Neighbourhood effect of two short dents on buckling behaviour of short thin stainless steel cylindrical shells

B. Prabu* and A.V. Raviprakash

Department of Mechanical Engineering,
Pondicherry Engineering College,
Puducherry-605014, India
E-mail: bp_pec@yahoo.com
E-mail: avrp@sify.com
*Corresponding author

A. Venkatraman

Rajiv Gandhi College of Engg. & Technology,
Puducherry-607402, India
E-mail: avr2s@rediffmail.com

Abstract: Generally, thin cylindrical shells are susceptible for geometrical imperfections like non-circularity, non-cylindricity, dents, swellings etc. All these geometrical imperfections decrease the static buckling strength of thin cylindrical shells. In this work, neighbourhood effect of two circumferential short dents on the buckling behaviour of thin short stainless steel cylindrical shell is studied in detail. The dents are modelled on the FE surface of perfect cylindrical shell at half the height of the cylindrical shell by varying the centre distance between the dents. These cylindrical shells are analysed using non linear FE static buckling analysis and their buckling behaviours are compared with that of cylindrical shell with a short circumferential dent. It is found that the effect of two short dents and its nearness effect seem to be negligible compared with the effect of single dent in reducing the buckling strength of cylindrical shell.

Keywords: thin cylindrical shell; buckling strength; geometrical imperfections; dents.


Biographical notes: Balakrishnan Prabu received the Master of Engineering degree in Engineering Design from Anna University, Chennai, India in 1993 and PhD degree in thin shell buckling from Pondicherry (Central) University, Pondicherry, India in 2008. Currently he is working as an Associate Professor in the Department of Mechanical Engineering, Pondicherry Engineering College, Pondicherry, India. His main research interest is in buckling behaviour of thin shell structures.

Copyright © 2012 Inderscience Enterprises Ltd.
Alwardoss Velayutham Raviprakash received his Master of Engineering degree in Production Engineering from Annamalai University, Chidambaram, India in 1983 and is currently pursuing his PhD degree in buckling behaviour of thin plate structures at Pondicherry (Central) University, Pondicherry, India. At present, he is working as an Associate Professor in the Department of Mechanical Engineering, Pondicherry Engineering College, Pondicherry, India.

Ananda Padmanabhan Venkatraman received his Master of Science (Engineering) degree in Machine Design from University of Madras, Chennai India in 1972 and PhD degree in gear dynamics from Indian Institute of Technology Madras, Chennai, India in 1994. Currently he is working as Principal at Rajiv Gandhi College of Engineering and Technology, Pondicherry, India. His main research interests are in vibration analysis, tribology and stability analysis of thin shell structures.

1 Introduction

Thin cylindrical shells are one of the commonly used structural elements in engineering structures such as aircrafts, missiles, silos, bio-digesters, pipelines, tanks, submarine structures and nuclear structures. Potential mode of failure of these structures is buckling, because during their service life these components are often subjected to axial compressive loading. But these shell structures are prone to a large number of imperfections, owing to their manufacturing difficulties, which affect the load carrying capacity of these structures. The imperfections which cause failure in thin cylindrical shells are grouped into three major groups and they are, geometrical (out-of-straightness, initial ovality and geometrical eccentricities, dents, swells, etc.), structural (residual stresses and material inhomogenities) and loading imperfections (non-uniform edge load distribution, unintended edge moments, load eccentricities and load misalignments as well as imperfect boundary conditions). Also the constructional defects, such as small holes, cutouts, rigid inclusions and delaminations, could be counted as structural imperfections. Out of all these imperfections, the geometrical imperfections are more dominant in determining the load carrying capacity of thin cylindrical shells. Reliable prediction of buckling strengths of these structures is important because the buckling failure is catastrophic in nature.

2 Literature review

Studies involving the effect of local geometrical imperfections on the buckling of cylindrical shells are limited in number as mentioned by Shen and Li (2002). Among those, Hutchinson et al. (1971) gave the analysis using asymptotic formula given by Amazigo and Budiansky (1972) for the compressive buckling strength of axially loaded cylindrical shells with an axisymmetric cosine dimple imperfections. Amazigo and Budiansky (1972) derived an asymptotic formula and gave imperfection sensitivity analysis for axially compressed cylindrical shells with localised axisymmetric exponentially decay form of imperfections using Koiter’s general theory. Blachut and Galletly (1990) and Galletly and Blachut (1991), in their work, considered dimple or flat
spot of varying magnitude as geometrical imperfections at the crown of the spherical shells and achieved good agreement with experimental results. The effect of multiple large diamond shaped dimples on the buckling behaviour and load carrying capacity of cylindrical shells under axial compression was investigated experimentally by Krishnakumar and Forster (1991). In the work of Guggenberger (1995), the effect of a single longitudinal initial dent on the buckling strength of thin cylindrical shell subjected to external pressure was studied. The predicted buckling pressure from FE analysis was compared with experimental results. Park and Kyriakides (1996) evaluated the reduction in collapse pressure of long stainless steel cylindrical tubes with dents through a combination of experiment and numerical analysis. The geometry of each dented cylindrical shells were recorded with an imperfection scanning system and then collapsed under external pressure. The experimental results are compared with numerical results. Hambly and Calladine (1996) took specimens of drinks cans (R/t = 350) with dents. Those specimens were tested under eccentric axial compression and it was found that the nominal stress level in the dent region on buckling was 0.24 times of classical buckling stress.

In the work of Holst et al. (1995), the effect of local depression due to rolling process of steel plate and/or shrinkage of weld were studied in detail and it was concluded that these local depressions have more detrimental effect on buckling strength of cylindrical shells. Pircher and Bridge (2001) studied about effect of localised geometrical imperfections induced by circumferential weld on buckling strength of thin walled steel silos and tanks. From the survey of imperfection measurements done by Ding et al. (1996), imperfection data was obtained and mapped to get the general shape of the imperfection to represent shape of imperfections due to circumferential weld (Pircher et al., 2001). Cai et al. (2002) presented the effect of imperfections on axial loading due to frictional traction on the walls of the thin cylindrical silos. Wullschleger and Piening Mayer (2002) in their work discussed about different methods of determining the static and dynamic buckling strengths of cylindrical shells and also considered both the geometrical imperfections spread over the entire surface of the thin carbon fibre reinforced composite (CFRC) cylindrical shell and the local imperfections called ‘single horizontal dent’ or ‘single buckle’ in circumferential direction at half the cylinder height for their numerical analysis.

Khamlichi et al. (2004) studied analytically, about the effect of localised axisymmetric initial imperfections on the critical load of thin elastic cylindrical shell subjected to axial compression. Schneider (2006) discussed about the effect of local axisymmetric concave and convex axisymmetric ring shaped imperfection patterns on buckling strength of cylindrical shell under axial compression and one of the conclusion was that as width of imperfection increases buckling strength increases whereas as depth of imperfection increases buckling strength decreases. Prabu et al. (2007) carried out parametric study about the effect of dent dimensions and its orientations on the buckling strength of short stainless steel cylindrical shells under axial compression and it was concluded that circumferential dents have more dominant effect than longitudinal dents in reducing the buckling strength of cylindrical shells. As an extension of the above work, this study investigates the nearness effect of two circumferential short dents on buckling strength of cylindrical shell by modelling two short dents at half the height of the perfect cylindrical shell model with varying centre distance between the dents.
3 Buckling analysis

Types of buckling analysis are
1. Eigen (or linear) buckling analysis
2. non-linear buckling analysis.

3.1 Eigen buckling analysis

Eigen buckling analysis predicts the theoretical buckling strength of an ideal linear elastic structure. This analysis is used to predict the bifurcation point using linearised model of elastic structure. It is a technique used to determine buckling loads – critical loads – at which a structure becomes unstable and buckled mode shapes – the characteristic shape associated with a structure's buckled response. The other name for this Eigen buckling analysis is 'bifurcation analysis'. The bifurcation buckling refers to unbounded growth of new deformation pattern. This analysis involves calculating the points at which the primary load deflection path is bifurcated by a secondary load deflection path as shown in Figure 1(a). ANSYS finite element software package is used to determine the buckling strength of the perfect cylindrical shell through Eigen buckling analysis and this phenomenon is explained in Figure 1(b). In Eigen buckling analysis, imperfections and non-linearities cannot be included. Sub-space iteration scheme can be used to extract the load factor or Eigen value.

Figure 1  (a) Bifurcation buckling (b) FE analysis of Eigen and non-linear buckling
The basic form of the Eigen buckling analysis is given by

\[ [K] \{ \phi_i \} = \lambda_i [S] \{ \phi_i \} \]  

where:

- \([K]\) = Structural stiffness matrix
- \(\{ \phi_i \}\) = Eigen vector
- \(\lambda_i\) = Eigen value
- \([S]\) = Stress stiffness matrix

### 3.2 Non-linear buckling analysis

This is a more accurate approach and since this finite element analysis has capability of analysing the actual structures with imperfections. This approach is highly recommended for design or evaluation of actual structures. This technique employs a non-linear structural analysis with gradually increasing loads to seek the load level at which the structure become unstable and this phenomenon is also explained in Figure 1(b). Using this non-linear technique, features such as initial imperfections, plastic behaviour etc., can be included in the model. In this analysis both geometrical and material non-linearities are utilised, because the thin shell structures are subjected to large deformations and also at some of the imperfection location(s) on the structures the stresses may exceed elastic limit due to imperfections present in that location(s). Here the material non-linearity is defined with kinematic hardening rule. A full incremental non-linear static stress analysis is used taking initial displacement (imperfections) matrix into account and applying load incrementally. In order to find the maximum load carrying

![Figure 1](image-url)
capacity of the structure accurately, snap through approach of the non-linear analysis has to be followed. While using Newton–Raphson iteration scheme to solve system of equation in non-linear analysis near to the critical load of structure, the tangential stiffness matrix may become singular and thereby further load step is not possible. In order to overcome this problem, arc tangent iteration scheme is adopted (Forde and Stiemer, 1987).

4 FE modelling

An eight noded quadrilateral shell element, SHELL93 of ANSYS is used for modelling the thin cylindrical shells with dents. This element can handle membrane, bending and transverse shear effect, and also able to form curvilinear surface satisfactorily. This element also has plasticity, stress stiffening, large deflection and large strain capabilities.

4.1 Verification of FE results

In order to validate the numerical results obtained in the present work, test steel cylindrical shell used by Jullien and Liman (1998) and by Han et al. (2006) for validation purposes of their FE models using Shell 181 element, is considered for validation of perfect cylindrical shell model in the present work. Taking a typical element size of 3 mm × 3 mm, both linear and non-linear analysis was carried out and the comparison of results is presented in Table 1.

<table>
<thead>
<tr>
<th>Sl. no.</th>
<th>Description of study</th>
<th>Buckling load (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Present work (linear analysis using Shell 93)</td>
<td>23.884</td>
</tr>
<tr>
<td>2</td>
<td>Present work (non-linear analysis using Shell 93)</td>
<td>22.967</td>
</tr>
<tr>
<td>3</td>
<td>Classical buckling load (Timoshenko and Gere, 1965)</td>
<td>23.883</td>
</tr>
<tr>
<td>4</td>
<td>Han et al. (linear analysis using Shell 181)</td>
<td>23.690</td>
</tr>
<tr>
<td>5</td>
<td>Han et al. (non-linear analysis using Shell 181)</td>
<td>22.210</td>
</tr>
<tr>
<td>6</td>
<td>Jullien and Liman (linear analysis using CASTEM 2000)</td>
<td>25.060</td>
</tr>
</tbody>
</table>

Validation of the developed imperfect cylindrical shell model (with distributed geometrical imperfections) is done with the published result of Jamal et al. (2003). For a mesh size of 4 mm × 4 mm, distributed geometrical imperfections given in the form in equation (32) of Jamal et al. (2003) are added on the perfect cylindrical shell model. For imperfection amplitude of 0.1 times thickness (t), elastic buckling strength obtained from non-linear FE analysis is 45.02 N/mm as against 44.59 N/mm obtained in Jamal et al. (2003) using ABACUS. Stress contour with fictitious deformation obtained at limit load condition from the present work is shown in Figure 2. By the above verifications, FE model generated is validated with other element (Shell 181) as well as with other published results. This comparison of results clearly indicates that the present non linear analysis using Shell 93 elements could adequately simulate the response.
4.2 Thin cylindrical shell model

The thin cylindrical model taken for study (Athiannan and Palaninathan, 2004) is:

\[
\begin{align*}
\text{Radius (R)} & = 350 \text{ mm} \\
\text{Length (Lc)} & = 340 \text{ mm} \\
\text{Thickness (t)} & = 1.25 \text{ mm}
\end{align*}
\]

4.3 Dent modelling

The dent modelled is shown in Figure 3 using similar equation given by Guggenberger (1995) such that along the longitudinal direction, sinusoidal shape is assumed and along the transverse direction of the dent, exponential decay shape on both sides is assumed. In this work, shape of the dent assumed is defined by the Equation 2.
where:

\[ \delta = t \times \exp \left( -\left( \frac{4y_2}{W} \right)^2 \right) \times \cos \left( \frac{\pi y_1}{L} \right) \]  \tag{2}

\[ \delta = \text{radial inward displacement of dent} \]

\[ y_1 = \text{distance from the centre of the dent to point on the dent along longitudinal axis of the dent, with} \]

\[ -\frac{L}{2} \leq y_1 \leq \frac{L}{2} \]

\[ y_2 = \text{distance from the centre of the dent to point on the dent along transverse axis of the dent, with} \]

\[ -\frac{W}{2} \leq y_2 \leq \frac{W}{2} \]

The length (L), width (W) and depth (D) of the dent is taken as 40t, 20t and 3t respectively based on references Guggenberger (1995) and Park and Kyriakides (1996).

Simply supported boundary conditions (only radial displacement restraints) are applied on both the edges of the cylindrical shell and the uniform displacement loading is applied from the top edge, and the bottom edge is restrained from moving along load direction (Han et al, 2006).

4.4 Material modelling

The important properties of stainless steel used in the analysis are

Young’s modulus (E) = 1.93 \times 10^5 \text{ MPa}

Yield stress (\sigma_y) = 205 \text{ MPa}

Poisson’s ratio (\gamma) = 0.305

Multi-linear kinematic hardening behaviour is considered for modelling the material behaviour of stainless steel and the material behaviour is approximated by Ramberg Osgood approximation Equation 3 and it is shown in Figure 4.

\[ \varepsilon = \frac{\sigma}{E} + 0.002 \left( \frac{\sigma}{\sigma_y} \right)^n \]  \tag{3}

where:

\[ \varepsilon = \text{Strain}, \]

\[ \sigma = \text{Stress}, \]

\[ n = \text{Strain hardening index} = 6 \text{ (Hautala, 2003).} \]
4.5 Mesh convergence study

The reliable prediction of the buckling strength is important because it is a failure of catastrophic nature. Hence, the analytical solution (Timoshenko and Gere, 1965) of the perfect thin cylindrical shell is compared with the FE Eigen buckling analysis result i.e., both buckling strength and Eigen modes (shown in Table 2 and Figure 5) and thus the perfect cylindrical shell is ensured. From the mesh convergence study, the optimum number of elements for the perfect cylindrical shell to predict the solution accurately is found to be $60 \times 37$ elements along circumferential and longitudinal directions respectively.

<table>
<thead>
<tr>
<th>$m$</th>
<th>$n$</th>
<th>Analytical solution (N/mm$^2$)</th>
<th>FEA result (N/mm$^2$)</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
<td>1134.3</td>
<td>1143.8</td>
<td>0.842</td>
</tr>
</tbody>
</table>

While modelling localised geometrical imperfections such as dents the following three points have to be considered carefully (Cai et al., 2002 and Song et al., 2004).
1 Modelling of dent shape accurately.

2 At the places where there is change of curvature present in the shell structures may cause local bending stress. Hence it is essential to model the surrounding region carefully.

3 Size of the element in the dent region. As per Song et al. (2004), when the element size is reduced to half the previous element size, the variations in the buckling strength should be less than 1%.

The finite element models of thin cylindrical shells with dents are modelled with 80 elements along the circumferential direction of the cylindrical shell. To capture the local bending stress or shape by FE models around the dent region, fine mesh should be used for a distance of half bending wave length (Equation 4) from the dent location. Accordingly in the dent quadrant of the cylindrical shell, taking the centre of the dent as origin, three concentric circles are drawn on the surface of the cylinder to model the single dent. The diameters of the concentric circles are 0.5 L, 1.1 L and 1.1 L + 2λb where λb – half bending wavelength given by Cai et al. (2002) and Song et al. (2004).

\[
λ_b = 2.44\sqrt{Rt}
\]  

(4)

Figure 6 shows the closeness of the elements used to model a dent. On the inner two circles, the element size is approximately maintained as 2.5 mm on their peripheries to model the dent geometry accurately. In the outer circle, the element size is maintained as 3 mm approximately on its periphery. The ‘auto mesh’ generation scheme of ANSYS is used to generate the FE mesh. In all other quadrants of the cylindrical shell, 60 elements are taken along the longitudinal direction.

**Figure 6** Close-up view of dent portion (see online version for colours)

In case of two dents, the outer circle’s centre is taken as the middle of the two dents and the radius is determined such that closed mesh region from the tips of the dents should be more than half bending wavelength λb. Figures 7(b) and 7(c), show the closeness of elements used to model the dents. In this work, the centre distance between the dents is varied as 60, 80, 100, 120, 150 and 200 mm.
5 Results and discussion

5.1 Progressive load – stress conditions

In case of thin cylindrical shells with a short dent of size $L=40t$, $W=20t$ and $D=3t$, the dent effective region (DER) consisting of two ridges each one adjacent to each dent tip and two inclined trough surfaces at each dent tip one above the dent and one below the dent and two bridge surfaces one above the dent and one below the dent are formed as shown in Figure 8. But in case of cylindrical shell with two circumferential short dents with gap more than $\lambda_b$ located at half the height of cylindrical shell has tendency to form a middle ridge trough surface formation in between the inner dent tips in addition to the ridge-trough surface formations at outer dent tips are shown in Figure 9.

The Figure 10 shows top view of the von Mises stress contours of cylindrical shell with 120 mm centre distance between two short dents at different load sub steps. The Figure 10(a) shows the stress contours superimposed on deformed cylindrical shell at the load sub-step of 0.21983.

Further, in this load sub step, stress build up on the trough surfaces can also be seen. In the next higher load sub step of 0.24453 as shown in Figure 10(b), the stress build up on the previous load sub step spread to the other region of the cylindrical shell. At load sub step 0.27930 as shown in Figure 10(c), once again higher stress build up occurs on the trough surfaces and spreading of this higher stress to the other regions of cylindrical shell can be seen at load sub steps of 0.29529, 0.30420 and 0.31396 as shown in Figures 10(d), 10(e) and 10(f). In Figures 10(g) and 10(h), stress building up on trough surfaces and spreading to other regions of cylindrical shell can be seen. Figure 10(h) shows the limit load condition. From load sub step of 0.27930 (as shown in Figure 10c) onwards, the plastic condition at dent tips progressively spread to entire dent geometry as the load applied on the cylindrical shell increases gradually.
Figure 8  (a) Top view of ridge-trough surfaces formation at load sub step of 0.28743 and (b) Line diagram to represent the terminologies used to explain the buckling behaviour (dent not to scale) (see online version for colours)
Figure 9  (a) and (b) Description of ridge-trough surfaces formation (dents not to scale)  
(see online version for colours)
5.2 Stress contours at limit load condition

Figure 11 shows the von Mises stress contours at their limit load condition of thin cylindrical shells with two dents of same size $L=40t$, $W=20t$ and $D=3t$ and the centre distance between dents is varied as 60 mm, 80 mm, 100 mm, 120 mm, 150 mm and 200 mm respectively. From Figures 11(a) and (b), it can be seen that two dents merge together and act as a single dent and in all the other cases in between inner dent tips a
new middle ridge-trough surface formation can be seen in Figures 11(c) to 11(f). From these figures it can be concluded that if the gap between inner edges of dents is less than half bending wave length, buckling behaviour of cylindrical shell has tendency to merge two dents as single dent.

**Figure 11** von Mises stress contours of thin cylindrical shells with two dents having varied centre distance (a) 60 mm (b) 80 mm (c) 100 mm (d) 120 mm (e) 150 mm (f) 200 mm (see online version for colours)
Figure 12 shows the variation of buckling strength ratio (BSR) (which is defined as the ratio between the buckling strength of imperfect cylindrical shell to that of perfect cylindrical shell) with respect to centre distance variation between two dents compared with BSR of cylindrical shell with single dent. The maximum reduction in buckling strength of a cylindrical shell with a dent of size $L = 40t$, $W = 20t$ and $D = 3t$ is 63.12% but with two circumferential short dents maximum reduction buckling strength is only 63.7%. From this it can be understood that the effect of two dents and its nearness effect seem to be negligible compared with effect of single dent in reducing the buckling strength of cylindrical shell.

In all the cases, it can be seen that the buckling strengths of the cylindrical shells with two dents are slightly lower than buckling strength of the cylindrical shell with single dent. This is because of the extent or circumferential width of DER is greater than that of DER of single dent.

Further, compared with BSR of cylindrical shell with a dent, a slight reduction in BSR of about 1.9% and 2.2% are noticed when the centre distance between dents are maintained as 60 mm and 80 mm respectively. This is because the two dents tend to merge together into single dent as illustrated in Figure 13 and thereby the DER is increased causing reduction in buckling strength.

Figure 12  BSR vs. centre distance between dents

Similar to the cylindrical shell with a dent shown in Figure 8(a), in the case of cylindrical shell with two short dents up to 80 mm centre distance also, the formation of rings of plastic zones excluding DER nearer the edges of the cylindrical shells are noticed before reaching the limit load condition of zero stiffness condition which can be seen in Figures 11(a) and (b). As shown in Figure 11(c), when the centre distance between dents is 100 mm, the buckling behaviour of the cylindrical shell has a tendency to form a middle ridge-trough surface in between the inner dent tips in addition to the ridge-trough surface formations at outer dent tips as shown in Figures 9(a) and (b). Because of the middle ridge-trough surface formation in the DER of dents this region can support more load than DER of dents without middle ridge formation and hence, proportion of load shared between DER and perfect portion of cylindrical shell is varied. Due to this, no formation of rings of plastic zones are seen in Figures 11(c) to (f) i.e., before the formation of rings of plastic zones, the limit load condition of zero stiffness is reached as
shown in Figure 14. Because of the formation of middle ridge-trough surface, the load carrying capacity of cylindrical shell increases slightly by 1.57% when compared to cylindrical shell with 80 mm centre distance between two dents.

**Figure 13** Illustration the progressive change of shape from two dent form to single dent form as the load sub step increases in the case of two dents with 80 mm centre distance (a) 0.03233 (b) 0.30414 (c) 0.32757 (d) 0.36069 (see online version for colours)

**Figure 14** Load sub-step value vs. edge displacement curve (stiffness curves) of stainless steel cylindrical shell with varied centre distance between the dents
Figure 15 shows the stiffness curve obtained from FE model of thin cylindrical shell with 80 mm centre distance between two circumferential short dents. A kink in the stiffness curve can be noticed. This is because at that load sub step two dents combined together and formed into a single dent.

Figure 15  Load sub-step value vs. edge displacement curve (stiffness curve) of stainless steel cylindrical shell with 80 mm centre distance between the dents

The stress condition inside the dent is decided by the load shared by the DER and this load is applied on the dent geometry via six trough surfaces. Since dent tips resist deformation, stresses are concentrated at dent tips. In Figures 11(c) to 11(f), four bridge surfaces, two above the dents and two below the dents are formed and also it can be noted that the stress induced in the bridge surface is lower because, the trough surfaces act as stiffeners and allow only less load via bridge surfaces and also the longitudinal edges of the dents provide lesser resistance to support the load by forming local deformation.

Figure 16  (a) Isometric view and (b) front view of displacement contours at limit load condition (see online version for colours)

Figure 16(a) shows isometric view of displacement contours at limit load condition and it can be seen that as expected the deformation is maximum at dent geometry and the
Neighbourhood effect of two short dents on buckling behaviour

displacement below the dent gradually tends to zero at the supporting edge. Due to presence of dent the displacement contours in and around the DER is getting distorted. Fig 16 (b) shows lobe formations of ridges and dent geometry in front view at limit load condition.

6 Conclusions

The following conclusions are made based on FE analysis carried on a thin short stainless steel cylindrical shell with two circumferential short dents at half of the height of cylindrical shell taken for study.

1 The effect of two short dents and its nearness effect seem to be negligible compared with the effect of single dent in reducing the buckling strength of cylindrical shell.

2 In case of cylindrical shell with two circumferential short dents, when the gap between two dents is more than half bending wavelength, because of the formation of additional middle ridge-trough surface, the strength of the DER is slightly more than that of the DER of single dent.

3 When the gap between two circumferential dents is less than half bending wavelength, the two dents merge together and form into a single long dent and fails at limit load condition. Before reaching the limit load condition, rings of plastic zones excluding DER are formed at the edges of the cylindrical shell.

4 In contrast, when the gap is more than half bending wavelength, cylindrical shell fails before forming the rings of plastic zones excluding DER at the edges of the cylindrical shell.

5 The buckling strength of cylindrical shell with dents mainly depends on the load carrying capacity of the perfect cylindrical shell portion other than the DER.

Acknowledgements

The authors would like to thank the management of Bharathiyar College of Engineering & Technology, Karaikal, INDIA for providing the necessary computing facilities.

References


Neighbourhood effect of two short dents on buckling behaviour


**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_c$</td>
<td>Diameter of the cylindrical shell</td>
</tr>
<tr>
<td>$L_c$</td>
<td>Length of the cylindrical shell</td>
</tr>
<tr>
<td>$R$</td>
<td>Radius of the cylindrical shell</td>
</tr>
<tr>
<td>$t$</td>
<td>Thickness of the cylindrical shell</td>
</tr>
<tr>
<td>$m$</td>
<td>Number of longitudinal lobes</td>
</tr>
<tr>
<td>$n$</td>
<td>Number of circumferential lobes</td>
</tr>
<tr>
<td>$L$</td>
<td>Length of the dent</td>
</tr>
<tr>
<td>$W$</td>
<td>Width of the dent</td>
</tr>
<tr>
<td>$D$</td>
<td>Depth of the dent</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Radial inward displacement for dent geometry</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Angle of inclination of dent</td>
</tr>
<tr>
<td>$E$</td>
<td>Young’s modulus</td>
</tr>
<tr>
<td>$E_T$</td>
<td>Tangential modulus</td>
</tr>
<tr>
<td>$\sigma_Y$</td>
<td>Yield stress</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density</td>
</tr>
<tr>
<td>$[S]$</td>
<td>Stress stiffness matrix</td>
</tr>
</tbody>
</table>
[k]  Structural stiffness matrix

\{ \phi_i \}  Eigen vector

\lambda_i  Eigen value

\lambda_b  Half bending wavelength

y_1  Distance from the centre of the dent to point on the dent along longitudinal axis

y_2  Distance from the centre of the dent to point on the dent along transverse axis