA from the National Renewable Energy Laboratory (NREL), U.S. Department of Energy. In Canada it can be obtained from The Meteorological Service of Canada.

nodes need to be solar-powered, and what the provisioning is for those nodes so that the total cost is minimized. Since solar powered node provisioning cost is a function of traffic flow through them, we need to flow the smallest amount of traffic through the assigned solar nodes. Given a traffic flow matrix, and a set of N mesh node locations with associated connectivity, we want to locate the positions of the solar powered nodes such that the total solar provisioning cost is minimized. We refer to this as the *Optimum Flow/Placement* (or OFP) problem.

We introduce a binary decision variable d_i which will indicate whether a location is solar (i.e., 1) or AC mains (continuously) powered (i.e., 0). This variable is a vector with dimension $N \times 1$ where N is the number of nodes. We have $(K = N - P)$ AC-powered nodes, and we would like to place them such that they carry as much of the traffic as possible.

The problem can be formulated as the following Mixed Integer Non-linear Programming (MINLP) problem,

$$\min_{\gamma_{ij}, d_i} C \tag{3}$$

such that

$$\gamma_{ij} \geq 0 \quad \forall i, j \in N \tag{4}$$

$$\gamma_{ij} \leq 1 \quad \forall i, j \in N \tag{5}$$

$$C = (N - P)C_{AC} + \sum_i (d_i) CS_i \quad i \in N \tag{6}$$

$$C_{AC} = F \tag{7}$$

$$CS_i = \Lambda(L_i, T) \tag{8}$$

$$L_i = \sum_{l \in N} \gamma_{il} + \sum_{h \in N} \gamma_{hi} \tag{9}$$

$$\sum_{l \in N} \gamma_{il} - \sum_{h \in N} \gamma_{hi} = D_i \quad \forall i \tag{10}$$

$$\sum_i d_i = P \tag{11}$$

$$d_i \in \{0, 1\}, \quad i = 1 \dots N \tag{12}$$

$$P_{Outi} = \Phi(L_i, T) \quad \forall i \tag{13}$$

$$P_{Outi} \leq T \quad \forall i \tag{14}$$

As seen in Equation 3, the objective is to minimize the total cost over the decision variable d_i and γ_{ij} where γ_{ij} represents the total flow on the link between nodes i and j. The constraint in Equation 4 ensures that the direction of flows on the link correspond to the link direction. Equation 5 represents the link capacity condition on the total flow through a link and Equation 6 gives the total cost as the sum of the solar and AC powered node costs. Equations 7 and 8 represent the assumed cost models for the AC nodes and the solar nodes. F represents the average cost of deploying an AC node. The optimization problem is independent of the total AC cost since it is fixed. In Equation 8 we see that $\Lambda(\cdot)$ models the functional dependency of the node cost, where L_i represent the sum of the incoming and outgoing flows for node i. The flow balance is shown in Equation 9 and T is the target outage probability which is assumed to be the same for all nodes. An additional flow balance is shown in Equation 10, where D_i represents the traffic demand sourced or requested by the node. This has a positive sign for sources and a negative sign for sinks, and is zero for relay nodes. The constraint in Equation 11 guarantees that the number of AC nodes is equal to P, while the condition shown in Equation 12 simply ensures that d_i is a binary vector. Equation 13 represents the non-linear dependency of the outage on L_i and T. Finally, Equation 14 shows that the outage target must not be exceeded.

The constraint in Equation 13 can be removed by noticing that the solar cost model guarantees that the target outage level is met if the load does not exceed that used in the design, and hence the problem can be simplified into a quadratic Mixed Integer Quadratic Problem (MIQP).

$$\min_{\gamma_{ij}, d_i} \sum_i d_i \cdot CS_i \quad i \in N \tag{15}$$

such that

$$\gamma_{ij} \geq 0 \quad \forall i, j \in N \tag{16}$$

$$\gamma_{ij} \leq 1 \quad \forall i, j \in N \tag{17}$$

$$CS_i = K_1 L_i + K_2 \tag{18}$$

$$L_i = \sum_{l \in N} \gamma_{il} + \sum_{h \in N} \gamma_{hi} \tag{19}$$

$$\sum_l \gamma_{il} - \sum_h \gamma_{hi} = D_i \quad \forall i \tag{20}$$

$$\textstyle\sum_i d_i = P \tag{21}$$

$$d_i \in \{0, 1\} \tag{22}$$

The objective is a quadratic combination of the variables while all of the constraints are linear. In the next section we consider the complexity of the defined problem.

VI. Problem Complexity

Theorem 1: The cost-optimal joint node placement and flow routing problem in hybrid powered wireless mesh networks is an NP-complete problem.

Proof: The theorem can be proven by reduction to the well-known minimum set cover problem [11]. In the set cover problem we are given a collection C of subsets of a finite set S, whose union is S. A set cover for S involves selecting a subset $C' \subseteq C$ such that every element in S belongs to at least one member of C'. The decision version of the problem can be described as follows. "Is there a set cover such that the cardinality of C', is less than or equal to K, i.e., $|C'| \leq K$?"

Assume we are given any instance of the minimum set cover problem, where $S = \{E_1, E_2, \dots, E_{n-1}, E_n\}$. We transform this into an instance of the optimal flow/placement problem from Section V as follows. A four-stage mesh network is formed as shown in Figure 2. The first stage is a source node, I, from which all traffic originates, and a unit traffic flow (chosen to be much smaller than the link capacity) is to be sent to each of the elements of the set, S. The nodes in stage 3 each represent one of the elements in S as shown. This traffic must be sent through stage 2 which consists of $|C'|$ nodes, each representing one of the subsets of S. The connectivity between stages 2 and 3 are defined by the entries

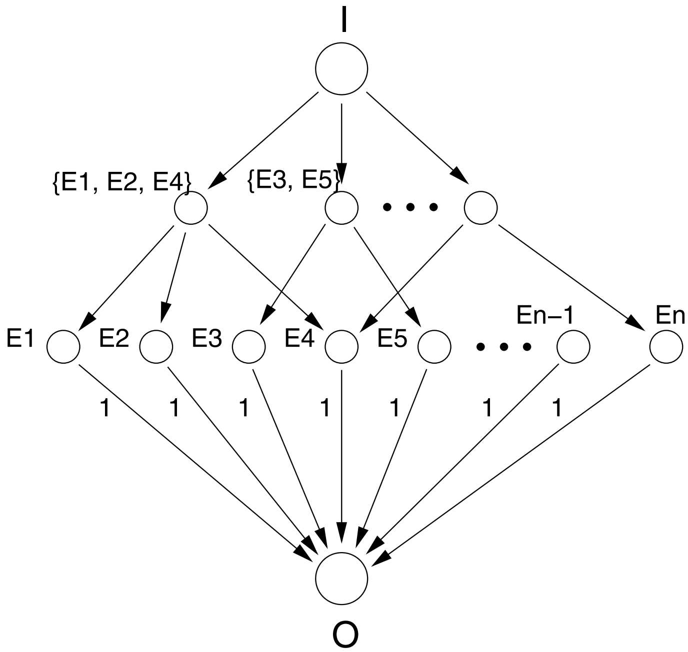

Fig. 2. Network Construction for NP-completeness Proof

in the provided subsets C, an example of which is shown in the figure. The traffic flow leaves the network via node O. The objective in this network is to find $K = N - P$ nodes such that all traffic flow demands are met and there is a zero solar provisioning cost for the P remaining nodes. In this case K represents the number of chosen intermediate nodes for traffic routing.

Now assume that we have a polynomial time algorithm that can solve the optimum flow/placement problem from Section V, and we input the network problem defined above (in Figure 2). The algorithm will find the best solar node placement/provisioning cost for P solar nodes. If this cost is zero, then the algorithm has found a subset of K second-stage nodes through which the traffic can be routed, leaving no traffic flow for the P remaining (solar) nodes (i.e., Their panel/battery provisioning cost is zero). It can be seen that this node selection provides a set cover for the original problem. It is also easy to see that if a set cover exists for the given set of parameters, our algorithm must be able to find one, otherwise the assumed optimality of our algorithm is violated. Running this procedure for all values of K will provide an answer to the decision version of the set covering problem. It follows from this that our problem is NP-complete. ■

VII. Solution Methodology

We now consider two different algorithms for performing the node assignment. The first relies on solving the underlying MIQP problem using the branch and bound approach [12]. Our experiments have shown that this achieves the optimal provisioning and assignment in networks which can be solved by exhaustive search. We compare the results for larger networks to a simple heuristic that uses flow routing followed by node type assignment.

The numerical solver in [12] relies on removing the integer constraints and solving the relaxed version of the problem by calling the quadratic programming solver in Matlab. The solver then uses branch and bound in order to search through the binary integer subspace in order to find a solution.

We also include the following design heuristic. Initially all nodes are assumed to be AC powered. Linear programming is then used to find the shortest path routing. Once this is done, the P nodes carrying the least traffic are assigned as solar nodes and the total network cost is then calculated. The advantage of this approach is that the linear program is solved once followed by a simple node flow ordering.

A. Performance Results

Our experiments have shown that the results obtained using the algorithms can significantly reduce solar provisioning costs. In the following, we show some typical examples using a 4×4 mesh network shown in Figure 5. We assume the following traffic matrix for this example. Node 16 is a gateway to which all nodes are sending traffic, we assume 0.6 units of flow from node 1, 0.2 from node 5 and 0.9 from node 15.

For the case where $P = 8$, the deployments are shown in Figures 3 and 4. The shaded nodes are the solar nodes and the unshaded nodes are the AC powered nodes. The flows are labeled on the arcs and arcs labeled with an X are not carrying traffic. We can see that the optimal routing sends as much traffic as possible through the AC nodes as opposed to the simple heuristic, as would be expected. For all values of P, the optimal placement outperforms the shortest path routing. This is shown in Figure 5 which plots the normalized cost of the network versus the number of solar nodes for the shortest path heuristic versus the optimal case. The normalized cost model used for this example assumes that the unit panel cost is double the unit battery cost, hence all of the values here are normalized to the unit battery cost. The cost results are also normalized to the case where $P_{Out} = 0.1$. For different outage probabilities, the costs can be easily scaled up. We can see from Figure 5 that when the number of solar nodes is low, the results for the branch and bound are similar to the proposed heuristic. However, when the number of nodes increases, the costs quickly diverge until P reaches 8. The cost for the branch and bound solution is almost 90 units while for the heuristic it is almost 175. The costs then start to converge again once the value of P approaches N. This behavior of the cost function is related to the path diversity available for the different values of P.

We also considered the case when certain solar nodes are pre-assigned their locations. This could occur if the nodes are deployed at locations where it is impossible to supply them with continuous power. For example, we consider the case where nodes 10, 11, and 12 are pre-assigned in this way. The optimal deployment will try to route traffic to avoid these nodes, and the results are shown in Figure 6. We can see that the gap between the optimal cost and the shortest path heuristic increases. This is due to the fact that the optimal

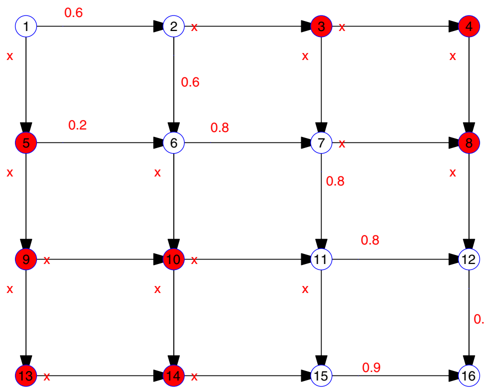

Fig. 3. Optimal Deployment Example for a 4×4 Mesh

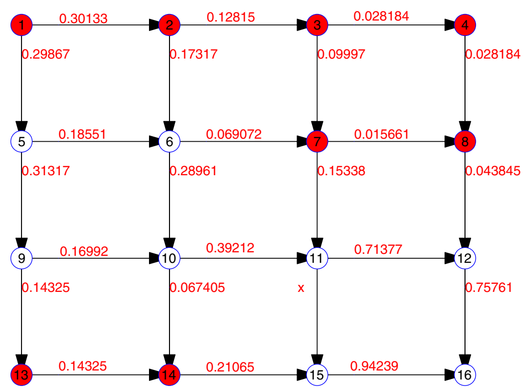

Fig. 4. Heuristic Deployment Example Using Shortest Path for a 4×4 Mesh

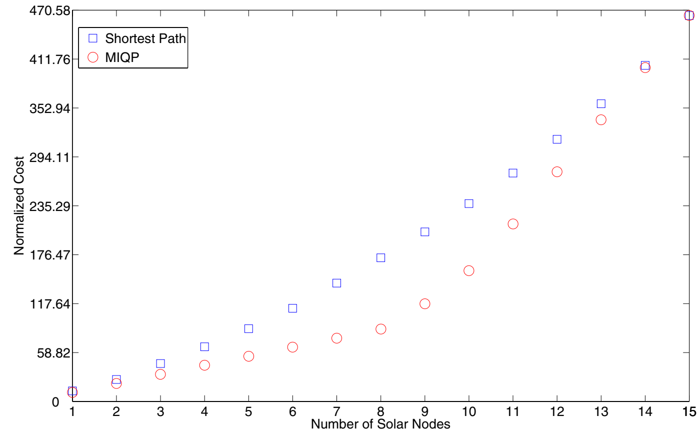

Fig. 5. Example of Total Solar Cost Versus Number of Solar Nodes for a 4×4 Mesh

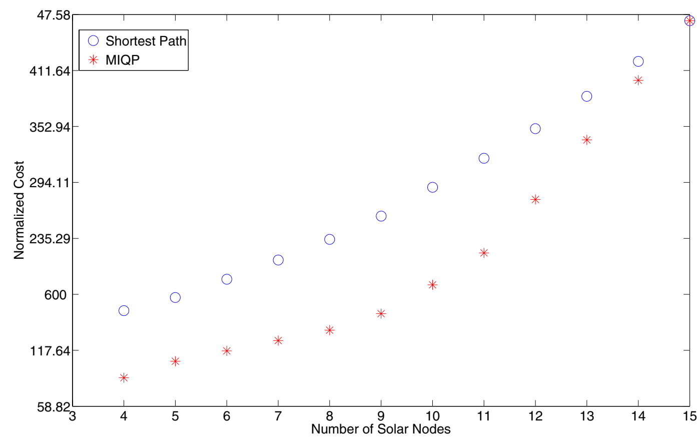

Fig. 6. Cost of Optimal Deployment Example for a 4×4 Mesh with Preassigned Solar Nodes

deployment must keep traffic away from these nodes. The shortest path heuristic is oblivious to the pre-assignment of the nodes and hence the cost of the network deployment increases when compared to the branch and bound solution that takes the nature of the nodes into account.

In the next set of results we introduce a second gateway into the network. We set Node 7 as a gateway in addition to Node 16 and include flows shown in the format *node(flow)*: 1(flow=1), 2(flow=0.2), 3(flow=0.01), 4(flow=0.15) and 5(flow=0.12). The algorithm will choose node 7 as an AC node and route traffic accordingly. The results are shown in Figure 7. We notice that the overall cost of the network deployment has increased. We also see that the shortest path results are close to the optimal due to the congestion that was already affecting this region of the network.

We also consider the case where nodes 9, 10, 13 and 14 are removed and the traffic matrix is 1(0.5), 4(0.3), 6 (0.6) with destination node 16. This corresponds to a more asymmetric case, and the normalized costs obtained are shown in Table II. As before, the costs for both algorithms are very close when P is high or low (when compared to N). However, when P is close to $N/2$ the shortest path heuristic does not perform as well as the optimal solution. For example, when $P = 8$ the cost rises from 119.41 to 152.53. We can see that the divergence of the costs is not high when compared to the previous cases since the route diversity and number of nodes has decreased.

VIII. Conclusions

In this paper we have considered the problem of cost-optimal solar powered node assignment in hybrid WLAN mesh networks. The problem formulation is a binary mixed-integer non-linear problem that can be simplified into a binary mixed-integer quadratic problem. It has been shown that the problem is NP-complete, and deployment results were shown using a branch and bound and shortest path heuristic. Our results show that there is a significant improvement in cost that can be obtained using the proposed algorithms, and that the branch

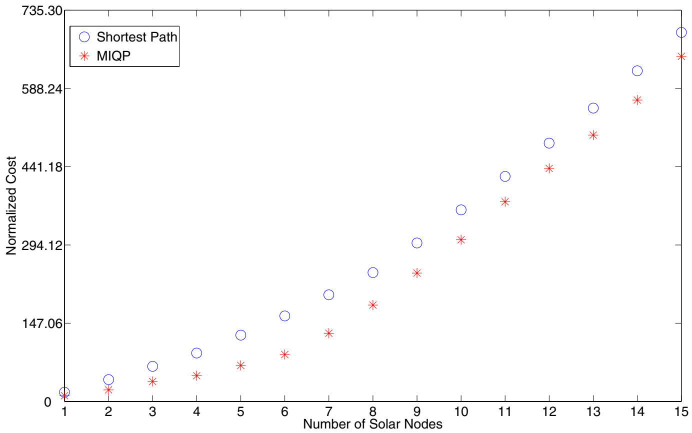

Fig. 7. Cost of Optimal Deployment Example for a 4×4 Mesh with an Additional Gateway at 7

P	Optimal	Shortest Path
1	10.882	10.882
2	21.765	21.765
3	32.647	32.647
4	43.529	43.529
5	54.412	62.987
6	75.000	90.046
7	92.353	117.91
8	119.41	152.53
9	156.18	187.19
10	192.94	226.10
11	236.18	271.93
12	292.35	326.09
13	348.53	382.27
14	424.12	440.46
15	499.71	506.98

TABLE II
NORMALIZED COST FOR ASYMMETRIC NETWORK

and bound approach achieves the optimum assignment for various network examples.

IX. ACKNOWLEDGMENTS

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REFERENCES

[1] A. Bogdanov, E. Maneva, and S. Riesenfeld. "Power-Aware Base Station Positioning for Sensor Networks.". In *Proceedings of INFOCOM*, 2004.

[2] Y. Shi, Y.T. Hou, and A. Efrat. "Design of Low-Complexity Approximation Algorithms for Base Station Placement Problems in Sensor Networks". In *Technical Report- Bradley Dept. of ECE Virginia Tech*, July 2005.

[3] Y. T. Hou, Y. Shi, H. D. Sherali, and S. F. Midkiff. "On Energy Provisioning and Relay Node Placement for Wireless Sensor Networks" . *IEEE Transactions on Wireless Communications*, 4(5), September 2005.

[4] W. Hu, C. Chou, and S. Jha. "Deploying Long-Lived and Cost-Effective Hybrid Sensor Networks". In *Proceedings of First International Conference on Broadband Networks, BroadNets*, 2004.

[5] R. Akl and S. Park. "Optimal Access Point Selection and Traffic Allocation in IEEE 802.11 Networks". In *Proceedings of 9th World Multiconference on Systemics, Cybernetics and Informatics (WMSCI 2005): Communication and Network Systems, Technologies and Applications*, volume 8, pages 75–79, July 2005.

[6] M. Galota, C. Glaßer, S. Reith, and H. Vollmer. "A Polynomial-time Approximation Scheme for Base Station Positioning in UMTS Networks". In *DIALM '01: Proceedings of the 5th International Workshop on Discrete Algorithms and Methods for Mobile Computing and Communications*, pages 52–59, 2001.

[7] C. Quinones, M. Vazqvez, A. del Prado, E. de la Cruz, and A. Garrido. "Technical and Economical Comparison of PhotoVoltaic Solar System and AC Mains Powering for Advanced Optical Network Units at Remote Sites". In *Twentieth International Telecommunications Energy Conference, INTELEC*, pages 790–796, October 1998.

[8] F.M. Safie. "Probabilistic Modeling of Solar Power Systems". In *Annual Proceedings of Reliability and Maintainability Symposium, 1989.*, pages 425–430, 1989.

[9] L. Narvarte and E. Lorenzo. "On the Usefulness of Stand-Alone PV Sizing Methods". *Progress in Photovoltaics: Research and Applications, Prog.Photovolt: Res.Appl.*, 8:391–409, 2000.

[10] H.A.M. Maghraby, M.H. Shwehdi, and G.K. Al-Bassam. "Probabilistic Assessment of Photovoltaic (PV) Generation System". *IEEE Transactions on Power Systems*, 17(1):205–208, 2002.

[11] M. Garey and D. Johnson. *"Computers and Intractability; A Guide to the Theory of NP-Completeness"*. W. H. Freeman & Co., New York, NY, USA, 1990.

[12] A. Bemporad. Hybrid Toolbox - User's Guide, 2004. http://www.dii.unisi.it/hybrid/toolbox.