Abstract—In earlier paper [1] we proposed a differential spatial multiplexing (SM) scheme based on complex square orthogonal designs, referred to as differential orthogonal spatial multiplexing (DOSM). The receiver of DOSM does not require estimation of channel fading coefficients, channel power, signal power, or noise power to decode the data symbols and the decision is based on the two consecutively received codewords. In [1], the transmission matrix of DOSM is based on complex square orthogonal designs which cannot achieve full data rate. Hence, we presented a constellation rotation strategy to enhance the rate of DOSM. In this paper, we construct the transmission matrix of DOSM from complex rectangular orthogonal designs and find that full-rate DOSM can be achieved with increased encoding block length. An upper bound of the pair-wise error probability (PEP) for DOSM in Rayleigh fading channels is derived. Simulation results show that the proposed DOSM outperforms the differential space-time block code (DSTBC) with full diversity and the existing differential SM schemes in terms of error-rate performance over quasi-static Rayleigh fading channels.

Index Terms—Multiple-input multiple output (MIMO) systems, differential space-time modulation, differential spatial multiplexing.

I. INTRODUCTION

Recently, great progress in code design for multiple-antenna transmission in a wireless environment has been made [2-5]. All the above work focuses on the assumption that the channel is known at the receiver, which is reasonable for most practical systems because the receiver has to estimate the channel for carrier frequency and carrier phase synchronization. However, this assumption is questionable in highly mobile communication environments, or when multiple transmit and receive antennas are deployed. Channel estimation may not be feasible or cost-effective in such environments [6, 7].

In [7-13] differential code designs for MIMO systems have been presented. They belong to two categories. The work in [7-10] are based on orthogonal or quasi-orthogonal space-time codes (STC) and those in [11-13] are based on spatial multiplexing (SM). The codes in [7-10] are designed to achieve full diversity, but capacity does not increase linearly with the number of transmit antennas and is limited to one data symbol per channel use over all transmit antennas. As the number of antennas increases, the effect of diversity gives diminishing returns in STC, and the capacity remains constant. On the other hand the capacity of SM increases linearly with the number of transmit antennas. This suggests that for large numbers of antennas SM will yield more benefits than STC, and hence work should be focused on implementation of SM schemes.

Several differential SM schemes [11-13] have recently been developed, aimed at achieving high data rate transmission and partial diversity. The main difference among [11-13] is in the construction of the transmitted codewords. The work in [11] employs non-orthogonal transmitted data matrices whose ranks and determinants decrease as frame length increases, and hence the performance of such a system suffer substantial degradation when long frames are used. The work in [12] can only achieve a partial data rate of \( N - (N - 1)N / (2T) \) symbols per channel use, where \( N \) and \( T \) are the number of transmit antennas and encoding block length, respectively. The authors in [13] present a hybrid system that combines differential space-time (DST) modulation [9] and spatial multiplexing (SM) [3]. It divides multiple transmit and receive antennas into \( G \) (\( G \leq N \)) groups and applies DST encoding and decoding or classical differential phase-shift keying (DPSK) individually to each group. At the receiver, it employs group interference suppression (GIS) filters to suppress the signal components originating from interfering groups. It can only achieve a partial diversity order of \( N \cdot M / G^2 \), where \( N \) and \( M \) are the number of transmit and receive antennas respectively, and a partial throughput of \( G \) symbols per channel use. In addition, the BER performance of such a hybrid system gets worse as \( G \) increases.

Unlike [1], in which the transmitted codewords are based on complex square orthogonal designs, a full-rate differential orthogonal spatial multiplexing (DOSM) based on complex rectangular orthogonal designs is proposed in this paper. We derive an upper bound based on the pair-wise error probability (PEP) for DOSM in Rayleigh fading channels. We compare our DOSM with full-diversity differential STBC [8], the differential BLAST [11], DSM scheme of [13]. Simulation results show that DOSM outperforms full-diversity differential STBC and the existing differential SM systems in terms of bit error rate (BER) performance. Moreover, DOSM has a comparable performance to coherent SM with estimated channel and greatly reduces the
receiver complexity and additional bandwidth requirement for channel estimation.

The rest of the paper is organized as follows. The system model is presented in Section II. The error bounds for DOSM are derived in Section III. Simulation results are given in Section IV, and finally conclusions are drawn in Section V.

II. DIFFERENTIAL ORTHOGONAL SPATIAL MULTIPLEXING

A. Rayleigh Flat-Fading Channel Model

Fig. 1 illustrates the transmitter and receiver structure of the proposed DOSM. We consider a wireless communication link comprising \( N \) transmit antennas and \( M \) receive antennas that operates in a Rayleigh flat-fading environment. At each time slot \( t \), signals \( s_n^t \), \( n = 1, 2, \ldots, N \) are drawn from a certain signal constellation with average energy \( E_s/N_0 \), where \( E_s \) is the total available transmit power independent of \( N \). The commonly defined transmit SNR, or alternatively the average received SNR per receive antenna, is \( E_s/N_0 \). The signal \( r_m^t \) received at time \( t \) at antenna \( m \) is modeled by

\[
r_m^t = \sum_{n=1}^{N} h_{n,m} s_n^t + n_m^t \tag{1}
\]

where the coefficient \( h_{n,m} \) is the path gain from transmit antenna \( n \) to receive antenna \( m \), and the coefficients are modeled as independent complex Gaussian random variables with variance 0.5 per real dimension. The noise variables \( n_m^t \) are independent samples of a zero-mean complex Gaussian random variable with variance \( 1/(2E_s/N_0) \) per dimension. The average signal energy per transmitted symbol is normalized to be 1/N. Hence, the average power of the received signal at each receive antenna is 1.

B. Encoding Algorithm

The differential encoding is done in blocks of \( T \) time slots. We can rewrite (1) in an equivalent matrix form:

\[
R_v = H S_v + N_v \tag{2}
\]

where \( R_v = \{r_m^t\} \) is the \( M \times T \) receive matrix, \( H_v = \{h_{n,m}\} \) is the \( M \times N \) fading matrix, \( S_v = \{s_n^t\} \) is the \( N \times T \) transmit matrix and \( N_v = \{n_n^t\} \) is the \( M \times T \) noise matrix, which are used for the \( v \)th block. The \( n \)th row of the above matrices represents the signals at the \( n \)th antenna, and the \( t \)th column of the matrices denotes the signals at time slot \( t \).

The transmit matrix \( S_v \) is obtained by differential encoding from the previous transmitted code block \( S_{v-1} \) and the \( T \times T \) unitary matrix \( Q_v \):

\[
S_v = S_{v-1} Q_v \tag{3}
\]

where \( Q_v \) is derived from the QR decomposition [14] of the \( T \times T \) lower triangular matrix \( C_v \), carrying information:

\[
C_v = \begin{bmatrix}
c_1 & 0 & \cdots & 0 \\
c_2 & c_{T+1} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
c_T & c_{2T+1} \cdots & c_{(T+1)T/2} & 0
\end{bmatrix} \tag{4}
\]

The QR decomposition can be calculated by means of Gram-Schmidt orthogonalization [14].

At each time slot, \( T(T+1)b/2 \) bits arrive at the encoder and select PSK \( 2^b \) constellation symbols \( c_1, \ldots, c_{T(T+1)/2} \). Since \( T \) time slots are used to transmit \( T(T+1)/2 \) symbols, the rate of the code is \( T(T+1)/2 \). Unlike [1], in which partial-rate DOSM is based on complex square orthogonal designs \( T = N \), full-rate DOSM can be achieved with \( T = 2N - 1 \).

C. Differential Decoding

The received signals for the \( v \)th block can be rewritten as:

\[
\begin{align*}
R_v &= H_v S_v + N_v \\
&= H_v S_{v-1} Q_v + N_v \\
&= (H_v - H_v S_{v-1} Q_v) Q_v + N_v \\
&= (R_v - N_v - N_v - N_v) Q_v + N_v
\end{align*} \tag{5}
\]

Since \( Q_v \) is a unitary matrix \( (Q_v^H Q_v = I) \). The noise variance of \( N_v - N_v - Q_v \) is doubled compared to coherent spatial multiplexing which leads to 3 dB loss in SNR.

The optimum detector recovers the input symbols by minimizing the decision metric over all codewords

\[
\left\| \begin{bmatrix} R_v - R_v - Q_v \end{bmatrix} \right\|^2 \tag{6}
\]

where \( \left\| \right\|^2 \) is the Frobenius-norm of a matrix.

Note that the decision metric in (6) is similar to that of coherent spatial multiplexing. Therefore, some near-optimal detection schemes such as sphere decoding [15] and the Sequential algorithm with Gaussian Approximation (SGA) [16] can be used to reduce computational complexity at the receiver.

The throughput and diversity order of DOSM are shown in Table I, compared with those of differential quasi-orthogonal STBC [10] and the existing differential SM systems. It can be seen that DOSM can achieve full rate and full receive diversity. DQOSTBC can achieve full diversity, however, its throughput is restricted to one data symbol per channel use. Differential BLAST can obtain full transmission rate, but its diversity order is 1. DSM [12, 13] can only achieve partial data rate and partial diversity.

III. PEP BOUND

We derive an upper bound based on the pairwise error probability (PEP) of DOSM. The probability that one given specific codeword \( Q_v \) will be decoded in error as another
particular word \( P_s \) is upper bounded (using the Chernoff bound) by [18]

\[
P(Q_s \rightarrow P_r) \leq \exp \left( -d^2(R_{\text{rel}}) \frac{E_s}{8N_0} \right) \quad (7)
\]

where

\[
d^2(R_{\text{rel}}) = \| R_{\text{rel}}(Q_s - P_r) \|^2 = \| R_{\text{rel}}D_r \|^2. \quad (8)
\]

and \( D_r \) is the difference between the two codewords. The elements of \( R_{\text{rel}} \) are independent complex Gaussian random variables with zero mean and variance \((1/N_0)/2\) per dimension. Note that there is 3 dB loss in SNR due to differential decoding. Since it is known that \( d^2(R_{\text{rel}}) \) is non-negative, we can tighten the bound in (7) to

\[
P(Q_s \rightarrow P_r) \leq \frac{1}{2} \exp \left( -d^2(R_{\text{rel}}) \frac{E_s}{8N_0} \right). \quad (9)
\]

The multiplication of unitary matrix \( S_0 \) to \( H_0 \) does not change the variance of \( H_0 \), and makes the variance of the elements of \( H, S, + N \) unity. The variance of the element of \( N \) is \((E_s/N_0)\), hence the variance of \( H, S, + N \) is \(1+1/(E_s/N_0)\).

Let us define the autocorrelation matrix of \( D_r \) as \( \Phi_{\text{dd}} = D_rD_r^H \). In the case of Rayleigh fading, (9) can be rewritten as [5]:

\[
P(Q_s \rightarrow P_r) \leq \frac{1}{2} \prod_{i=1}^{r} \left( 1 + 1 + \frac{E_s}{N_0} \right)^{-M} \quad (10)
\]

where \( r \) is the rank of \( \Phi_{\text{dd}} \), and \( \lambda_i, i = 1, 2, \cdots, n \), are the eigenvalues of \( \Phi_{\text{dd}} \).

Given the PEP in (10) the word error probability (WEP) can be upper bounded using the union bound [18]:

\[
P_{\text{we}} \leq \sum_{Q_s \in \Xi} \sum_{P_r \in \Xi} P(Q_s \rightarrow P_r) = \frac{1}{RN} \sum_{Q_s \in \Xi} \sum_{P_r \in \Xi} P(Q_s \rightarrow P_r) \quad (11)
\]

where \( \Xi \) is the entire code space consisting of \( RN \) equally likely codewords, and \( P(Q_s) = 1/\Xi \) is the probability of transmitting the codeword \( Q_s \).

Given the number of transmit and receive antennas to be used and the frame error rate (FER) performance requirement, the design criteria for DOSM may be different in the low, medium and high SNR ranges. We assume that \( E_s/(8N_0) \ll 1 \) refers to a low SNR regime, and \( E_s/(8N_0) \gg 1 \) to a high SNR regime. The intermediate regime, \( E_s/(8N_0) = 1 \) covers only a very narrow range of SNR, and will be neglected here. For \( E_s/(8N_0) \ll 1 \), when the term \((1 + \lambda_i (1 + E_s/N_0)/8)^{-M}\) in (10) is expanded, we may neglect terms in \((\lambda_i (1 + E_s/N_0)/8)^2\) and higher powers, so that the upper bound becomes

\[
P(Q_s \rightarrow P_r) \leq \frac{1}{2} \left( 1 + \frac{8}{E_s} \right)^{-M} \quad (12)
\]

From this bound, we can see that in the code design for a low SNR regime, the minimum trace should be maximized. For \( E_s/(8N_0) \gg 1 \), the upper bound becomes

\[
P(Q_s \rightarrow P_r) \leq \frac{1}{2} \left( \prod_{i=1}^{r} \lambda_i \right)^{-M} \left( \frac{E_s}{8N_0} \right)^{-M} \quad (13)
\]

It is obvious that the minimum rank and minimum determinant (the product of the eigenvalues) should be maximized. However, since DOSM aims at maximising transmission rate, it is not usually feasible for \( \Phi_{\text{dd}} \) to achieve full rank for all pairs of codewords. In fact, the minimum rank and minimum determinant of \( \Phi_{\text{dd}} \) are in general 1 and 0, respectively. In this case, there exists only one non-zero eigenvalue \( \lambda_1 \) which is also the trace of \( \Phi_{\text{dd}} \cdot \text{trace}(\Phi_{\text{dd}}) = \lambda_1 \). Equation (13) then becomes

\[
P(Q_s \rightarrow P_r) \leq \frac{1}{2} \left( \frac{E_s}{8N_0} \right)^{-M} \quad (14)
\]

Therefore, for \( E_s/(8N_0) \gg 1 \) the bound in (13) is dominated by the PEP given in (16):

\[
P_{\text{we}} \leq \sum_{Q_s \in \Xi} \sum_{P_r \in \Xi} P(Q_s \rightarrow P_r) \quad (15)
\]

where \( \Theta \) is the code space of the codeword pairs having minimum rank of \( \Phi_{\text{dd}} \).

IV. SIMULATION RESULTS

The performance of the proposed DOSM is studied in comparison with coherent SM, DSTBC with full diversity [8] and other differential SM schemes such as differential BLAST in [11] and DSM in [13].

The transmission starts by sending an identity matrix \( S_0 \) as the reference matrix. DSTBC [8] can achieve full diversity, however, its throughput is limited to 1 and \( \frac{1}{4} \) data symbols per channel use for \( N = M = 2 \) and \( N = M = 3 \), respectively. In order to match the data rate of DOSM and the other differential SM systems which use QPSK constellation, DSTBC must employ significantly expanded constellations (16-QAM for \( N = M = 2 \) and 256-QAM for \( N = M = 3 \)) which greatly degrade the system performance and increase the decoding complexity.
Quasi-static Rayleigh flat fading with uncorrelated fading gains is assumed. For coherent SM we test the BER performance of such a system assuming a perfectly known channel and an estimated channel with least square channel estimation (LS-CE) [17]. 2 and 4 pilot symbols per transmit antenna are used in a frame when $N = M = 2$ and $N = M = 3$, respectively. A pilot symbol has the same energy as that of a data symbol. However, neither the transmitter nor the receiver requires channel knowledge for any of the differential SM systems. For all systems the detector performs exhaustive search for all possible transmitted codewords. The frame length is 256 symbols long.

A. Validation of the Error Bound

Fig. 2 shows the average BER for DOSM, calculated using the union bound (15). The bound is rapidly convergent and is only 2 decibels away from the simulation results. In addition, it is computationally efficient. It only requires the calculation of ranks and eigenvalues of the codeword difference matrices. It is estimated by the bound that DOSM is about 3-4 dB poorer in performance than coherent SM with perfect channel information (PCI) in a 2×2 system at high SNR.

B. Comparison to Other Differential SM Schemes

It can be seen from Fig. 3 that the proposed DOSM scheme outperforms other differential SM systems. Differential BLAST shows a poor BER performance. The non-unitary transmission matrices used in differential BLAST lead to decreasing ranks and determinants for the transmission matrices, and therefore the performance of differential BLAST gets worse as the frame length increases. In [11] short frames (frame length=40) are used, and such a system shows a relatively better performance. The diversity order of DSM [13] is 1 because it applies DPSK modulation individually to each group and employs a set of group interference suppression (GIS) filters to suppress the DPSK signal components originating from interfering groups. No transmit and receive diversity gain is achieved in this case. DOSM has an SNR gain over DSTBC when $BER \geq 10^{-4}$.

Fig. 4 shows the performance of DOSM in a 3×3 system. The performance gain of DOSM over DSTBC and other differential SM systems increases as the number of antennas. Note that DSTBC [8] can achieve full diversity, however, its throughput is limited to one data symbol per channel use over all transmit antennas. In order to match the data rate of DOSM, the scheme in [8] requires large constellation expansion which greatly degrades the system performance and increases the decoding complexity. DOSM has a comparable performance to coherent SM with estimated channel and greatly reduces the receiver complexity and additional bandwidth requirement for channel estimation.

It is obvious from the above figures that $M$-level receive diversity is achieved for DOSM, which confirms the prediction made by the error bound in (14).

V. CONCLUSIONS

A full-rate DOSM based on complex rectangular orthogonal designs is proposed in this paper. The proposed DOSM does not require estimation of channel fading coefficients, channel power, signal power, or noise power to decode the signal and the decision is based on the two consecutively received codewords. An upper bound based on the PEP is derived and used to analyze the system performance.

Simulation results show that DOSM outperforms full-diversity DSTBC and the existing differential SM systems in quasi-static Rayleigh fading channels. It is shown that DOSM is about 3-4 dB poorer in performance than the corresponding coherent SM system with perfect channel information. Moreover, DOSM has a comparable performance to coherent SM with estimated channel and greatly reduces the receiver complexity and additional bandwidth requirement for channel estimation.

REFERENCES


Table 1. Throughput and diversity order of differential MIMO systems

<table>
<thead>
<tr>
<th>Differential MIMO system</th>
<th>Throughput (symbol per channel use)</th>
<th>Diversity order</th>
</tr>
</thead>
<tbody>
<tr>
<td>DOSM</td>
<td>$N$</td>
<td>$M$</td>
</tr>
<tr>
<td>DQOSTBC [10]</td>
<td>1</td>
<td>$N \cdot M$</td>
</tr>
<tr>
<td>DSM [12]</td>
<td>$N(\frac{N-1}{N(2T)})$</td>
<td>Up to $M$</td>
</tr>
<tr>
<td>DSM [13]</td>
<td>$G \cdot (G \leq \min{N,M})$</td>
<td>$N \cdot M / G^2$</td>
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Fig. 1. Block diagram of differential orthogonal spatial multiplexing.

Fig. 2. Error bounds of DOSM for QPSK with $N=2$ and $M=2$ in quasi-static Rayleigh fading channels, compared with simulations.

Fig. 3. Performance of DOSM for QPSK with $N=2$ and $M=2$ in quasi-static Rayleigh fading channels, $R=4$ bps/Hz.

Fig. 4. Performance of DOSM for QPSK with $N=3$ and $M=3$ in quasi-static Rayleigh fading channels, $R=6$ bps/Hz.